LABORATORY MANUAL AND PROBLEMS FOR ELEMENTS OF STATISTICAL METHOD

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Laboratory Manual and Problems for Elements of Statistical Method

BY
ALBERT E. WAUGH
Professor of Economics, University of Connecticut

SECOND EDITION
THIRD IMPRESSION

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LABORATORY MANUAL AND PROBLEMS
FOR ELEMENTS OF STATISTICAL METHOD

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To the Memory of

John Waugh
1687–1781

Alexander Waugh
1729–1810

Dan Waugh
1767–1812

Dan Waugh II
1801–1878

Albert Freeman Waugh
1837–1913

Frank Albert Waugh
1869–1943

PREFACE TO THE SECOND EDITION

This second edition of the Laboratory Manual differs from the first mainly in the addition of new tables. The tables of sums and of sums of squares of natural numbers have been expanded to include the sums of the first six powers of these numbers. A table of natural logarithms, useful in the z-transformation and other recent statistical innovations, has been added. Several tables for use in connection with the application of the chi-square test are now included, as are tables of the F- and t-functions. A short table of natural trigonometric functions, a table of Julian days, a table of logarithmic factorials, a table of common statistical constants, and a table of random numbers complete the additions to the tables, which should now be sufficient for most practical laboratory work. The effort has been made to continue, as far as possible, the plan of the first edition, in which all parts of the same table appear on facing pages.

I am deeply indebted to those authors and publishers who have permitted me to use here tables that have appeared elsewhere. I am indebted to Prof. R. A. Fisher and Dr. F. Yates, also to Messrs. Oliver & Boyd, Ltd., of Edinburgh, for permission to reprint Tables A24 and A26 from their book "Statistical Tables for Biological, Agricultural and Medical Research." Professor George W. Snedecor and the Collegiate Press have kindly consented to the reprinting of Table A25 from their "Statistical Methods," 3d ed. Table A23 is Elderton's table, reprinted here with permission of the *Biometrika* office.

A few new problems have been added at the end of almost every section. Some of them illustrate methods not heretofore covered in the manual. Almost all of them are actual data from recent statistical studies, and, as before, the attempt has been made to include data from many and varied fields. Almost every student and teacher should find here problems from familiar fields.

ALBERT E. WAUGH.

UNIVERSITY OF CONNECTICUT,
STORRS, CONN.,
December, 1943.

PREFACE TO THE FIRST EDITION

No one ever becomes a good statistician until he has developed a real feeling for and appreciation of statistical data. Such feeling and appreciation cannot be developed by reading about statistical problems in the abstract; it is necessary to work with actual data until acquaintance has deepened into intimacy and recognition has become insight. It is for this reason that laboratory work is an indispensable part of any beginning course in statistical method. Every important concept touched on in the classroom should be illustrated by one or more numerical problems in the laboratory until the student can visualize the relationships between the characteristics of the raw data and the nature of the final results.

The collection of laboratory material offered here has the merit that the data are in most cases those of actual problems. The cases have been carefully selected with the aim of minimizing the drudgery of arithmetical computation without oversimplifying the problems beyond all semblance to reality. The author has found that students understand methods much more easily and completely when they are applied to real rather than to hypothetical problems. In such cases it is also true that the interpretation of results seems more natural and is more quickly grasped.

Many of the difficulties faced by the beginning student in statistics, however, are not numerical. For most such students the concepts dealt with are entirely new. The student has to learn, not only what to do, but why to do it and how to interpret the results. For this reason a considerable number of discussion or thought questions have been included in this manual. Many of these will require no arithmetical work at all, but they will force the student to think in terms of the concepts and the relationships involved in statistical theory and methodology.

An effort has been made to include enough examples of each type of problem to furnish the teacher with illustrative lecture material and with several alternative sets of laboratory problems.

In this way the problems can be adapted to the interests of the class, the student can understand easily the tremendous range of problems to which statistical methods are applicable, and the teacher may vary the problems from section to section or from semester to semester for pedagogical purposes. The problems are so keyed that they may easily be referred to by letter and number.

The wise statistician soon learns that there are various methods by which he can reduce the amount of purely arithmetical work which it is necessary for him to do. Among the timesavers are computing machines, slide rules, nomographs, and books of tables. In the earlier sections of this manual are given a number of the most useful statistical tables and formulas which are of help not alone to embryo statisticians but to everyone who carries out statistical computations. It is, of course, impossible to make the list of formulas complete, but an attempt has been made to include those which are most often needed. Each formula is accompanied by a page number which refers to the place in the author's "Elements of Statistical Method" where the use of the formula is explained.

Among the most useful timesavers are various statistical tables. Included here are those tables which are most often needed and which make the greatest savings in the time of computation. The author has computed several of the tables especially for this manual. Tables A3 and A4, condensed from Salvosa's tables, appear here, so far as he knows, for the first time in book form. Table A11 appears here, he believes, for the first time in print. Table A21* is much more complete than other tables of the Poisson proportions which have appeared. The uses of the tables, with short illustrative examples, are explained in a short section that immediately precedes them.

This manual should prove useful in any statistics laboratory, whether used in conjunction with a course in statistics or not. The problems are obviously designed for the usual course in elementary statistics, and can be used in any such course regardless of the text that is adopted. The manual follows the order of the author's "Elements of Statistical Method," and references to the "text" are intended to indicate that book. In selecting material for the problems great care has been used to draw from

*This became Table A22 of the present edition.

sources of such a diverse nature that almost any statistics course, specialized or general, will find appropriate illustrative material here.

References to the sources of materials used in this manual are made in footnotes at the proper points. The author is greatly indebted to the authors and publishers mentioned for permission to use these materials. He is also under great obligation to his colleagues and to his former students who have called to his attention many of the interesting problems included here.

The author will appreciate the comments of his fellows, either in the classroom or in the research laboratory, with regard to the arrangement and treatment of these materials and their usefulness.

ALBERT E. WAUGH.

CONNECTICUT STATE COLLEGE,
STORRS, CONNECTICUT,
July, 1938.

CONTENTS

SECTION A

STATISTICAL TABLES

In using a statistical or mathematical table we ordinarily start with a known value of one variable and wish to find the unknown corresponding value of another variable. For example, we wish to find the logarithm of 17. The known number is 17, and the unknown corresponding value is the logarithm of 17, or 1.23045. Technically we call the known value with which one enters a table the *argument*, and the unknown corresponding value for which one is searching is called the *entry*. These terms will be used over and over again in describing the following tables.

Experience shows that the student or research man working on a statistical problem usually needs many values of a given kind on any given problem. For example, he may have to look up the squares of 40 numbers, or their reciprocals. For this reason this manual has been arranged as far as possible so that all reciprocals can be found on two facing pages, all constants for the unit normal curve on two facing pages, and so on. It is hoped that this concentration of material will prove more useful than the usual table in which squares, square roots, cubes, cube roots, etc., are all entered together.

Most of the tables given here carry the entries to but four or five significant figures. In practice the statistician seldom needs more, for the inaccuracies of his original data usually make it impossible for him to put more accurate tables to good use. Naturally the shorter tables are easier and faster to use when extreme accuracy is unnecessary.

The following short explanations are intended to point out the commonest uses of the tables, but familiarity with statistical procedures will bring to light many uses in addition to those listed.

Table A1. Areas under the Unit Normal Curve. This table shows the proportion (percentage) of the total area under a normal curve which lies between a perpendicular erected at the

mean and a perpendicular erected at various distances from the mean. The argument is the distance from the mean (measured in units of the standard deviation). The entry is the percentage of all the cases in the normal distribution lying between the mean and the given number of standard deviations above (or below) the mean. *Example:* Smith's height is 5 ft. 7 in. The average height for men of his weight is 5 ft. 5 in., and the standard deviation is 1.5 in. What proportion of the men exceed Smith's height? *Ans.:* Smith is 2 in. above the mean in height. This is 2/1.5 or 1.33 standard deviations above the mean height. In a normal distribution 50 per cent of the cases lie below the mean. The table shows that another 40.82 per cent lie above the mean but within 1.33 standard deviation from the mean. We thus account for 90.82 per cent, and the other 9.18 per cent exceed Smith in height.

Table A2. Ordinates of the Unit Normal Curve. This table shows the height of a normal curve at various distances from the mean, the distances being measured in units of the standard deviation. The figures in this table must be multiplied by a constant to find the expected frequencies for any particular problem. The constant for any given problem is found by multiplying the class interval by the number of cases, and dividing the product so obtained by the standard deviation. *Example:* A given frequency table lists 763 cases in classes with a class interval of 8. The arithmetic mean is 97.5 and the standard deviation is 14.4. How many cases would be expected in the class which has its mid-point at 94.5? (Since the class interval is 8 this class would include all cases with values from 91 to 98.) *Ans.:* The constant for our problem is $763(8)/14.4 = 423.8$. The class mark differs from the mean by 3.0, or by $3.0/14.4 = 0.21$ standard deviations. Using 0.21 as the argument we find the entry .3902. This we multiply by our constant to get $(423.8)(.3902) = 165.3$. We would expect to find 165.3 cases in the class from 91 to 98.

Table A3. Areas under Skewed Curves. This table shows the proportions of the areas under skewed curves lying below various numbers of standard deviations from the arithmetic mean. The amount of skewness is measured by α_3 and appears at the top of the table. *Example:* Alice is 5 ft. 7 in.

tall when the average height of women is 5 ft. 5 in., the standard deviation is 2.8 in., and the skewness is +0.7. What proportion of the women exceed Alice in height? *Ans.:* Alice's height differs from the mean by +2 in., or by $+\frac{2}{2.8} = +0.71\sigma$. Opposite the argument +0.71 we find the entry .7803 (interpolating in the column under a skewness of 0.7). We conclude that Alice exceeds 78 per cent of the women and that the other 22 per cent exceed Alice.

Had the skewness been −0.7 instead of +0.7, we should have been forced to reverse the table and find what proportion of the women were farther from the mean than -0.71σ (instead of $+0.71\sigma$). In this case the answer would have been that 24.12 per cent of the women exceeded Alice in height.

Table A4. Ordinates of Skewed Curves. This table is similar to Table A2 but gives data for skewed curves with the amounts of skewness (measured in terms of α_3) given at the tops of the columns. Using the problem with which we illustrated Table A2, and assuming a skewness of 0.5, we proceed as follows: Class mark is at -0.21σ. Using as our argument −0.21, we find the entry .4093 (interpolating between the entries at −0.20 and −0.25). Multiplying this entry by the constant for this problem, we get (423.8)(.4093) = 173.5. Hence we expect to get 173.5 cases in this class. Had the skewness been negative, we should have used as our argument +0.21, and we should have found the entry .3690. The computed frequency would have been (423.8)(.3690) = 156.4.

Table A5. Squares of Numbers. With any number N as argument, find its square N^2 as entry. *Example:* What is the square of 256? *Ans.:* Find the argument 256. The entry is 65,536, which is the required square. Moving the decimal point *one* place in either direction in the argument moves it *two* places in the same direction in the entry. *Example:* What is the square of 25.6? *Ans.:* 655.36.

Table A6. Square roots of Numbers from 10 to 100. Take any number between 10 and 100 as the argument, and find the square root of the number as the entry. *Example:* What is the square root of 35.4? Take 35.4 as the argument, and find as the entry 5.950. This is the required square root correct to four

significant figures. Moving the decimal point *two* places in the argument moves the decimal point *one* place in the same direction in the entry. *Example:* What is the square root of .354? *Ans.:* .5950.

Table A7. Square Roots of Numbers from 100 to 1000. Take any number between 100 and 1000 as the argument, and find its square root as the entry. *Example:* What is the square root of 473? Take 473 as the argument, and find the entry 21.75. This is the required square root. As in the preceding table, moving the decimal point *two* places in the argument moves it *one* place in the same direction in the entry. *Example:* The square root of 4.73 is 2.175.

Table A8. Reciprocals of Numbers. Take any number N as the argument, and its reciprocal $1/N$ is the entry. *Example:* what is the reciprocal of 2.78? *Ans.:* Taking 2.78 as the argument, we find .3597 as the entry. Therefore the reciprocal of 2.78 is .3597. Moving the decimal point one place in the argument moves it one place *in the opposite direction* in the entry. *Example:* The reciprocal of 27.8 is .03597. The reciprocal of 0.278 is 3.597.

Table A9. Conversion from Probable Error to Standard Error. If we know the probable error of a distribution and want the standard error of the same distribution, we take the known probable error as argument, and find the standard error as entry. *Example:* The probable error of a mean is 3.74 lb. What is the standard error of the mean? *Ans.:* Take 3.74 as the argument, and find the entry 5.545. Therefore the standard error is 5.545 lb. Moving the decimal point *one* place in the argument moves it *one* place in the same direction in the entry. *Example:* The probable error of a coefficient of correlation is .144. What is the standard error? Take the argument .144, and find the entry .2135.

Table A10. Conversion of Standard Error to Probable Error. If we know the standard error of a distribution and want the probable error of the same distribution, we take the known standard error as the argument, and find the probable error as the entry. *Example:* The standard error of a difference is \$4.65. What is the probable error of the difference? *Ans.:* Take as the argument 4.65, and find the entry 3.136. The

required probable error is \$3.136. Moving the decimal point *one* place in the argument moves it *one* place in the same direction in the entry. *Example:* The standard error of a coefficient of correlation is .104. What is the probable error? *Ans.:* Taking .104 as the argument, we find .07015 as the required probable error.

Table A11. Values of z Corresponding to Given Values of r. To find the value of z which corresponds to any given value of r, take the value of r as the argument, and find the required value of z as the entry. *Example:* What value of z corresponds to $r = .784$? *Ans.:* Take .784 as the argument, and find as the entry that z is 1.0557.

Table A12. Values of r Corresponding to Given Values of z. To find the value of r which corresponds to any given value of z, take the value of z as the argument, and find the required value of r as the entry. *Example:* What value of r corresponds to $z = 1.16$? *Ans.:* Take 1.16 as the argument, and find as the entry that r is .8210.

Table A13. Values of $\sqrt{pq}$ When $p + q = 1$. This table is useful in probability problems in solving the formulas

$$\sigma = \sqrt{n}(\sqrt{pq})$$

$$\sigma_{\%} = \frac{\sqrt{n}}{\sqrt{pq}}$$

The value of p (or q) is the argument, and the value of $\sqrt{pq}$ is the entry. *Example:* If $p = 0.484$, what is the value of $\sqrt{pq}$? *Ans.:* Take 0.484 as the argument, and find the required value of $\sqrt{pq}$ as .4997 as the entry.

Table A14. Four-place Common Logarithms. For rough work this table is far faster than Table A15, but somewhat less accurate. The number is the argument, and its logarithm is the entry. *Example:* What is the logarithm of 3.14? *Ans.:* Take 3.14 as the argument, and find its logarithm, .4969 as the entry. Students unfamiliar with logarithms should consult an elementary textbook on algebra, referring especially to the method of determining the characteristics of logarithms.

Table A15. Five-place Common Logarithms. This is a more extensive table of the same type of information as that given in Table A14. *Example:* What is the logarithm of 473.8? *Ans.:*

Take 4738 as the argument, and find the mantissa of the logarithm 0.67560 as the entry. The characteristic is always a number smaller by unity than the number of digits to the left of the decimal point in the original number. Since our original number 473.8 has three digits to the left of the decimal point, the characteristic is 2, and the entire logarithm is 2.67560.

Table A16. Natural or Napierian Logarithms. For much work in statistical theory it is convenient to use logarithms based on the constant e, which is approximately equal to 2.71828, rather than on the value 10, which is the base of common logarithms. In this table we have various numbers as arguments, and their natural logarithms as the corresponding entries. *Example:* What is the natural logarithm of 3.14? *Ans.:* Take 3.14 for the argument. This will be found at the right of 3.1 in the left margin and beneath the number 4 at the head of a column. The corresponding entry is 1.1442. This is the natural logarithm of 3.14. Only the latter part (.1442) appears in the table, but inspection of the table will disclose that the number 1 is to be added at the beginning of all entries throughout this part of the table. Similarly we find that the natural logarithm of 9.42 is 2.2428.

To find the logarithms of numbers not given as arguments, we make use of the following rule:

$$\log_e (10n) = \log_e 10 + \log_e n$$
$$\log_e (100n) = \log_e 100 + \log_e n$$
$$\text{etc.}$$

Example: What is the natural logarithm of 314? *Ans.:* Find as before that $\log_e 3.14$ is 1.1442. Note at the end of the table, on page 65, that $\log_e 100$ is 4.6052. Hence $\log_e 314$ is $1.1442 + 4.6052 = 5.7494$.

Table A17. The Sums of the First Six Powers of the Natural Numbers from 1 to n. This table is especially useful in fitting various trends by the methods of least squares. In the column headed $\Sigma(n)$ are the sums of the numbers 1, 2, 3, . . . n. For example, opposite the number 12 in the first column, we find the entry 78 in the second column. This tells us that the sum of the 12 numbers, 1, 2, 3, 4, . . . 10, 11, 12 is 78. The sum of the squares of these 12 numbers is given in the next column, headed $\Sigma(n^2)$, as 650. The sum of the fifth powers of the numbers from 1 to 6 inclusive (namely, the sum of $1^5 + 2^5 + 3^5 + 4^5 + 5^5 + 6^5$)

is found, in the column headed $\Sigma(n^5)$ and opposite the number 6 at the left, to be 12,201. As one further example, we desire the sum of the cubes of the numbers from 1 to 21, inclusive. *Ans.:* Opposite the argument 21 and under the column heading $\Sigma(n^3)$ find the answer, 53,361.

Table A18. The Sums of the First Six Powers of the Natural Odd Numbers from 1 to o. In this table small o is taken to represent the particular odd number in question. When the number of periods studied is even and one is fitting a trend by the method of least squares, it is often advantageous to take the origin at the center and number the periods with the successive odd numbers. In this table we find the sum of the odd numbers, the sum of their squares, and the sums of their other powers up to the sixth. The sum of the odd numbers themselves is given in the column headed $\Sigma(o)$. For example, what is the sum of the odd numbers from 1 to 9 inclusive? We look under the column heading $\Sigma(o)$ and opposite the argument 9 to find the answer, 25. The sum of the squares of the odd numbers from 1 to 19 is found, in the column headed $\Sigma(o^2)$ and opposite the argument 19, to be 1,330. The sum of the fifth powers of the odd numbers from 1 to 21 (that is, the sum of the numbers $1^5 + 3^5 + 5^5 + 7^5 + \cdots + 17^5 + 19^5 + 21^5$) is found, under the column heading $\Sigma(o^5)$ and opposite the argument 21, to be 9,351,001.

Table A19. Percentage Which the Standard Error of Estimate Is of the Standard Deviation of the Dependent Variable. Given the value of the coefficient of correlation r as the argument, we find the required percentage as the entry. *Example:* If the relationship between height and weight in a group of school boys is found to be such that $r = 0.762$, how large will the standard error of estimating the boys' heights be as compared with the standard deviation of their heights? *Ans.:* Take 0.762 for the argument, and find as the corresponding entry the figure 64.8. Therefore the standard error of estimating the boys' heights will be 64.8 per cent of the standard deviation of their heights. Obviously we have 64.8 per cent as much error in estimates from the regression equation as we would have if we guessed at the heights. Correlation procedures have reduced our error by 35.2 per cent.

Table A20. Chances of Differing from the Mean by Given Numbers of Standard Deviations. The argument is the distance

from the mean, measured in units of the standard deviation. The entry is the probability that an item picked entirely at random from a normal distribution will lie as far from the mean as the stated number of standard deviations (or farther). *Example:* What are the chances that an item picked at random from a normal distribution will lie 2.2σ from the arithmetic mean, or more? *Ans.:* Take 2.2 as the argument, and find as the corresponding entry the number 0.0277. Therefore the probability of picking an item this far from the mean is 0.0277, or 277 chances in ten thousand.

Table A21. Chances of Differing from the Mean in a Given Direction by More Than Given Numbers of Standard Deviations. The argument is the distance from the mean, measured in units of the standard deviation. The entry is the probability that an item chosen entirely at random from a normal distribution will lie on a given side of the mean and as far from the mean as the stated number of standard deviations. *Example:* What are the chances that an item chosen at random from a normal distribution will lie 2.2σ or more *above* the arithmetic mean? *Ans.:* Take as the argument the number 2.2. Find the corresponding entry, 0.0139. Therefore the probability of picking an item by chance this far *above* the mean (or this far *below* the mean) is 0.0139, or 139 chances out of ten thousand.

Table A22. Proportion of Cases in Class 0 of Poisson Distribution. In fitting a Poisson curve to a distribution, the frequency classes are numbered 0, 1, 2, 3, etc. The average of the distribution is computed on the basis of these class values. This table shows, for distributions with various averages, the percentage of the cases which fall in the first class, called Class 0. *Example:* In a Poisson distribution with an average of 0.48, we shall find 61.88 per cent of the cases in Class 0. In a distribution with an average of 10.3, we shall find .00336 per cent of the cases in Class 0.

The proportion of cases in each of the other classes in such a distribution can be easily computed from the proportion in Class 0. To find the proportion in Class 1, multiply the proportion in Class 0 by the mean. To find the proportion in Class 2, multiply the proportion in Class 1 by half the mean. To find the proportion in Class 3, multiply the proportion in Class 2 by one-third of the mean. In general, the proportion in Class n is the proportion in the preceding class (Class $n - 1$) multiplied by the

quotient obtained when the mean is divided by n; that is,

$$p_1 = p_0(A); \qquad p_2 = p_1\left(\frac{A}{2}\right); \qquad p_3 = p_2\left(\frac{A}{3}\right)$$

$$p_n = p_{n-1}\left(\frac{A}{n}\right)$$

Table A23. Table of P for the Chi-square Test of Goodness of Fit. In testing various statistical hypotheses it is often desirable to compute the results that would be found if one's theory were correct and to compare these computed theoretical results with the actual ones. In such cases one does not expect to get complete and precise agreement between the actual and the expected results, but from the differences between the actual and the expected results he can compute a value known as χ^2. Using this value as the argument, along with the number of "degrees of freedom" (the latter being roughly an adjustment to make allowance for the fact that computed results may be "forced," by the mathematical processes followed, to agree to some extent with the actual data), we find as the entry the probability that one would get by pure chance an agreement as poor as or poorer than the one studied. For example, if we compute chi square in a given problem, and find it to be 11 when the number of degrees of freedom is 15, we look in this table opposite the argument 11 in the left-hand column and in the column headed $n = 15$. Here we discover that P is .752594. If we round this off to 0.75, we can say that by pure chance about 75 per cent of the time one would get less agreement than there was in the case studied. In other words, the hypothesis is a tenable one, although by no means proved. The chi-square test never proves that a hypothesis is either correct or incorrect. The results are always in probability terms. Very small values of P mean that one would be unlikely by chance to get as much disagreement between actual and computed data as he had obtained in the problem studied. Very large values of P mean that the agreement between the theoretical and actual data is very good—too good, perhaps, to be true. Thus one might question the data if he is told that six coins thrown over and over always yield exactly three heads and three tails. Such agreement between theory and practice is too good to be true. Values of P in the middle ground (neither very high nor very low) lead one to conclude that his hypothesis is reasonable, although there

is no indication that the hypothesis is necessarily correct. To take one more example from the table, if we find a value of 20 for χ^2 when there are 4 degrees of freedom, P is .000499. One would get such great disagreement by chance only about five times in ten thousand, and so it seems likely that the disagreement is the result of some fault in the hypothesis rather than the result of chance.

Table A24. Table of Chi Square. This table is used much like Table A23. Given values of chi square and the number of degrees of freedom, one looks for his value of P as follows: Take the number of degrees of freedom as the argument n in the first column. Run across the corresponding row of the table toward the right looking for the value of chi square. Probably it will lie between two entries in the table. In that case, P lies between the two column headings involved. *Example:* With 16 degrees of freedom, chi square is 8.342. What is the value of P? *Ans.:* Starting with number 16 in the first column, read across to find that 8.342 lies between the entries 7.962 and 9.312. Hence P is between 0.95 and 0.90. This would be interpreted as indicating close agreement between the hypothesis and the data. For fuller interpretation, see the explanation of Table A23 above.

Table A25. Table of F. This table is used in interpreting the analysis of variance. Given two variances with their degrees of freedom, find the number of degrees of freedom attached to the larger variance at the top of the table. Find the number of degrees of freedom attached to the smaller variance at the left. Where the column and the row cross will be found the 5 per cent point and the 1 per cent point of F. F is the quotient found by dividing the larger variance by the smaller. For example, if we have a variance of 68.2 with 6 degrees of freedom, and a second variance of 14.8 with 8 degrees of freedom, we enter the table under $n_1 = 6$ and at the right of $n_2 = 8$. In this case, F is $68.2/14.8 = 4.6$. We discover that the 5 per cent point is 3.58 and the 1 per cent point is 6.37. Since our value, 4.6, lies between them, its level of significance is between 0.05 and 0.01.

Table A26. Fisher's Table of the Distribution of t for Certain Probability Levels. This table is used with small samples when deviations are judged in terms of the standard deviation of the sample rather than the standard deviation of the universe. The table gives values of t as entries. Starting with a known number of degrees of freedom at the left-hand column, we find at the

right the value of t for our problem. We then discover from the column headings the probability of obtaining by chance alone a value of t as large as the one in question. For example, a difference divided by its standard error yields a quotient of 0.952. The number of degrees of freedom is 20. In this case t is 0.952 and n is 20. We enter the table opposite the value 20 in the left-hand column. We do not find the value $t = 0.952$ anywhere, but we know that it lies between the entries $t = 0.860$ and $t = 1.064$. Hence we say that the probability of our getting a difference as large as this one by pure chance, from data essentially similar, is between 0.4 and 0.3. As P gets smaller and smaller, it becomes more and more likely that the difference (or other measure) is "significant" in the statistical sense.

Table A27. Natural Trigonometric Functions. This table gives the values of the sine, cosine, tangent, and cotangent of angles of whole numbers of degrees and half degrees in the first quadrant. When reading degrees from the left, use the column headings at the top; when reading degrees from the right, use the column footings at the bottom. *Examples:* $\sin 17° = .292$; $\tan 26°30' = .499$; $\cos 70° = .342$. For other functions: $\operatorname{cosec} x = 1/\sin x$; $\sec x = 1/\cos x$; $\operatorname{vers} x = 1 - \cos x$; $\operatorname{covers} x = 1 - \sin x$. If, in a right triangle, C is the right angle, A and B are the other two angles, and sides a, b, and c lie opposite angles A, B, and C respectively, then $\sin A = a/c$, $\cos A = b/c$, $\tan A = a/b$, $\cot A = b/a$, $\sec A = c/b$, and $\operatorname{cosec} A = a/c$. Sin and cosec are positive in the first and second quadrants and negative elsewhere. Cos and sec are positive in the first and fourth quadrants and negative elsewhere. Tan and cot are positive in the first and third quadrants and negative elsewhere. In the second quadrant the functions are found thus:

$$\sin (90° + A) = \cos A \qquad \tan (90° + A) = -\cot A$$
$$\cos (90° + A) = -\sin A \qquad \cot (90° + A) = -\tan A$$

In the third quadrant:

$$\sin (180° + A) = -\sin A \qquad \tan (180° + A) = \tan A$$
$$\cos (180° + A) = -\cos A \qquad \cot (180° + A) = \cot A$$

In the fourth quadrant:

$$\sin (270° + A) = -\cos A \qquad \tan (270° + A) = -\cot A$$
$$\cos (270° + A) = \sin A \qquad \cot (270° + A) = -\tan A$$

These equations would be used as follows:

Wanted: tan 197°

$$197^\circ = 180^\circ + 17^\circ$$
$$\tan\,(180^\circ + 17^\circ) = \tan 17^\circ = .306$$

Wanted: sin 300°

$$300^\circ = 270^\circ + 30^\circ$$
$$\sin\,(270^\circ + 30^\circ) = -\cos 30^\circ = -.866.$$

Table A28. Perpetual Calendar. In problems of business statistics it is often necessary to find the number of Sundays or the number of Saturdays in a given month. This table makes it possible to find the calendar for any month from 1700–2099. *Example:* How many Sundays were there in the month of April, 1929? *Ans.:* Under the general heading 1900 find the figure 29. This represents the year 1929. Move across the row containing the number 29 until, at the right, the number 2 is found under April. This indicates that calendar 2 is the correct one. Consulting calendar 2, we find that April 1929 began on Monday and contained 4 Sundays and 4 Saturdays. Note that each of the calendars contains 31 days. In practice the student will have to realize that the month stopped earlier than the calendar given when the month contained 28, 29, or 30 days.

Example: How many Saturdays were there in June, 1933? Under June and opposite 1933 we find the number 5. The fifth calendar shows 5 Saturdays, but the last one is the 31st. Since June contains but 30 days there were but 4 Saturdays in June, 1933.

The calendar can be extended indefinitely. The twenty-second century will be exactly like the eighteenth century in the Gregorian calendar; that is, the year 2144 will be exactly like the year 1744, etc. Similarly the twenty-third century will duplicate the nineteenth century, the twenty-fourth century will duplicate the twentieth century, etc.

Table A29. The Julian Day. In working with data that are given on a daily basis it is often necessary to find in days the time that elapsed between given dates. This is especially true when dealing with cyclical data. Such computation can be carried out directly by adding up the years, allowing for leap years, and adding in the odd months and days. The work is much simplified, however, by the use of a table of Julian days.

Each day is numbered, and by finding the difference between their numbers one can ascertain at once the time elapsed. *Example:* How many days elapsed from July 8, 1869, to Mar. 20, 1943? *Ans.:* The Julian day corresponding to each date is found first, as follows:

1869	2403698	1943	2430725
July	181	March	59
Day	8	Day	20
Julian day	2403887		2430804

Subtracting the former day from the latter, we find:

	2430804
	2403887
Days elapsed	26917

For leap year, use the month numbers in the second column at the lower right.

Table A30. Commonly Used Constants. Many statistical formulas include among their terms letters that represent definite numbers. Most common are various multiples of π, the ratio of the circumference of a circle to its diameter, and of e, which is defined as

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

This latter number is very useful in the calculus and is the base of the natural or Napierian logarithms. When one is manipulating the statistical formulas themselves, it is commonly more convenient to use the letters than to use the corresponding irrational numbers; but, when one occasionally wishes to solve one of the formulas, it becomes necessary to use the numbers themselves. For this reason the values of the more common constants and of their more commonly used multiples are given in this table with the corresponding logarithms.

Table A31. The Greek Alphabet. Many statistical formulas are written in Greek letters. The alphabet is given on page 84 with the English names of the Greek letters.

Table A32. Logarithmic Factorials. Factorial n (usually written $n!$) indicates the product obtained by multiplying together the successive natural integers from 1 to n; thus, $4!$ means $1 \times 2 \times 3 \times 4 = 24$. As n increases, $n!$ will very

quickly become unwieldy in size, but its logarithm may be used to find its approximate size and the first few of its significant figures. For example, what is the value of 48!? This indicates the product of the numbers $1 \times 2 \times 3 \times \cdots 46 \times 47 \times 48$. Looking in Table A32, we find the logarithm of 48! to be 61.09391. From Table A15, on page 46, we discover that the first four digits of the antilogarithm will be 1241. The characteristic of the logarithm, 61, tells us that the number has 62 digits to the left of the decimal point. Hence the number is approximately 1.241×10^{61}. This is our answer with four significant figures. Interpolation in the logarithmic table might yield another significant figure.

Table A33. Short Table of Random Sampling Numbers. There are several methods for increasing the likelihood that the selection of a sample is random, but by far the easiest is the use of one of the tables of random sampling numbers. For any extended sampling problem one should use one of the large published tables,[1] but a small table is given here for practice. Two examples of the use of the table follow: First, to select at random 10 items from the 170 numbers of Prob. F14, we number the items from 1 to 170, starting at the upper left, running across the top row, and then taking the second and third rows in order, always reading from left to right. Thus the tenth item is 48 and the eleventh item is 49. In order to select 170 items, we shall need random numbers of but three digits. The three-digit numbers run from 000 to 999, making 1000 numbers. If we let each of our positions in the original problem be represented by 5 random numbers, we shall utilize 5×170, or 850, numbers. We can start anywhere in Table A33. Suppose we start at the top of the second column and read down, using just the first three digits of each number. We get the numbers 752, 529, 256, etc. Since we are assigning 5 numbers to each position in the table, we can divide these random numbers by 5, discarding any remainder no matter how large. This will give us the positions in Prob. F14. Dividing 752, 529, 256, etc., by 5 and discarding

[1] See, for example, L. H. C. Tippett, Random Sampling Numbers No. XV, *Tracts for Computers*, Cambridge University Press, London, 1927; M. G. Kendall and B. B. Smith, Tables of Random Sampling Numbers No. XXIV, *Tracts for Computers*, Cambridge University Press, London, 1939; R. A. Fisher and F. Yates, "Statistical Tables for Biological, Agricultural and Medical Research," pp. 82–87, Oliver & Boyd, Edinburgh, 1938.

remainders, we get the ten quotients: 150, 105, 051, 082, 016, 114, 112, 137, 011, and 088. These tell us to select from Prob. F14 the 150th item, the 105th item, the 51st item, etc. Our ten values, selected at random, are 51, 50, 51, 55, 53, 48, 47, 51, 49, and 54. For our second illustration we select at random 12 items from the table of barometric heights in Prob. F13*u*. There are 2922 items in the table, and so we shall have to use four-digit random numbers. But since there are 10,000 of these numbers (counting both 0000 and 9999 and all between), we can assign three random numbers to each item in the table. The single item in the first class will thus be assigned the three random numbers 0, 1, and 2. The two items of the next class will receive the six succeeding random numbers, 3, 4, 5, 6, 7, and 8. The eight items of the next class get 24 random numbers, etc. We thus find that the random numbers assigned to each class are as follows:

Height of Barometer	Number of Cases	Random Numbers	Height of Barometer	Number of Cases	Random Numbers
28.35	1	0–2	29.75	509	1947–3473
28.55	2	3–8	29.95	656	3474–5441
28.75	8	9–32	30.15	580	5442–7181
28.95	30	33–122	30.35	353	7182–8240
29.15	74	123–344	30.55	140	8241–8660
29.35	166	345–842	30.75	30	8661–8750
29.55	368	843–1946	30.95	5	8751–8765

Now we take random numbers from Table A33 in any prearranged order. For example, if we start at the bottom of the second column and read up, we get the random numbers (see page 86) 9130, 7497, 7150, 1667, 3087, 3340, 2217, 2129, 2963, 3469, 4750, and 4972. Looking at our table, we see that the first value, 9130, does not appear at all; so we skip it. The next value, 7497, corresponds to a barometric height of 30.35. Similarly we get the other barometric values, until our sample of 12 items is 30.35, 30.15, 29.55, 29.75, 29.75, 29.75, 29.75, 29.75, 29.75, 29.95, 29.95, and 30.55. The last of these values corresponds to the random number 8311, which is a 13th random number that we had to use because we skipped the first one. Thus we have 12 items selected from the table. We can use the table of random numbers for selection either from frequency tables or from ungrouped items merely by assigning numbers to the groups or to the individual items in advance according to some plan and then by using random numbers to determine which of the items or groups will be chosen.

Table A1. Areas under the Normal Curve

Fractional parts of the total area (1.000) under the normal curve between the mean and a perpendicular erected at various numbers of standard deviations (x/σ) from the mean.[1] To illustrate the use of the table, 39.065 per cent of the total area under the curve will lie between the mean and a perpendicular erected at a distance of 1.23σ from the mean.

Each figure in the body of the table is preceded by a decimal point.

x/σ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	00000	00399	00798	01197	01595	01994	02392	02790	03188	03586
0.1	03983	04380	04776	05172	05567	05962	06356	06749	07142	07535
0.2	07926	08317	08706	09095	09483	09871	10257	10642	11026	11409
0.3	11791	12172	12552	12930	13307	13683	14058	14431	14803	15173
0.4	15554	15910	16276	16640	17003	17364	17724	18082	18439	18793
0.5	19146	19497	19847	20194	20450	20884	21226	21566	21904	22240
0.6	22575	22907	23237	23565	23891	24215	24537	24857	25175	25490
0.7	25804	26115	26424	26730	27035	27337	27637	27935	28230	28524
0.8	28814	29103	29389	29673	29955	30234	30511	30785	31057	31327
0.9	31594	31859	32121	32381	32639	32894	33147	33398	33646	33891
1.0	34134	34375	34614	34850	35083	35313	35543	35769	35993	36214
1.1	36433	36650	36864	37076	37286	37493	37698	37900	38100	38298
1.2	38493	38686	38877	39065	39251	39435	39617	39796	39973	40147
1.3	40320	40490	40658	40824	40988	41149	41308	41466	41621	41774
1.4	41924	42073	42220	42364	42507	42647	42786	42922	43056	43189
1.5	43319	43448	43574	43699	43822	43943	44062	44179	44295	44408
1.6	44520	44630	44738	44845	44950	45053	45154	45254	45352	45449
1.7	45543	45637	45728	45818	45907	45994	46080	46164	46246	46327
1.8	46407	46485	46562	46638	46712	46784	46856	46926	46995	47062
1.9	47128	47193	47257	47320	47381	47441	47500	47558	47615	47670
2.0	47725	47778	47831	47882	47932	47982	48030	48077	48124	48169
2.1	48214	48257	48300	48341	48382	48422	48461	48500	48537	48574
2.2	48610	48645	48679	48713	48745	48778	48809	48840	48870	48899
2.3	48928	48956	48983	49010	49036	49061	49086	49111	49134	49158
2.4	49180	49202	49224	49245	49266	49286	49305	49324	49343	49361
2.5	49379	49396	49413	49430	49446	49461	49477	49492	49506	49520
2.6	49534	49547	49560	49573	49585	49598	49609	49621	49632	49643
2.7	49653	49664	49674	49683	49693	49702	49711	49720	49728	49736
2.8	49744	49752	49760	49767	49774	49781	49788	49795	49801	49807
2.9	49813	49819	49825	49831	49836	49841	49846	49851	49856	49861
3.0	49865									
3.5	4997674									
4.0	4999683									
4.5	4999966									
5.0	4999997133									

[1] This table has been adapted, by permission, from F. C. Kent, "Elements of Statistics," McGraw-Hill Book Company, Inc., 1924.

TABLE A2. ORDINATES OF THE NORMAL CURVE

Ordinates (heights) of the unit normal curve.[1] The height (y) at any number of standard deviations (x) from the mean is

$$y = 0.3989e^{-x^2/2}$$

To obtain answers in units of particular problems, multiply these ordinates by $N(Ci)/\sigma$ where N is the number of cases, Ci the class interval, and σ the standard deviation.

Each figure in the body of the table is preceded by a decimal point.

x/σ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	39894	39892	39886	39876	39862	39844	39822	39797	39767	39733
0.1	39695	39654	39608	39559	39505	39448	39387	39322	39253	39181
0.2	39104	39024	38940	38853	38762	38667	38568	38466	38361	38251
0.3	38139	38023	37903	37780	37654	37524	37391	37255	37115	36973
0.4	36827	36678	36526	36371	36213	36053	35889	35723	35553	35381
0.5	35207	35029	34849	34667	34482	34294	34105	33912	33718	33521
0.6	33322	33121	32918	32713	32506	32297	32086	31874	31659	31443
0.7	31225	31006	30785	30563	30339	30114	29887	29658	29430	29200
0.8	28969	28737	28504	28269	28034	27798	27562	27324	27086	26848
0.9	26609	26369	26129	25888	25647	25406	25164	24923	24681	24439
1.0	24197	23955	23713	23471	23230	22988	22747	22506	22265	22025
1.1	21785	21546	21307	21069	20831	20594	20357	20121	19886	19652
1.2	19419	19186	18954	18724	18494	18265	18037	17810	17585	17360
1.3	17137	16915	16694	16474	16256	16038	15822	15608	15395	15183
1.4	14973	14764	14556	14350	14146	13943	13742	13542	13344	13147
1.5	12952	12758	12566	12376	12188	12001	11816	11632	11450	11270
1.6	11092	10915	10741	10567	10396	10226	10059	09893	09728	09566
1.7	09405	09246	09089	08933	08780	08628	08478	08329	08183	08038
1.8	07895	07754	07614	07477	07341	07206	07074	06943	06814	06687
1.9	06562	06438	06316	06195	06077	05959	05844	05730	05618	05508
2.0	05399	05292	05186	05082	04980	04879	04780	04682	04586	04491
2.1	04398	04307	04217	04128	04041	03955	03871	03788	03706	03626
2.2	03547	03470	03394	03319	03246	03174	03103	03034	02965	02898
2.3	02833	02768	02705	02643	02582	02522	02463	02406	02349	02294
2.4	02239	02186	02134	02083	02033	01984	01936	01888	01842	01797
2.5	01753	01709	01667	01625	01585	01545	01506	01468	01431	01394
2.6	01358	01323	01289	01256	01223	01191	01160	01130	01100	01071
2.7	01042	01014	00987	00961	00935	00909	00885	00861	00837	00814
2.8	00792	00770	00748	00727	00707	00687	00668	00649	00631	00613
2.9	00595	00578	00562	00545	00530	00514	00499	00485	00470	00457
3.0	00443									
3.5	0008727									
4.0	0001338									
4.5	0000160									
5.0	000001487									

[1] This table adapted, by permission, from Kent, "Elements of Statistics."

Ordinates may also be computed from the equation $\log y = 9.600910065820942 - 10 - 0.217147240951626x^2$.

TABLE A3. AREAS UNDER SKEWED CURVES[1]

	Amount of Skewness (value of α_3)					
x/σ	0.1	0.3	0.5	0.7	0.9	1.1
−3.65	.0001					
−3.60	.0001					
−3.55	.0001					
−3.50	.0001					
−3.45	.0001					
−3.40	.0002					
−3.35	.0002					
−3.30	.0002					
−3.25	.0003					
−3.20	.0004	.0001				
−3.15	.0005	.0001				
−3.10	.0006	.0001				
−3.05	.0007	.0001				
−3.00	.0008	.0002				
−2.95	.0010	.0002				
−2.90	.0012	.0003				
−2.85	.0014	.0004				
−2.80	.0017	.0005				
−2.75	.0020	.0007	.0001			
−2.70	.0024	.0009	.0001			
−2.65	.0029	.0011	.0002			
−2.60	.0034	.0014	.0002			
−2.55	.0040	.0017	.0004			
−2.50	.0047	.0022	.0005			
−2.45	.0055	.0027	.0007			
−2.40	.0064	.0033	.0010			
−2.35	.0075	.0040	.0014	.0001		
−2.30	.0087	.0049	.0018	.0002		
−2.25	.0101	.0059	.0024	.0003		
−2.20	.0116	.0071	.0031	.0006		
−2.15	.0134	.0085	.0041	.0009		
−2.10	.0153	.0101	.0052	.0014		
−2.05	.0175	.0120	.0066	.0021	.0001	
−2.00	.0200	.0142	.0082	.0030	.0002	
−1.95	.0227	.0166	.0102	.0043	.0005	
−1.90	.0258	.0194	.0125	.0059	.0010	
−1.85	.0291	.0225	.0152	.0079	.0018	
−1.80	.0329	.0260	.0184	.0103	.0031	
−1.75	.0370	.0300	.0220	.0133	.0050	.0001

[1] This table condensed by permission from Tables of Pearson's Type III Function, R. L. Salvosa, *Annals of Mathematical Statistics*, Vol. I, No. 2, May, 1930.

TABLE A3. AREAS UNDER SKEWED CURVES.—(*Continued*)

	Amount of Skewness (value of α_3)					
x/σ	0.1	0.3	0.5	0.7	0.9	1.1
−1.70	.0415	.0344	.0262	.0169	.0075	.0006
−1.65	.0464	.0393	.0309	.0212	.0107	.0018
−1.60	.0518	.0447	.0362	.0262	.0148	.0039
−1.55	.0576	.0507	.0421	.0319	.0199	.0072
−1.50	.0639	.0572	.0487	.0384	.0259	.0118
−1.45	.0708	.0643	.0560	.0457	.0330	.0179
−1.40	.0782	.0720	.0641	.0540	.0413	.0255
−1.35	.0861	.0803	.0728	.0631	.0506	.0347
−1.30	.0947	.0894	.0823	.0731	.0611	.0453
−1.25	.1038	.0990	.0926	.0840	.0727	.0575
−1.20	.1135	.1094	.1037	.0959	.0854	.0712
−1.15	.1238	.1204	.1155	.1086	.0992	.0862
−1.10	.1348	.1321	.1281	.1223	.1140	.1024
−1.05	.1464	.1445	.1415	.1367	.1298	.1199
−1.00	.1586	.1576	.1556	.1521	.1466	.1384
−0.95	.1714	.1714	.1704	.1681	.1642	.1579
−0.90	.1848	.1858	.1859	.1850	.1826	.1782
−0.85	.1989	.2009	.2022	.2025	.2017	.1992
−0.80	.2135	.2165	.2190	.2207	.2214	.2208
−0.75	.2288	.2328	.2364	.2394	.2417	.2429
−0.70	.2446	.2496	.2544	.2586	.2624	.2654
−0.65	.2609	.2670	.2728	.2784	.2835	.2882
−0.60	.2778	.2848	.2917	.2984	.3049	.3111
−0.55	.2951	.3031	.3110	.3188	.3266	.3341
−0.50	.3129	.3218	.3306	.3395	.3484	.3572
−0.45	.3312	.3408	.3506	.3604	.3702	.3801
−0.40	.3497	.3602	.3707	.3813	.3921	.4029
−0.35	.3687	.3798	.3910	.4024	.4139	.4255
−0.30	.3879	.3996	.4114	.4234	.4355	.4478
−0.25	.4074	.4196	.4319	.4444	.4570	.4698
−0.20	.4270	.4396	.4524	.4653	.4782	.4913
−0.15	.4468	.4598	.4728	.4860	.4992	.5125
−0.10	.4667	.4799	.4931	.5064	.5198	.5332
−0.05	.4867	.5000	.5133	.5267	.5400	.5535
0.00	.5066	.5199	.5333	.5466	.5599	.5732
+0.05	.5266	.5398	.5530	.5661	.5793	.5924
0.10	.5464	.5594	.5724	.5853	.5982	.6110
0.15	.5660	.5788	.5915	.6041	.6166	.6291
0.20	.5855	.5979	.6102	.6224	.6345	.6465
0.25	.6047	.6167	.6285	.6403	.6519	.6635

TABLE A3. AREAS UNDER SKEWED CURVES.—(*Continued*)

x/σ	Amount of Skewness (value of α_3) 0.1	0.3	0.5	0.7	0.9	1.1
0.30	.6237	.6351	.6465	.6577	.6688	.6798
0.35	.6423	.6532	.6639	.6745	.6851	.6955
0.40	.6606	.6708	.6809	.6909	.7008	.7107
0.45	.6784	.6879	.6974	.7067	.7160	.7253
0.50	.6959	.7046	.7133	.7220	.7307	.7393
0.55	.7128	.7208	.7288	.7368	.7448	.7527
0.60	.7293	.7365	.7437	.7510	.7583	.7656
0.65	.7453	.7516	.7581	.7646	.7713	.7779
0.70	.7607	.7662	.7719	.7778	.7837	.7897
0.75	.7756	.7803	.7852	.7903	.7956	.8010
0.80	.7899	.7938	.7979	.8024	.8070	.8118
0.85	.8037	.8067	.8101	.8139	.8179	.8221
0.90	.8169	.8191	.8218	.8249	.8282	.8319
0.95	.8294	.8309	.8329	.8353	.8381	.8412
1.00	.8414	.8422	.8435	.8453	.8475	.8501
1.05	.8529	.8529	.8536	.8548	.8565	.8586
1.10	.8637	.8630	.8631	.8638	.8650	.8666
1.15	.8739	.8727	.8722	.8724	.8731	.8743
1.20	.8836	.8818	.8808	.8805	.8809	.8815
1.25	.8928	.8904	.8889	.8882	.8880	.8884
1.30	.9014	.8985	.8966	.8954	.8949	.8949
1.35	.9094	.9062	.9038	.9023	.9014	.9011
1.40	.9170	.9134	.9107	.9088	.9076	.9069
1.45	.9241	.9201	.9171	.9149	.9134	.9125
1.50	.9306	.9264	.9231	.9206	.9189	.9177
1.55	.9368	.9323	.9288	.9261	.9241	.9227
1.60	.9425	.9378	.9341	.9312	.9290	.9274
1.65	.9477	.9429	.9390	.9359	.9336	.9318
1.70	.9526	.9477	.9437	.9404	.9379	.9359
1.75	.9571	.9521	.9480	.9446	.9420	.9399
1.80	.9612	.9562	.9520	.9486	.9458	.9436
1.85	.9650	.9600	.9558	.9523	.9494	.9471
1.90	.9685	.9636	.9593	.9557	.9528	.9504
1.95	.9717	.9668	.9626	.9590	.9560	.9535
2.00	.9746	.9698	.9656	.9620	.9589	.9564

TABLE A3. AREAS UNDER SKEWED CURVES.—(*Continued*)

	Amount of Skewness (value of α_3)					
x/σ	0.1	0.3	0.5	0.7	0.9	1.1
2.1	.9797	.9751	.9710	.9674	.9645	.9618
2.2	.9838	.9796	.9756	.9722	.9691	.9665
2.3	.9873	.9833	.9796	.9762	.9733	.9706
2.4	.9900	.9864	.9830	.9798	.9769	.9743
2.5	.9922	.9890	.9858	.9828	.9801	.9776
2.6	.9940	.9911	.9882	.9854	.9828	.9804
2.7	.9954	.9929	.9903	.9877	.9852	.9829
2.8	.9965	.9943	.9920	.9896	.9873	.9851
2.9	.9973	.9955	.9934	.9912	.9891	.9870
3.0	.9980	.9964	.9946	.9926	.9906	.9887
3.1	.9985	.9972	.9956	.9938	.9920	.9902
3.2	.9989	.9978	.9964	.9948	.9931	.9915
3.3	.9992	.9983	.9970	.9956	.9941	.9926
3.4	.9994	.9986	.9976	.9964	.9950	.9936
3.5	.9996	.9989	.9981	.9970	.9957	.9944
3.6	.9997	.9992	.9984	.9975	.9964	.9952
3.7	.9998	.9994	.9987	.9979	.9969	.9958
3.8	.9998	.9995	.9990	.9982	.9974	.9964
3.9	.9999	.9996	.9992	.9985	.9978	.9969
4.0	.9999	.9997	.9993	.9988	.9981	.9973
4.2		.9998	.9996	.9992	.9986	.9980
4.4		.9999	.9997	.9994	.9990	.9985
4.6		.9999	.9998	.9996	.9993	.9989
4.8			.9999	.9997	.9995	.9992
5.0			.9999	.9998	.9996	.9994
5.2				.9999	.9997	.9995
5.4				.9999	.9998	.9997
5.6				.9999	.9999	.9998
5.8					.9999	.9998
6.0					.9999	.9999
6.2						.9999
6.4						.9999
6.6						.9999

TABLE A4. ORDINATES OF SKEWED CURVES[1]

x/σ	0.1	0.3	0.5	0.7	0.9	1.1
−3.9	.0001					
−3.8	.0001					
−3.7	.0002					
−3.6	.0003					
−3.5	.0005					
−3.4	.0007	.0001				
−3.3	.0010	.0002				
−3.2	.0015	.0003				
−3.1	.0022	.0006				
−3.0	.0031	.0010	.0001			
−2.95	.0037	.0013	.0001			
−2.90	.0044	.0017	.0002			
−2.85	.0052	.0021	.0003			
−2.80	.0061	.0027	.0004			
−2.75	.0072	.0034	.0006			
−2.70	.0084	.0042	.0009			
−2.65	.0097	.0052	.0013			
−2.60	.0113	.0063	.0019			
−2.55	.0130	.0077	.0026	.0001		
−2.50	.0150	.0093	.0036	.0002		
−2.45	.0173	.0112	.0048	.0004		
−2.40	.0197	.0134	.0063	.0007		
−2.35	.0225	.0160	.0082	.0014		
−2.30	.0256	.0189	.0105	.0023		
−2.25	.0291	.0222	.0132	.0037		
−2.20	.0328	.0259	.0165	.0057		
−2.15	.0370	.0301	.0204	.0083		
−2.10	.0416	.0348	.0250	.0118		
−2.05	.0465	.0400	.0302	.0161	.0016	
−2.00	.0519	.0458	.0361	.0215	.0039	
−1.95	.0578	.0522	.0428	.0280	.0077	
−1.90	.0641	.0591	.0503	.0356	.0133	
−1.85	.0709	.0666	.0586	.0445	.0210	
−1.80	.0782	.0748	.0678	.0546	.0309	.0003
−1.75	.0859	.0835	.0777	.0659	.0430	.0048
−1.70	.0942	.0929	.0885	.0783	.0572	.0157
−1.65	.1029	.1029	.1000	.0920	.0734	.0323
−1.60	.1121	.1134	.1123	.1066	.0913	.0537
−1.55	.1218	.1245	.1253	.1222	.1108	.0789
−1.50	.1319	.1361	.1389	.1386	.1315	.1069

[1] This table condensed by permission from Pearson's Type III Function, R. L. Salvosa, *Annals of Mathematical Statistics*, Vol. I, No. 2, May, 1930.

TABLE A4. ORDINATES OF SKEWED CURVES.—(*Continued*)

x/σ	0.1	0.3	0.5	0.7	0.9	1.1
−1.45	.1424	.1482	.1530	.1557	.1532	.1367
−1.40	.1534	.1607	.1676	.1733	.1755	.1674
−1.35	.1647	.1736	.1827	.1913	.1981	.1983
−1.30	.1763	.1868	.1979	.2096	.2208	.2288
−1.25	.1882	.2002	.2134	.2278	.2432	.2584
−1.20	.2004	.2139	.2289	.2460	.2652	.2866
−1.15	.2127	.2276	.2444	.2638	.2864	.3131
−1.10	.2252	.2413	.2598	.2812	.3067	.3376
−1.05	.2377	.2550	.2749	.2981	.3258	.3600
−1.00	.2503	.2686	.2896	.3142	.3437	.3802
−0.95	.2628	.2819	.3038	.3294	.3602	.3980
−0.90	.2751	.2948	.3174	.3438	.3751	.4135
−0.85	.2872	.3074	.3304	.3570	.3885	.4267
−0.80	.2991	.3195	.3426	.3691	.4003	.4376
−0.75	.3106	.3310	.3539	.3801	.4104	.4463
−0.70	.3216	.3418	.3643	.3897	.4189	.4528
−0.65	.3322	.3519	.3738	.3981	.4257	.4574
−0.60	.3422	.3613	.3822	.4052	.4309	.4600
−0.55	.3516	.3698	.3895	.4109	.4346	.4608
−0.50	.3603	.3774	.3957	.4154	.4367	.4600
−0.45	.3682	.3841	.4008	.4185	.4374	.4577
−0.40	.3753	.3897	.4047	.4203	.4367	.4540
−0.35	.3816	.3944	.4075	.4210	.4348	.4590
−0.30	.3869	.3981	.4092	.4204	.4316	.4429
−0.25	.3914	.4006	.4097	.4186	.4273	.4358
−0.20	.3949	.4022	.4092	.4158	.4220	.4278
−0.15	.3974	.4027	.4076	.4119	.4157	.4190
−0.10	.3989	.4022	4050	.4071	.4086	.4096
−0.05	.3994	.4007	.4014	.4014	.4008	.3995
−0.00	.3989	.3982	.3969	.3949	.3923	.3890
0.05	.3974	.3947	.3915	.3876	.3832	.3782
0.10	.3949	.3904	.3853	.3797	.3736	.3670
0.15	.3915	.3851	.3784	.3711	.3635	.3556
0.20	.3871	.3791	.3707	.3621	.3532	.3440
0.25	.3819	.3723	.3625	.3526	.3425	.3324
0.30	.3758	.3648	.3537	.3426	.3316	.3207
0.35	.3690	.3566	.3444	.3324	.3206	.3090
0.40	.3614	.3478	.3347	.3219	.3095	.2974
0.45	.3530	.3386	.3247	.3113	.2983	.2858
0.50	.3441	.3288	.3143	.3004	.2872	.2745

TABLE A4. ORDINATES OF SKEWED CURVES.—(*Continued*)

x/σ	0.1	0.3	0.5	0.7	0.9	1.1
0.55	.3346	.3187	.3037	.2895	.2761	.2632
0.60	.3246	.3082	.2930	.2786	.2651	.2522
0.65	.3141	.2975	.2821	.2677	.2542	.2414
0.70	.3033	.2866	.2711	.2568	.2434	.2309
0.75	.2922	.2755	.2602	.2460	.2329	.2206
0.80	.2808	.2643	.2492	.2354	.2225	.2106
0.85	.2693	.2531	.2384	.2249	.2124	.2009
0.90	.2576	.2419	.2276	.2146	.2026	.1914
0.95	.2459	.2307	.2170	.2045	.1930	.1823
1.00	.2341	.2197	.2066	.1946	.1837	.1735
1.05	.2225	.2088	.1964	.1850	.1746	.1650
1.10	.2109	.1980	.1864	.1757	.1659	.1567
1.15	.1995	.1875	.1766	.1667	.1574	.1488
1.20	.1883	.1773	.1672	.1579	.1493	.1412
1.25	.1773	.1673	.1580	.1494	.1414	.1339
1.30	.1666	.1575	.1491	.1413	.1339	.1270
1.35	.1562	.1481	.1406	.1334	.1266	.1203
1.40	.1461	.1391	.1323	.1259	.1197	.1138
1.45	.1364	.1303	.1244	.1186	.1131	.1077
1.50	.1270	.1219	.1168	.1117	.1067	.1019
1.55	.1181	.1139	.1095	.1051	.1006	.0963
1.60	.1095	.1062	.1026	.0988	.0948	.0909
1.65	.1014	.0989	.0960	.0927	.0893	.0859
1.70	.0936	.0920	.0897	.0870	.0841	.0810
1.75	.0863	.0854	.0837	.0815	.0791	.0764
1.80	.0793	.0791	.0780	.0764	.0743	.0721
1.85	.0728	.0732	.0727	.0715	.0698	.0679
1.90	.0667	.0677	.0676	.0668	.0656	.0640
1.95	.0609	.0624	.0628	.0624	.0615	.0603
2.00	.0556	.0575	.0583	.0583	.0577	.0567
2.05	.0506	.0529	.0541	.0544	.0541	.0534
2.10	.0459	.0486	.0501	.0507	.0507	.0502
2.15	.0416	.0446	.0463	.0472	.0474	.0472
2.20	.0376	.0408	.0428	.0439	.0444	.0444
2.25	.0340	.0374	.0396	.0409	.0415	.0417
2.30	.0306	.0341	.0365	.0380	.0388	.0391
2.35	.0275	.0311	.0336	.0353	.0363	.0367
2.40	.0247	.0284	0310	.0328	.0339	.0345
2.45	.0221	.0258	.0285	.0304	.0316	.0324
2.50	.0198	.0235	.0262	.0282	.0295	.0303

TABLE A4. ORDINATES OF SKEWED CURVES.—(*Continued*)

x/σ	0.1	0.3	0.5	0.7	0.9	1.1
2.55	.0176	.0213	.0241	.0261	.0275	.0285
2.60	.0157	.0193	.0221	.0242	.0257	.0267
2.65	.0139	.0175	.0203	.0224	.0239	.0250
2.70	.0124	.0158	.0186	.0207	.0223	.0234
2.75	.0109	.0143	.0170	.0191	.0208	.0219
2.80	.0097	.0129	.0155	.0177	.0193	.0205
2.85	.0085	.0116	.0142	.0163	.0180	.0192
2.90	.0075	.0104	.0130	.0151	.0167	.0180
2.95	.0064	.0092	.0116	.0139	.0155	.0168
3.00	.0058	.0084	.0108	.0128	.0144	.0157
3.1	.0044	.0068	.0089	.0109	.0125	.0138
3.2	.0033	.0054	.0074	.0092	.0107	.0120
3.3	.0025	.0043	.0061	.0078	.0092	.0105
3.4	.0019	.0034	.0050	.0066	.0079	.0091
3.5	.0014	.0027	.0041	.0055	.0068	.0080
3.6	.0010	.0021	.0034	.0046	.0058	.0069
3.7	.0008	.0016	.0027	.0039	.0050	.0060
3.8	.0005	.0013	.0022	.0033	.0043	.0052
3.9	.0004	.0010	.0018	.0027	.0037	.0045
4.0	.0003	.0008	.0015	.0023	.0031	.0039
4.2	.0001	.0005	.0010	.0016	.0023	.0030
4.4	.0001	.0003	.0006	.0011	.0016	.0022
4.6		.0002	.0004	.0007	.0012	.0017
4.8		.0001	.0002	.0005	.0008	.0012
5.0			.0002	.0003	.0006	.0009
5.2			.0001	.0002	.0004	.0007
5.4			.0001	.0002	.0003	.0005
5.6				.0001	.0002	.0004
5.8				.0001	.0002	.0003
6.0					.0001	.0002
6.2					.0001	.0002
6.4					.0001	.0001
6.6						.0001
6.8						.0001

TABLE A5. SQUARES OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9
100	10000	10201	10404	10609	10816	11025	11236	11449	11664	11881
110	12100	12321	12544	12769	12996	13225	13456	13689	13924	14161
120	14400	14641	14884	15129	15376	15625	15876	16129	16384	16641
130	16900	17161	17424	17689	17956	18225	18496	18769	19044	19321
140	19600	19881	20164	20449	20736	21025	21316	21609	21904	22201
150	22500	22801	23104	23409	23716	24025	24336	24649	24964	25281
160	25600	25921	26244	26569	26896	27225	27556	27889	28224	28561
170	28900	29241	29584	29929	30276	30625	30976	31329	31684	32041
180	32400	32761	33124	33489	33856	34225	34596	34969	35344	35721
190	36100	36481	36864	37249	37636	38025	38416	38809	39204	39601
200	40000	40401	40804	41209	41616	42025	42436	42849	43264	43681
210	44100	44521	44944	45369	45796	46225	46656	47089	47524	47961
220	48400	48841	49284	49729	50176	50625	51076	51529	51984	52441
230	52900	53361	53824	54289	54756	55225	55696	56169	56644	57121
240	57600	58081	58564	59049	59536	60025	60516	61009	61504	62001
250	62500	63001	63504	64009	64516	65025	65536	66049	66564	67081
260	67600	68121	68644	69169	69696	70225	70756	71289	71824	72361
270	72900	73441	73984	74529	75076	75625	76176	76729	77284	77841
280	78400	78961	79524	80089	80656	81225	81796	82369	82944	83521
290	84100	84681	85262	85849	86436	87025	87616	88209	88804	89401
300	90000	90601	91204	91809	92416	93025	93636	94249	94864	95481
310	96100	96721	97344	97969	98596	99225	99856	100489	101124	101761
320	102400	103041	103684	104329	104976	105625	106276	106929	107584	108241
330	108900	109561	110224	110889	111556	112225	112896	113569	114244	114921
340	115600	116281	116964	117649	118336	119025	119716	120409	121104	121801
350	122500	123201	123904	124609	125316	126025	126736	127449	128164	128881
360	129600	130321	131044	131769	132496	133225	133956	134689	135424	136161
370	136900	137641	138384	139129	139876	140625	141376	142129	142884	143641
380	144400	145161	145924	146689	147456	148225	148996	149769	150544	151321
390	152100	152881	153664	154449	155236	156025	156816	157609	158404	159201
400	160000	160801	161604	162409	163216	164025	164836	165649	166464	167281
410	168100	168921	169744	170569	171396	172225	173056	173889	174724	175561
420	176400	177241	178084	178929	179776	180625	181476	182329	183184	184041
430	184900	185761	186624	187489	188356	189225	190096	190969	191844	192721
440	193600	194481	195364	196249	197136	198025	198916	199809	200704	201610
450	202500	203401	204304	205209	206116	207025	207936	208849	209764	210681
460	211600	212521	213444	214369	215296	216225	217156	218089	219024	219961
470	220900	221841	222784	223729	224676	225625	226576	227529	228484	229441
480	230400	231361	232324	233289	234256	235225	236196	237169	238144	239121
490	240100	241081	242064	243049	244036	245025	246016	247009	248004	249001
500	250000	251001	252004	253009	254016	255025	256036	257049	258064	259081
510	260100	261121	262144	263169	264196	265225	266256	267289	268324	269361
520	270400	271441	272484	273529	274576	275625	276676	277729	278784	279841
530	280900	281961	283024	284089	285156	286225	287296	288369	289444	290521
540	291600	292681	293764	294849	295936	297025	298116	299209	300304	301401

TABLE A5. SQUARES OF NUMBERS.—(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
550	302500	303601	304704	305809	306916	308025	309136	310249	311364	312481
560	313600	314721	315844	316969	318096	319225	320356	321489	322624	323761
570	324900	326041	327184	328329	329476	330625	331776	332929	334084	335241
580	336400	337561	338724	339889	341056	342225	343396	344569	345744	346921
590	348100	349281	350464	351649	352836	354025	355216	356409	357604	358801
600	360000	361201	362404	363609	364816	366025	367236	368449	369664	370881
610	372100	373321	374544	375769	376996	378225	379456	380689	381924	383161
620	384400	385641	386884	388129	389376	390625	391876	393129	394384	395641
630	396900	398161	399424	400689	401956	403225	404496	405769	407044	408321
640	409600	410881	412164	413449	414736	416025	417316	418609	419904	421201
650	422500	423801	425104	426409	427716	429025	430336	431649	432964	434281
660	435600	436921	438244	439569	440896	442225	443556	444889	446224	447561
670	448900	450241	451584	452929	454276	455625	456976	458329	459684	461041
680	462400	463761	465124	466489	467856	469225	470596	471969	473344	474721
690	476100	477481	478864	480249	481636	483025	484416	485809	487204	488601
700	490000	491401	492804	494209	495616	497025	498436	498849	501264	502681
710	504100	505521	506944	508369	509796	511225	512656	514089	515524	516961
720	518400	519841	521284	522729	524176	525625	527076	528529	529984	531441
730	532900	534361	535824	537289	538756	540225	541696	543169	544644	546121
740	547600	549081	550564	552049	553536	555025	556516	558009	559504	561001
750	562500	564001	565504	567009	568516	570025	571536	573049	574564	576081
760	577600	579121	580644	582169	583696	585225	586756	588289	589824	591361
770	592900	594441	595984	597529	599076	600625	602176	603729	605284	606841
780	608400	609961	611524	613089	614656	616225	617796	619369	620944	622521
790	624100	625681	627264	628849	630436	632025	633616	635209	636804	638401
800	640000	641601	643204	644809	646416	648025	649636	651249	652864	654481
810	656100	657721	659344	660969	662596	664225	665856	667489	669124	670761
820	672400	674041	675684	677329	678976	680625	682276	683929	685584	687241
830	688900	690561	692224	693889	695556	697225	698896	700569	702244	703921
840	705600	707281	708964	710649	712336	714025	715716	717409	719104	720801
850	722500	724201	725904	727609	729316	731025	732736	734449	736164	737881
860	739600	741321	743044	744769	746496	748225	749956	751689	753424	755161
870	756900	758641	760384	762129	763876	765625	767376	769129	770884	772641
880	774400	776161	777924	779689	781456	783225	784996	786769	788544	790321
890	792100	793881	795664	797449	799236	801025	802816	804609	806404	808201
900	810000	811801	813604	815409	817216	819025	820836	822649	824464	826281
910	828100	829921	831744	833569	835396	837225	839056	840889	842724	844561
920	846400	848241	850084	851929	853776	855625	857476	859329	861184	863041
930	864900	866761	868624	870489	872356	874225	876096	877969	879844	881721
940	883600	885481	887364	889249	891136	893025	894916	896809	898704	900601
950	902500	904401	906304	908209	910116	912025	913936	915849	917764	919681
960	921600	923521	925444	927369	929296	931225	933156	935089	937024	938961
970	940900	942841	944784	946729	948676	950625	952576	954529	956484	958441
980	960400	962361	964324	966289	968256	970225	972196	974169	976144	978121
990	980100	982081	984064	986049	988036	990025	992016	994009	996004	998001

TABLE A6. SQUARE ROOTS OF NUMBERS FROM 10 TO 100

N	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10	3.162	3 178	3.194	3.209	3 225	3.240	3.256	3.271	3 286	3 302
11	3 317	3 332	3 347	3.362	3 376	3.391	3.406	3 421	3 435	3.450
12	3 464	3.479	3.493	3.507	3 521	3 536	3.550	3 564	3 578	3.592
13	3.606	3 619	3.633	3.647	3.661	3.674	3.688	3 701	3.715	3.728
14	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3 847	3.860
15	3.873	3 886	3.899	3.912	3.924	3.937	3.950	3.962	3 975	3 987
16	4.000	4 012	4 025	4.037	4.050	4.062	4.074	4 087	4 099	4.111
17	4.123	4.135	4.147	4.159	4.171	4.183	4 195	4 207	4 219	4.231
18	4 243	4 254	4 266	4.278	4.290	4.301	4 313	4.324	4.336	4 347
19	4 359	4 370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4 461
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4 561	4.572
21	4.583	4.593	4.604	4.615	4.626	4.637	4 648	4.658	4.669	4 680
22	4 690	4 701	4.712	4.722	4.733	4.742	4 754	4.764	4.775	4 785
23	4.796	4 806	4.817	4.827	4.837	4.848	4 858	4.868	4 879	4.889
24	4 899	4.909	4.919	4.930	4.940	4.950	4.960	4 970	4 980	4 990
25	5 000	5 010	5.020	5.030	5.040	5.050	5.060	5 070	5 079	5.089
26	5.099	5 109	5 119	5.128	5 138	5 148	5.158	5.167	5.177	5 187
27	5 196	5.206	5.215	5 225	5.234	5 244	5 254	5.263	5 273	5 282
28	5 292	5 301	5.310	5.320	5.329	5 339	5.348	5 357	5 367	5 376
29	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5 468
30	5 477	5.486	5.495	5.505	5.514	5.523	5 532	5.541	5 550	5.559
31	5.568	5 577	5.586	5.595	5 604	5.612	5 621	5.630	5.639	5.648
32	5 657	5 666	5 674	5 683	5.692	5.701	5.710	5.718	5.727	5 736
33	5.745	5 753	5 762	5 771	5.779	5 788	5.797	5 805	5.814	5.822
34	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5 891	5.899	5 908
35	5 916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5 992
36	6 000	6.008	6.017	6 025	6 033	6 042	6.050	6.058	6.066	6 075
37	6 083	6.091	6.099	6 107	6.116	6 124	6.132	6.140	6 148	6 156
38	6.164	6.173	6.181	6 189	6.197	6 205	6.213	6.221	6 229	6.237
39	6.245	6.253	6.261	6.269	6.277	6 285	6.293	6.301	6 309	6 317
40	6 325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6 395
41	6.403	6.411	6.419	6 427	6 434	6 442	6.450	6 458	6 465	6 473
42	6.481	6 488	6.496	6 504	6.512	6.519	6.527	6.535	6 542	6 550
43	6 557	6,565	6.573	6.580	6.588	6.595	6 603	6.611	6 618	6.626
44	6.633	6.641	6.648	6.656	6.663	6 671	6.678	6.686	6.693	6.701
45	6.708	6.716	6.723	6.731	6 738	6.745	6.753	6.760	6 768	6.775
46	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6 841	6.848
47	6.856	6.863	6.870	6.878	6.885	6.892	6.899	6.907	6 914	6 921
48	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6 979	6 986	6.993
49	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064
50	7.071	7.078	7.085	7 092	7.099	7 106	7.113	7.120	7.127	7.134
51	7.141	7.148	7.155	7.162	7 169	7 176	7 183	7 190	7.197	7 204
52	7.211	7 218	7.225	7.232	7 239	7.246	7.253	7.259	7 266	7.273
53	7.280	7.287	7.294	7.301	7.308	7 314	7 321	7 328	7.335	7 342
54	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7 409

TABLE A6. SQUARE ROOTS OF NUMBERS FROM 10 TO 100.—(*Continued*)

N	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
55	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7 477
56	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543
57	7 550	7.556	7.563	7.570	7.576	7.582	7.589	7.596	7.603	7 609
58	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7 675
59	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740
60	7 746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804
61	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868
62	7.874	7 880	7 887	7.893	7.899	7.906	7.912	7.918	7.925	7,931
63	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7 981	7 987	7.994
64	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056
65	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118
66	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8 173	8.179
67	8 185	8.191	8.198	8 204	8.210	8.216	8.222	8 228	8 234	8.240
68	8.246	8 252	8 258	8.264	8 270	8.276	8.283	8 289	8.295	8 301
69	8.307	8 313	8.319	8.325	8.331	8.337	8.343	8.349	8 355	8.361
70	8 367	8 373	8.379	8.385	8.390	8 396	8.402	8.408	8.414	8 420
71	8.426	8 432	8.438	8.444	8.450	8 456	8.462	8.468	8 473	8.479
72	8.485	8 491	8.497	8.503	8 509	8.515	8.521	8.526	8.532	8 538
73	8 544	8.550	8.556	8 562	8 567	8 573	8 579	8.585	8.591	8 597
74	8.602	8.608	8.614	8.620	8 626	8.631	8.637	8.643	8 649	8 654
75	8.660	8.666	8.672	8.678	8 683	8.689	8.695	8.701	8.706	8 712
76	8.718	8.724	8.730	8.735	8.741	8.746	8.752	8.758	8.764	8 769
77	8 775	8.781	8 786	8.792	8.798	8 803	8.809	8.815	8 820	8 826
78	8 832	8 837	8.843	8 849	8.854	8 860	8.866	8.871	8 877	8.883
79	8 888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8 933	8 939
80	8 944	8 950	8 955	8.961	8.967	8.972	8.978	8.983	8.989	8 994
81	9 000	9 006	9 011	9.017	9 022	9.028	9.033	9.039	9.044	9.050
82	9.055	9 061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105
83	9 110	9.116	9 121	9 127	9.132	9.138	9.143	9.149	9.154	9 160
84	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214
85	9 220	9 225	9 230	9 236	9.241	9.247	9 252	9.257	9.263	9 268
86	9 274	9.279	9 284	9.290	9.295	9.301	9.306	9.311	9.317	9.322
87	9 327	9.333	9 338	9.343	9 349	9.354	9.359	9.365	9.370	9.376
88	9 381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9 429
89	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.463	9.482
90	9 487	9.492	9 497	9.503	9.508	9.513	9.518	9.524	9.529	9.534
91	9 539	9 545	9.550	9 555	9.560	9 566	9.571	9.576	9.581	9 586
92	9 592	9.597	9.602	9 607	9.612	9.618	9 623	9 628	9.633	9 638
93	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9 685	9 690
94	9.695	9.701	9.706	9 711	9.716	9.721	9.726	9.731	9.737	9.742
95	9 747	9.752	9.757	9 762	9.767	9 772	9.778	9.783	9.788	9.793
96	9 798	9.803	9 808	9 813	9.818	9 823	9.829	9.834	9.839	9.844
97	9 849	9 854	9.859	9 864	9 869	9 874	9 879	9.884	9.889	9.894
98	9 899	9.905	9.910	9.915	9 920	9.925	9.930	9.935	9.940	9.945
99	9.950	9 955	9.960	9 965	9.970	9.975	9.980	9.985	9.990	9 995

TABLE A7. SQUARE ROOTS OF NUMBERS FROM 100 TO 1000

N	0	1	2	3	4	5	6	7	8	9
100	10.00	10.05	10.10	10.15	10.20	10.25	10.30	10 34	10.39	10 44
110	10.49	10.54	10 58	10.63	10.68	10.72	10.77	10 82	10.86	10.91
120	10.95	11.00	11.05	11.09	11.14	11.18	11.22	11 27	11 31	11 36
130	11.40	11.45	11.49	11.53	11.58	11.62	11.66	11 70	11.75	11.79
140	11.83	11.87	11.92	11.96	12.00	12.04	12.08	12.12	12.17	12 21
150	12.25	12.29	12.33	12.37	12.41	12.45	12.49	12.53	12 57	12 61
160	12 65	12.69	12.73	12.77	12 81	12.85	12.88	12 92	12 96	13 00
170	13 04	13.08	13.11	13 15	13.19	13.23	13.27	13 30	13 34	13 38
180	13.42	13.45	13.49	13.53	13.56	13 60	13.64	13 67	13 71	13.75
190	13.78	13.82	13.86	13.89	13.93	13.96	14.00	14.04	14 07	14.11
200	14.14	14.18	14.21	14.25	14.28	14.32	14.35	14 39	14 42	14 46
210	14.49	14.53	14.56	14.59	14.63	14 66	14.70	14 73	14 76	14 80
220	14.83	14 87	14.90	14.93	14.97	15.00	15.03	15 07	15 10	15 13
230	15.17	15.20	15.23	15.26	15.30	15.33	15.36	15 39	15 43	15.46
240	15.49	15.52	15.56	15.59	15.62	15.65	15.68	15.72	15 75	15.78
250	15.81	15.84	15.87	15.91	15.94	15.97	16.00	16 03	16 06	16.09
260	16 12	16.16	16.19	16.22	16.25	16.28	16 31	16 34	16 37	16.40
270	16 43	16.46	16.49	16.52	16.55	16.58	16.61	16.64	16 67	16 70
280	16.73	16 76	16.79	16.82	16.85	16.88	16 91	16 94	16 97	17.00
290	17.03	17.06	17.09	17.12	17.15	17.18	17.20	17.23	17 26	17 29
300	17.32	17.35	17.38	17.41	17.44	17.46	17.49	17 52	17 55	17.58
310	17.61	17.64	17.66	17.69	17.72	17.75	17.78	17 80	17 83	17.86
320	17.89	17.92	17 94	17.97	18.00	18.03	18.06	18 08	18 11	18 14
330	18.17	18.19	18.22	18.25	18 28	18.30	18.33	18 36	18 38	18.41
340	18 44	18.47	18.49	18.52	18.55	18.57	18.60	18.63	18 65	18.68
350	18.71	18.74	18.76	18.79	18.81	18.84	18.87	18.89	18 92	18 95
360	18.97	19.00	19.03	19.05	19 08	19.10	19 13	19 16	19 18	19.21
370	19.24	19 26	19.29	19 31	19.34	19 36	19.39	19.42	19 44	19 47
380	19.49	19.52	19.54	19.57	19.60	19.62	19 65	19.67	19 70	19.72
390	19.75	19.77	19.80	19.82	19.85	19.87	19.90	19.92	19.95	19.98
400	20.00	20.02	20.05	20 07	20.10	20 12	20 15	20.17	20 20	20.22
410	20 25	20.27	20.30	20.32	20.35	20.37	20.40	20 42	20 44	20.47
420	20.49	20.52	20.54	20.57	20.59	20 62	20 64	20.66	20 69	20.71
430	20 74	20.76	20.78	20.81	20.83	20.86	20.88	20.90	20 93	20.95
440	20.98	21.00	21.02	21.05	21.07	21.10	21.12	21.14	21.17	21.19
450	21 21	21.24	21.26	21.28	21 31	21.33	21.35	21.38	21 40	21.42
460	21 45	21.47	21.49	21.52	21.54	21.56	21.59	21.61	21 63	21.66
470	21.68	21.70	21 73	21.75	21.77	21.79	21.82	21.84	21 86	21.89
480	21.91	21.93	21.95	21.98	22.00	22.02	22.05	22.07	22 09	22.11
490	22.14	22.16	22.18	22.20	22.23	22.25	22.27	22.29	22.32	22.34
500	22.36	22.38	22.41	22.43	22.45	22 47	22.49	22.52	22 54	22 56
510	22.58	22 61	22 63	22.65	22 67	22 69	22.72	22 74	22 76	22 78
520	22.80	22.83	22 85	22.87	22.89	22 91	22 93	22.96	22 98	23 00
530	23.02	23.04	23.07	23 09	23.11	23.13	23.15	23 17	23.19	23 22
540	23.24	23.26	23 28	23.30	23 32	23 35	23.37	23.39	23 41	23.43
550	23.45	23.47	23.49	23 52	23.54	23.56	23 58	23 60	23 62	23 64

TABLE A7. SQUARE ROOTS OF NUMBERS FROM 100 TO 1000.—(*Continued*)

N	0	1	2	3	4	5	6	7	8	9
550	23 45	23.47	23.49	23.52	23.54	23.56	23.58	23.60	23.62	23 64
560	23.66	23.69	23.71	23.73	23.75	23.77	23.79	23.81	23.83	23.85
570	23.87	23.90	23.92	23.94	23.96	23.98	24.00	24.02	24.04	24.06
580	24 08	24.10	24.12	24.15	24.17	24.19	24.21	24.23	24.25	24.27
590	24.29	24.31	24.33	24.35	24 37	24.39	24.41	24.43	24.45	24.47
600	24 49	24.52	24.54	24 56	24.58	24.60	24.62	24.64	24.66	24.68
610	24 70	24.72	24.74	24.76	24.78	24.80	24.82	24.84	24.86	24.88
620	24 90	24.92	24.94	24.96	24.98	25.00	25.02	25.04	25.06	25.08
630	25.10	25.12	25.14	25.16	25 18	25.20	25.22	25.24	25.26	25.28
640	25 30	25.32	25.34	25 36	25.38	25.40	25.42	25.44	25.46	25.48
650	25.50	25.51	25 53	25.55	25.57	25.59	25.61	25.63	25.65	25.67
660	25 69	25 71	25.73	25.75	25.77	25.79	25.81	25.83	25.85	25.86
670	25 88	25 90	25 92	25 94	25.96	25.98	26.00	26.02	26.04	26.06
680	26 08	26.10	26.12	26.13	26 15	26.17	26.19	26.21	26.23	26.25
690	26.27	26 29	26.31	26.32	26 34	26.36	26.38	26 40	26.42	26.44
700	26 46	26 48	26.50	26.51	26.53	26 55	26.57	26 59	26.61	26.63
710	26 65	26.66	26.68	26.70	26.72	26.74	26.76	26.78	26.80	26.81
720	26 83	26 85	26.87	26.89	26.91	26.93	26.94	26.96	26.98	27.00
730	27 02	27.04	27.06	27.07	27.09	27.11	27.13	27 15	27.17	27.18
740	27.20	27.22	27.24	27.26	27.28	27.29	27 31	27.33	27.35	27.37
750	27 39	27 40	27.42	27.44	27.46	27.48	27.50	27.51	27.53	27.55
760	27.57	27.59	27.60	27.62	27.64	27.66	27.68	27.69	27.71	27.73
770	27 75	27 77	27.78	27.80	27.82	27.84	27.86	27.87	27.89	27.91
780	27 93	27.95	27.96	27.98	28.00	28.02	28.04	28.05	28.07	28.09
790	28.11	28.12	28 14	28.16	28.18	28.20	28 21	28.23	28.25	28.27
800	28 28	28 30	28 32	28 34	28 35	28 37	28 39	28.41	28.43	28.44
810	28.46	28.48	28.50	28.51	28.53	28 55	28.57	28.58	28 60	28.62
820	28 64	28 65	28 67	28.69	28.71	28.72	28 74	28.76	28.78	28.79
830	28.81	28 83	28.84	28.86	28 88	28 90	28 91	28 93	28.95	28.97
840	28 98	29.00	29 02	29 03	29.05	29.07	29.09	29.10	29.12	29.14
850	29.15	29.17	29.19	29.21	29.22	29.24	29.26	29.27	29.29	29.31
860	29.33	29.34	29.36	29.38	29 39	29.41	29.43	29.44	29.46	29.48
870	29.50	29 51	29.53	29.55	29.56	29.58	29.60	29.61	29.63	29.65
880	29.66	29.68	29.70	29.72	29.73	29.75	29.77	29.78	29.80	29.82
890	29.83	29.85	29.87	29.88	29.90	29.92	29.93	29.95	29.97	29.98
900	30 00	30 02	30.03	30.05	30.07	30.08	30.10	30.12	30 13	30.15
910	30 17	30.18	30.20	30.22	30.23	30.25	30.27	30.28	30 30	30.32
920	30.33	30.35	30.36	30.38	30.40	30.41	30.43	30.45	30.46	30.48
930	30.50	30 51	30.53	30 54	30.56	30.58	30.59	30 61	30.63	30 64
940	30 66	30.68	30.69	30.71	30 72	30.74	30.76	30.77	30.79	30.81
950	30.82	30.84	30.85	30.87	30 89	30.90	30.92	30.94	30.95	30.97
960	30.98	31.00	31.02	31.03	31.05	31.06	31.08	31.10	31.11	31.13
970	31.14	31.16	31 18	31.19	31 21	31.22	31 24	31.26	31.27	31 29
980	31.30	31.32	31.34	31.35	31.37	31.38	31.40	31.42	31 43	31.45
990	31.46	31.48	31.50	31.51	31 53	31.54	31.56	31.58	31.59	31.61

TABLE A8. RECIPROCALS OF NUMBERS

N	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.00	1.0000	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174
1.10	.9091	.9009	.8929	.8850	.8772	.8696	.8621	.8547	8475	.8403
1.20	.8333	.8264	.8197	.8130	.8065	.8000	.7937	.7874	7812	.7752
1.30	.7692	.7634	.7576	.7519	.7463	.7407	.7353	.7299	.7246	.7194
1.40	.7143	.7092	.7042	.6993	.6944	.6897	.6849	.6803	.6757	.6711
1.50	.6667	.6623	.6579	.6536	.6494	.6452	.6410	.6369	.6329	6289
1.60	6250	.6211	.6173	.6135	.6098	.6061	.6024	.5988	.5952	.5917
1.70	.5882	5848	.5814	.5780	.5747	.5714	.5682	.5650	5618	.5587
1.80	.5556	.5525	.5495	.5464	.5435	.5405	.5376	.5348	.5319	.5291
1.90	5263	.5236	.5208	.5181	.5155	.5128	.5102	.5076	.5051	.5025
2.00	.5000	.4975	.4950	.4926	.4902	.4878	.4854	.4831	.4808	.4785
2.10	4762	.4739	.4717	.4694	.4673	.4651	.4630	.4608	.4587	.4566
2 20	.4545	.4525	.4504	.4484	.4464	.4444	.4425	4405	.4386	4367
2.30	.4348	.4329	.4310	.4292	.4274	.4255	.4237	.4219	.4202	.4184
2.40	.4167	.4149	.4132	.4115	.4098	.4082	.4065	.4049	.4032	.4016
2.50	.4000	3984	.3968	.3953	.3937	3922	.3906	.3891	.3876	3861
2.60	3846	.3831	.3817	.3802	.3788	.3774	3759	.3745	3731	3717
2 70	.3704	.3690	.3676	.3663	.3650	.3636	3623	.3610	3597	3584
2.80	.3571	3559	.3546	.3534	.3521	.3509	.3496	.3484	3472	.3460
2.90	.3448	.3436	.3425	.3413	.3401	.3390	.3378	.3367	.3356	.3344
3 00	.3333	.3322	.3311	.3300	.3289	3279	.3268	.3257	.3247	.3236
3.10	.3226	.3215	.3205	.3195	.3185	.3175	3165	.3155	3145	3135
3.20	3125	.3115	.3106	.3096	.3086	.3077	.3067	.3058	.3049	.3040
3.30	.3030	3021	.3012	.3003	.2994	.2985	.2976	.2967	2959	2950
3.40	.2941	.2933	.2924	.2915	.2907	.2899	.2890	.2882	2874	2865
3.50	.2857	.2849	.2841	.2833	.2825	.2817	.2809	.2801	.2793	.2786
3.60	.2778	.2770	.2762	.2755	.2747	.2740	2732	.2725	.2717	.2710
3.70	.2703	.2695	.2688	.2681	.2674	.2667	.2660	.2653	.2646	.2639
3.80	.2632	.2625	.2618	.2611	.2604	.2597	.2591	.2584	.2577	.2571
3.90	.2564	.2558	.2551	.2545	.2538	.2532	.2525	.2519	.2513	.2506
4.00	.2500	.2494	.2488	.2481	.2475	.2469	.2463	.2457	.2451	.2445
4.10	.2439	.2433	.2427	.2421	.2415	.2410	.2404	.2398	.2392	.2387
4.20	.2381	.2375	.2370	.2364	.2358	.2353	.2347	.2342	.2336	.2331
4.30	2326	.2320	.2315	.2309	.2304	.2299	.2294	.2288	.2283	.2278
4.40	.2273	.2268	.2262	.2257	.2252	.2247	.2242	.2237	.2232	.2227
4.50	.2222	.2217	.2212	.2208	.2203	.2198	.2193	.2188	.2183	.2179
4.60	2174	2169	.2164	.2160	.2155	.2151	.2146	.2141	.2137	.2132
4.70	.2128	2123	.2119	.2114	.2110	.2105	.2101	.2096	.2092	.2088
4 80	2088	.2079	.2075	.2070	.2066	.2062	.2058	.2053	.2049	.2045
4.90	2041	.2037	.2033	.2028	.2024	.2020	.2016	.2012	.2008	.2004
5.00	.2000	.1996	.1992	.1988	.1984	.1980	.1976	.1972	.1968	.1965
5.10	1961	.1957	.1953	.1949	.1946	.1942	.1938	.1934	.1930	.1927
5 20	.1923	.1919	.1916	.1912	.1908	.1905	.1901	.1898	.1894	.1890
5.30	.1887	.1883	.1880	.1876	.1873	.1869	.1866	.1862	.1859	.1855
5.40	.1852	.1848	.1845	.1842	.1838	.1835	.1832	.1828	.1825	.1821

TABLE A8. RECIPROCALS OF NUMBERS.—(*Continued*)

N	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
5.50	.1818	.1815	.1812	.1808	.1805	.1802	.1799	.1795	.1792	.1789
5.60	.1786	.1783	.1779	.1776	.1773	.1770	.1767	.1764	.1761	.1757
5.70	.1754	.1751	.1748	.1745	.1742	.1739	.1736	.1733	.1730	.1727
5.80	.1724	.1721	.1718	.1715	.1712	.1709	.1706	.1704	.1701	.1698
5.90	.1695	.1692	.1689	.1686	.1684	.1681	.1678	.1675	.1672	.1669
6 00	.1667	.1664	.1661	.1658	.1656	.1653	.1650	.1647	.1645	.1642
6.10	.1639	.1637	.1634	.1631	.1629	.1626	.1623	.1621	.1618	.1616
6.20	1613	.1610	.1608	.1605	.1603	.1600	.1597	.1595	.1592	.1590
6.30	.1587	.1585	.1582	.1580	.1577	.1575	.1572	.1570	.1567	.1565
6.40	.1562	.1560	.1558	.1555	.1553	.1550	.1548	.1546	.1543	.1541
6 50	1538	.1536	.1534	.1531	.1529	.1527	.1524	.1522	.1520	.1517
6.60	.1515	.1513	.1511	.1508	.1506	.1504	.1502	.1499	.1497	.1495
6 70	1493	.1490	.1488	.1486	.1484	.1481	.1479	.1477	.1475	1473
6.80	1471	.1468	.1466	.1464	.1462	.1460	.1458	.1456	.1453	.1451
6.90	1449	.1447	.1445	.1443	.1441	.1439	.1437	.1435	.1433	.1431
7.00	.1429	.1427	.1424	.1422	.1420	.1418	.1416	.1414	.1412	.1410
7.10	1408	.1406	1404	.1403	.1401	.1399	.1397	.1395	.1393	.1391
7.20	1389	.1387	.1385	.1383	.1381	.1379	.1377	.1376	.1374	.1372
7 30	.1370	.1368	1366	.1364	.1362	.1361	.1359	.1357	.1355	.1353
7.40	1351	.1350	.1348	.1346	.1344	.1342	.1340	.1339	.1337	.1335
7.50	.1333	.1332	.1330	.1328	.1326	.1324	.1323	.1321	.1319	.1318
7.60	1316	.1314	.1312	.1311	.1309	.1307	.1305	.1304	.1302	.1300
7.70	.1299	.1297	.1295	.1294	.1292	.1290	.1289	.1287	.1285	.1284
7.80	.1282	.1280	.1279	.1277	.1276	.1274	.1272	.1271	.1269	.1267
7.90	1266	.1264	.1263	.1261	.1259	.1258	.1256	.1255	.1253	.1252
8.00	1250	.1248	.1247	.1245	.1244	.1242	.1241	.1239	.1238	.1236
8 10	1235	1233	.1232	.1230	.1228	.1227	.1225	.1224	.1222	.1221
8 20	1220	.1218	.1217	.1215	.1214	.1212	.1211	.1209	.1208	.1206
8 30	1205	1203	.1202	.1200	.1199	.1198	.1196	.1195	.1193	.1192
8.40	.1190	.1189	.1188	.1186	.1185	.1183	.1182	.1181	.1179	.1178
8 50	1176	.1175	.1174	.1172	.1171	.1170	.1168	.1167	.1166	.1164
8 60	1163	1161	.1160	.1159	.1157	.1156	.1155	.1153	.1152	.1151
8.70	1149	.1148	.1147	.1145	.1144	.1143	.1142	.1140	.1139	1138
8 80	.1136	.1135	.1134	.1132	.1131	.1130	.1129	.1127	.1126	.1125
8.90	1124	.1122	.1121	.1120	.1119	.1117	.1116	.1115	.1114	.1112
9.00	1111	.1110	.1109	.1107	.1106	.1105	.1104	.1103	.1101	.1100
9.10	.1099	.1098	.1096	.1095	.1094	.1093	.1092	.1091	.1089	.1088
9.20	.1087	.1086	.1085	.1083	.1082	.1081	.1080	.1079	.1078	1076
9.30	1075	.1074	.1073	.1072	.1071	.1070	.1068	.1067	.1066	.1065
9.40	.1064	.1063	.1062	.1060	.1059	.1058	.1057	.1056	.1055	.1054
9.50	.1053	.1052	.1050	.1049	.1048	.1047	.1046	.1045	.1044	.1043
9.60	.1042	.1041	.1040	.1038	.1037	.1036	.1035	.1034	1033	.1032
9.70	.1031	.1030	.1029	.1028	.1027	.1026	.1025	.1024	.1022	.1021
9.80	.1020	.1019	.1018	.1017	.1016	.1015	.1014	.1013	1012	.1011
9.90	1010	.1009	.1008	.1007	.1006	.1005	.1004	.1003	.1002	.1001

TABLE A9. CONVERSION FROM PROBABLE ERROR TO STANDARD ERROR

P.E.	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1.00	1.483	1.497	1.512	1.527	1.542	1.557	1.572	1.586	1.601	1.616
1.10	1.631	1.646	1.661	1.675	1.690	1.705	1.720	1 735	1.749	1.764
1.20	1.779	1.794	1.809	1.824	1.838	1.853	1 868	1.883	1.898	1.913
1.30	1.927	1.942	1.957	1.972	1.987	2.002	2 016	2.031	2.046	2.061
1.40	2.076	2.090	2.105	2.120	2.135	2.150	2.165	2.179	2 194	2.209
1.50	2 224	2.239	2.254	2.268	2.283	2.298	2.313	2.328	2.343	2 357
1.60	2 372	2.387	2.402	2.417	2.431	2.446	2.461	2.476	2.491	2.506
1.70	2.520	2.535	2.550	2.565	2.580	2.595	2 609	2.624	2.639	2 654
1.80	2.669	2.684	2.698	2.713	2.728	2.743	2.758	2.772	2 787	2.802
1.90	2 817	2.832	2.847	2.861	2.876	2.891	2.906	2.921	2 936	2 950
2.00	2.965	2.980	2.995	3.010	3.025	3.039	3 054	3 069	3.084	3 099
2.10	3 113	3.128	3.143	3.158	3.173	3.188	3.202	3.217	3 232	3.247
2.20	3 262	3 277	3 291	3.306	3.321	3 336	3 351	3 366	3 380	3 395
2.30	3 410	3.425	3.440	3.454	3.469	3.484	3.499	3.514	3.529	3 543
2.40	3.558	3.573	3.588	3.603	3.618	3.632	3.647	3.662	3 677	3 692
2.50	3.707	3.721	3.736	3 751	3.766	3.781	3.795	3.810	3.825	3.840
2.60	3.855	3.870	3.884	3.899	3.914	3.929	3.944	3.959	3.973	3 988
2 70	4 033	4 018	4.033	4.048	4.062	4.077	4.092	4.107	4.122	4 136
2.80	4 151	4 166	4.181	4.196	4.211	4.225	4.240	4.255	4 270	4 285
2.90	4 300	4.314	4.329	4.344	4.359	4.374	4.389	4.403	4.418	4 433
3.00	4 448	4.463	4.477	4.492	4.507	4.522	4.537	4.552	4 566	4.581
3.10	4 596	4.611	4.626	4.641	4.655	4.670	4.685	4.700	4.715	4.730
3.20	4.744	4.759	4.774	4.789	4.804	4.818	4.833	4.848	4 863	4.878
3.30	4 893	4.907	4 922	4.937	4 952	4.967	4.982	4.996	5 011	5 026
3.40	5 041	5.056	5.070	5.085	5.100	5.115	5.130	5.145	5.159	5.174
3.50	5 189	5.204	5.219	5.234	5.248	5.263	5.278	5.293	5.308	5.323
3 60	5 337	5.352	5.367	5 382	5.397	5.411	5.426	5.441	5.456	5 471
3.70	5 486	5.500	5.515	5.530	5.545	5.560	5.575	5.589	5.604	5 619
3.80	5.634	5.649	5.664	5 678	5 693	5.708	5.723	5.738	5.752	5.767
3.90	5 782	5.797	5.812	5.827	5.841	5.856	5.871	5.886	5.901	5.916
4.00	5.930	5.945	5.960	5.975	5.990	6.005	6.019	6.034	6.049	6.064
4.10	6.079	6.093	6.108	6.123	6.138	6.153	6 168	6.182	6.197	6.212
4.20	6 227	6.242	6.257	6.271	6.286	6.301	6 316	6.331	6.346	6.360
4.30	6.375	6.390	6.405	6.420	6.434	6.449	6.464	6.479	6.494	6.509
4.40	6 523	6.538	6.553	6.568	6.583	6.598	6.612	6.627	6.642	6 657
4.50	6.672	6.687	6.701	6.716	6.731	6.746	6.761	6.775	6.790	6.805
4.60	6 820	6.835	6.850	6.864	6.879	6.894	6.909	6.924	6.939	6.953
4.70	6 968	6.983	6.998	7.013	7.028	7.042	7.057	7.072	7.087	7.102
4.80	7.116	7.131	7.146	7.161	7.176	7.191	7.205	7.220	7.235	7.250
4.90	7 265	7.280	7.294	7.309	7.324	7.339	7.354	7.369	7.383	7.398
5.00	7.413	7.428	7.443	7.457	7.472	7.487	7.502	7.517	7.532	7.546
5.10	7.561	7.576	7.591	7.606	7.621	7 635	7.650	7.665	7.680	6.695
5.20	7.710	7.724	7.739	7.754	7.769	7.784	7 798	7 813	7.828	7.843
5.30	7.858	7.873	7.887	7.902	7.917	7.932	7.947	7.962	7.976	7.991
5.40	8.006	8.021	8.036	8.051	8.065	8.080	8.095	8 110	8 125	8.139

PE = $.67448975\sigma$.

TABLE A9. CONVERSION FROM PROBABLE ERROR TO STANDARD ERROR.—(*Continued*)

P.E.	.00	.01	.02	.03	04	.05	.06	.07	.08	.09
5.50	8.154	8.169	8.184	8.199	8 214	8.228	8.243	8.258	8.273	8.288
5 60	8 303	8.317	8.332	8.347	8 362	8.377	8.392	8 406	8.421	8 436
5.70	8.451	8 466	8.480	8.495	8.510	8.525	8.540	8.555	8 569	8.584
5.80	8 599	8.614	8.629	8.644	8.658	8.673	8.688	8.703	8.718	8.733
5.90	8.747	8.762	8.777	8.792	8.807	8.821	8.836	8.851	8.866	8.881
6.00	8.896	8.910	8.925	8.940	8.955	8 970	8.985	8.999	9.014	9.029
6.10	9.044	9.059	9.074	9 088	9.103	9.118	9.133	9.148	9.162	9.177
6 20	9.192	9.207	9.222	9.237	9.251	9.266	9.281	9.296	9 311	9 326
6 30	9 340	9.355	9.370	9.385	9.400	9.415	9.429	9.444	9.459	9 474
6.40	9.489	9.503	9.518	9.533	9.548	9.563	9.578	9.592	9.607	9.622
6.50	9.637	9.652	9.667	9.681	9.696	9.711	9.726	9.741	9.756	9.770
6.60	9.785	9.800	9.815	9.830	9.844	9.859	9.874	9.889	9.904	9 919
6.70	9.933	9.948	9 963	9.978	9.993	10.01	10.02	10.04	10.05	10.07
6.80	10.08	10.10	10.11	10.13	10.14	10.16	10.17	10.18	10.20	10.22
6.90	10.23	10.24	10.26	10.27	10.29	10.30	10.32	10.33	10.35	10.36
7.00	10.38	10.39	10.41	10 42	10 44	10.45	10.47	10.48	10 50	10.51
7 10	10 53	10.54	10 56	10.57	10.59	10.60	10.62	10.63	10.64	10.66
7.20	10 68	10.69	10.70	10.72	10 73	10.75	10.76	10.78	10.79	10.81
7.30	10.82	10.84	10.85	10.87	10.88	10.90	10.91	10.93	10.94	10.96
7.40	10.97	10.99	11.00	11.02	11.03	11.04	11.06	11.08	11.09	11.10
7.50	11.12	11.13	11.15	11.16	11.18	11.19	11.21	11.22	11.24	11.25
7 60	11.27	11.28	11.30	11.31	11.33	11.34	11 36	11.37	11 39	11.40
7 70	11.42	11.43	11.45	11.46	11.48	11.49	11.50	11.52	11.54	11.55
7.80	11.56	11.58	11.59	11.61	11 62	11 64	11.65	11 67	11 68	11 70
7.90	11.71	11.73	11.74	11.76	11.77	11.79	11.80	11.82	11.83	11 85
8 00	11.86	11.88	11.89	11 90	11 92	11.94	11.95	11.96	11.98	11.99
8.10	12 01	12.02	12.04	12.05	12.07	12 08	12.10	12.11	12.13	12.14
8.20	12.16	12.17	12.19	12 20	12.22	12.23	12 25	12.26	12.28	12.29
8.30	12.31	12.32	12.34	12.35	12.36	12.38	12.40	12.41	12.42	12.44
8 40	12.45	12.47	12.48	12.50	12.51	12.53	12.54	12.56	12.57	12.59
8 50	12.60	12.62	12.63	12.65	12.66	12.68	12.69	12 71	12.72	12.74
8.60	12.75	12.76	12.78	12.80	12.81	12.82	12.84	12.85	12.87	12.88
8.70	12 90	12.91	12.93	12.94	12.96	12.97	12.99	13.00	13.02	13.03
8.80	13.05	13.06	13.08	13.09	13.11	13.12	13.14	13.15	13.17	13.18
8.90	13.20	13.21	13.22	13.24	13.25	13.27	13.28	13.30	13.31	13.33
9.00	13.34	13.36	13.37	13.39	13.40	13.42	13.43	13.45	13.46	13 48
9.10	13.49	13.51	13.52	13.54	13.55	13.57	13 58	13.60	13.61	13.62
9.20	13.64	13.66	13.67	13.68	13.70	13.71	13.73	13.74	13.76	13.77
9.30	13.79	13.80	13.82	13 83	13.85	13.86	13.88	13.89	13.91	13.92
9.40	13 94	13.95	13.97	13.98	14.00	14.01	14.02	14.04	14.06	14.07
9.50	14.08	14.10	14.11	14.13	14.14	14.16	14.17	14.19	14.20	14.22
9 60	14.23	14.25	14.26	14.28	14.29	14.31	14.32	14.34	14.35	14.37
9 70	14.38	14.40	14.41	14.43	14.44	14.46	14.47	14.48	14.50	14.52
9.80	14.53	14.54	14.56	14.57	14.59	14.60	14.62	14.63	14.65	14 66
9.90	14.68	14.69	14.71	14.72	14.74	14.75	14.77	14.78	14.80	14.81

TABLE A10. CONVERSION OF STANDARD ERRORS TO PROBABLE ERRORS

σ	.00	.01	02	.03	04	05	06	.07	.08	.09
1.00	.6745	.6812	.6880	.6948	.7015	.7082	7150	.7218	.7285	.7352
1.10	.7420	7487	.7555	.7622	.7690	7757	7824	7892	.7959	.8027
1.20	.8094	.8162	.8229	.8297	.8364	.8431	8499	8566	8634	8701
1.30	.8769	.8836	.8904	.8971	.9038	.9106	.9173	9240	9308	9376
1.40	.9443	.9511	.9578	.9646	.9713	.9780	.9848	.9915	.9982	1.005
1.50	1.012	1.018	1.025	1.032	1.039	1.045	1.052	1 059	1 066	1 072
1.60	1.079	1.086	1.093	1.099	1.106	1.113	1.120	1.126	1 133	1 140
1 70	1.147	1.153	1.160	1.167	1.174	1.180	1.187	1 194	1 201	1 207
1 80	1.214	1.221	1.228	1.234	1.241	1 248	1.255	1.261	1 268	1 275
1.90	1.282	1.288	1.295	1.302	1.309	1.315	1.322	1.329	1 335	1 342
2.00	1.349	1.356	1.362	1.369	1.376	1.383	1.389	1.396	1 403	1 410
2 10	1.416	1.423	1.430	1.437	1.443	1 450	1.457	1 464	1 470	1.477
2 20	1.484	1 491	1.497	1.504	1.511	1.518	1.524	1.531	1.538	1.545
2 30	1.551	1.558	1.565	1.572	1.578	1.585	1.592	1.599	1.605	1 612
2 40	1.619	1 626	1 632	1.639	1.646	1.652	1.659	1.666	1 673	1.679
2 50	1 686	1 693	1 700	1.706	1.713	1 720	1.727	1 733	1 740	1 747
2.60	1 754	1 760	1 767	1 774	1 781	1 787	1.794	1 801	1 808	1.814
2 70	1.821	1 828	1 835	1.841	1 848	1.855	1 862	1 868	1 875	1 882
2.80	1.889	1.895	1.902	1 909	1.916	1 922	1.929	1 936	1 943	1.949
2 90	1 956	1 963	1.970	1 976	1 983	1.990	1 996	2 003	2 010	2 017
3 00	2.023	2.030	2.037	2.044	2.050	2 057	2 064	2.071	2 077	2 084
3 10	2.091	2 098	2.104	2.111	2.118	2.125	2 131	2 138	2 145	2 152
3 20	2.158	2.165	2 172	2 179	2.185	2 192	2.199	2 206	2 212	2.219
3 30	2.226	2 233	2 239	2 246	2 253	2 260	2 266	2 273	2 280	2 287
3 40	2.293	2.300	2.307	2 313	2 320	2.327	2 334	2 340	2 347	2 354
3.50	2 361	2.367	2 374	2.381	2 388	2 394	2 401	2 408	2 415	2.421
3 60	2.428	2 435	2 442	2 448	2 455	2.462	2 469	2 475	2 482	2 489
3.70	2.496	2.502	2.509	2 516	2 523	2.529	2 536	2 543	2 550	2 556
3 80	2.563	2.570	2.577	2 583	2.590	2.597	2 604	2 610	2 617	2 624
3.90	2.631	2 637	2 644	2 651	2.657	2 664	2.671	2 678	2 684	2 691
4 00	2 698	2.705	2.711	2.718	2 725	2.732	2 738	2 745	2 752	2 759
4.10	2.765	2.772	2.779	2.786	2.792	2.799	2.806	2.813	2 819	2 826
4.20	2.833	2.840	2.846	2.853	2.860	2.867	2.873	2 880	2 887	2.894
4.30	2.900	2 907	2.914	2.921	2.927	2.934	2.941	2 948	2 954	2 961
4.40	2.968	2.974	2.981	2.988	2.955	3.001	3.008	3 015	3 022	3 028
4 50	3.035	3.042	3.049	3.055	3.062	3.069	3.076	3.082	3 089	3 096
4 60	3.103	3.109	3.116	3.123	3.130	3.136	3 143	3 150	3.157	3 163
4 70	3 170	3 177	3.184	3.190	3.197	3.204	3 211	3.217	3.224	3 231
4 80	3.238	3.244	3.251	3.258	3.265	3.271	3.278	3.285	3 292	3 298
4.90	3.305	3.312	3.318	3.325	3.332	3 339	3.345	3.352	3 359	3 366
5.00	3 372	3.379	3 386	3.393	3 399	3.406	3.413	3.420	3.426	3.433
5.10	3.440	3.447	3.453	3 460	3.467	3 474	3.480	3.487	3 494	3 501
5 20	3.507	3.514	3.521	3.528	3.534	3.541	3 548	3 555	3 561	3 568
5.30	3.575	3.582	3 588	3 595	3 602	3 609	3.615	3 622	3 629	3 635
5.40	3.642	3.649	3.656	3 662	3.669	3 676	3.683	3.689	3.696	3.703

$\sigma = 1.4826022$ PE.

TABLE A10. CONVERSION OF STANDARD ERRORS TO PROBABLE ERRORS.—(*Continued*)

σ	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
5.50	3.710	3.716	3.723	3.730	3.737	3.743	3.750	3.757	3.764	3.770
5.60	3.777	3.784	3.791	3.797	3.804	3.811	3.818	3.824	3.831	3.838
5.70	3.845	3.851	3.858	3.865	3.872	3.878	3.885	3.892	3.899	3.905
5.80	3.912	3.919	3.926	3.932	3.939	3.946	3.953	3.959	3.966	3.973
5.90	3.979	3.986	3.993	4.000	4.006	4.013	4.020	4.027	4.033	4.040
6.00	4.047	4.054	4.060	4.067	4.074	4.081	4.087	4.094	4.101	4.108
6.10	4.114	4.121	4.128	4.135	4.141	4.148	4.155	4.162	4.168	4.175
6.20	4.182	4.189	4.195	4.202	4.209	4.216	4.222	4.229	4.236	4.243
6.30	4.249	4.256	4.263	4.270	4.276	4.283	4.290	4.296	4.303	4.310
6.40	4.317	4.323	4.330	4.337	4.344	4.350	4.357	4.364	4.371	4.377
6.50	4.384	4.391	4.398	4.404	4.411	4.418	4.425	4.431	4.438	4.445
6.60	4.452	4.458	4.465	4.472	4.479	4.485	4.492	4.499	4.506	4.512
6.70	4.519	4.526	4.533	4.539	4.546	4.553	4.560	4.566	4.573	4.580
6.80	4.587	4.593	4.600	4.607	4.614	4.620	4.627	4.634	4.640	4.647
6.90	4.654	4.661	4.667	4.674	4.681	4.688	4.694	4.701	4.708	4.715
7.00	4.721	4.728	4.735	4.742	4.748	4.755	4.762	4.769	4.775	4.782
7.10	4.789	4.796	4.802	4.809	4.816	4.823	4.829	4.836	4.843	4.850
7.20	4.856	4.863	4.870	4.877	4.883	4.890	4.897	4.904	4.910	4.917
7.30	4.924	4.931	4.937	4.944	4.951	4.957	4.964	4.971	4.978	4.984
7.40	4.991	4.998	5.005	5.011	5.018	5.025	5.032	5.038	5.045	5.052
7.50	5.059	5.065	5.072	5.079	5.086	5.092	5.099	5.106	5.113	5.119
7.60	5.126	5.133	5.140	5.146	5.153	5.160	5.167	5.173	5.180	5.187
7.70	5.194	5.200	5.207	5.214	5.221	5.227	5.234	5.241	5.248	5.254
7.80	5.261	5.268	5.275	5.281	5.288	5.295	5.301	5.308	5.315	5.322
7.90	5.328	5.335	5.342	5.349	5.355	5.362	5.369	5.376	5.382	5.389
8.00	5.396	5.403	5.409	5.416	5.423	5.430	5.436	5.443	5.450	5.457
8.10	5.463	5.470	5.477	5.484	5.490	5.497	5.504	5.511	5.517	5.524
8.20	5.531	5.538	5.544	5.551	5.558	5.565	5.571	5.578	5.585	5.592
8.30	5.598	5.605	5.612	5.618	5.625	5.632	5.639	5.645	5.652	5.659
8.40	5.666	5.672	5.679	5.686	5.693	5.699	5.706	5.713	5.720	5.726
8.50	5.733	5.740	5.747	5.743	5.760	5.767	5.774	5.780	5.787	5.794
8.60	5.801	5.807	5.814	5.821	5.828	5.834	5.841	5.848	5.855	5.861
8.70	5.868	5.875	5.882	5.888	5.895	5.902	5.909	5.915	5.922	5.929
8.80	5.936	5.942	5.949	5.956	5.962	5.969	5.976	5.983	5.989	5.996
8.90	6.003	6.010	6.016	6.023	6.030	6.037	6.043	6.050	6.057	6.064
9.00	6.070	6.077	6.084	6.091	6.097	6.104	6.111	6.118	6.124	6.131
9.10	6.138	6.145	6.151	6.158	6.165	6.172	6.178	6.185	6.192	6.199
9.20	6.205	6.212	6.219	6.226	6.232	6.239	6.246	6.253	6.259	6.266
9.30	6.273	6.279	6.286	6.293	6.300	6.306	6.313	6.320	6.327	6.333
9.40	6.340	6.347	6.354	6.360	6.367	6.374	6.381	6.387	6.394	6.401
9.50	6.408	6.414	6.421	6.428	6.435	6.441	6.448	6.455	6.462	6.468
9.60	6.475	6.482	6.489	6.495	6.502	6.509	6.516	6.522	6.529	6.536
9.70	6.543	6.549	6.556	6.563	6.570	6.576	6.583	6.590	6.597	6.603
9.80	6.610	6.617	6.623	6.630	6.637	6.644	6.650	6.657	6.664	6.671
9.90	6.677	6.684	6.691	6.698	6.704	6.711	6.718	6.725	6.731	6.738

TABLE A11. VALUES OF z FOR GIVEN VALUES OF r

r	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.000	.0000	.0010	.0020	.0030	.0040	.0050	.0060	.0070	.0080	.0090
.010	.0100	.0110	.0120	.0130	.0140	.0150	.0160	.0170	.0180	.0190
.020	.0200	.0210	.0220	.0230	.0240	.0250	.0260	.0270	.0280	.0290
.030	.0300	.0310	.0320	.0330	.0340	.0350	.0360	.0370	.0380	.0390
.040	.0400	.0410	.0420	.0430	.0440	.0450	.0460	.0470	.0480	.0490
.050	.0501	.0511	.0521	.0531	.0541	.0551	.0561	.0571	.0581	.0591
.060	.0601	.0611	.0621	.0631	.0641	.0651	0661	.0671	.0681	0691
.070	.0701	.0711	.0721	.0731	.0741	.0751	.0761	.0771	.0782	.0792
.080	.0802	.0812	.0822	.0832	.0842	.0852	.0862	.0872	.0882	.0892
.090	.0902	.0912	.0922	.0933	.0943	.0953	.0963	.0973	.0983	.0993
.100	.1003	.1013	.1024	.1034	.1044	.1054	.1064	.1074	.1084	.1094
.110	.1105	.1115	.1125	.1135	.1145	.1155	1165	.1175	.1185	.1195
.120	.1206	.1216	.1226	.1236	.1246	.1257	.1267	.1277	.1287	.1297
.130	.1308	.1318	.1328	.1338	.1348	.1358	.1368	.1379	.1389	.1399
.140	.1409	.1419	.1430	.1440	.1450	.1460	.1470	.1481	.1491	.1501
.150	.1511	.1522	.1532	.1542	.1552	.1563	.1573	.1583	.1593	.1604
.160	.1614	.1624	.1634	.1654	.1655	.1665	.1676	.1686	.1696	.1706
.170	.1717	.1727	.1737	.1748	.1758	.1768	.1779	.1789	.1799	.1810
.180	.1820	.1830	.1841	.1851	.1861	.1872	.1882	.1892	.1903	.1913
.190	.1923	.1934	.1944	.1954	.1965	.1975	.1986	.1996	.2007	.2017
.200	.2027	.2038	.2048	.2059	.2069	.2079	.2090	.2100	.2111	2121
.210	.2132	.2142	.2153	.2163	.2174	.2184	.2194	.2205	.2215	.2226
.220	.2237	.2247	.2258	.2268	.2279	.2289	.2300	.2310	.2321	.2331
.230	.2342	.2353	.2363	.2374	.2384	.2395	.2405	.2416	.2427	.2437
.240	.2448	.2458	.2469	.2480	.2490	.2501	.2511	.2522	.2533	.2543
.250	.2554	.2565	.2575	.2586	.2597	.2608	.2618	.2629	.2640	.2650
.260	.2661	.2672	.2682	.2693	.2704	.2715	.2726	.2736	.2747	.2758
.370	.2769	.2779	.2790	.2801	.2812	.2823	.2833	.2844	.2855	.2866
.280	.2877	.2888	.2898	.2909	.2920	.2931	.2942	.2953	.2964	.2975
.290	.2986	.2997	.3008	.3019	.3029	.3040	.3051	.3062	.3073	3084
.300	.3095	.3106	.3117	.3128	.3139	.3150	.3161	.3172	.3183	.3195
.310	.3206	.3217	.3228	.3239	.3250	.3261	.3272	.3283	.3294	.3305
.320	.3317	.3328	.3339	.3350	.3361	.3372	.3384	.3395	.3406	3417
.330	.3428	.3439	.3451	.3462	.3473	.3484	.3496	.3507	.3518	.3530
.340	.3541	.3552	.3564	.3575	.3586	.3597	.3609	.3620	.3632	.3643
.350	.3654	.3666	.3677	.3689	.3700	.3712	.3723	.3734	.3746	.3757
.360	.3769	.3780	.3792	.3803	.3815	.3826	.3838	.3850	.3861	.3873
.370	.3884	.3896	.3907	.3919	.3931	.3942	.3954	.3966	.3977	.3989
.380	.4001	.4012	.4024	.4036	.4047	.4059	.4071	.4083	.4094	.4106
.390	.4118	.4130	.4142	.4153	.4165	.4177	.4189	.4201	.4213	.4225
.400	.4236	.4248	.4260	.4272	.4284	.4296	.4308	.4320	.4332	.4344
.410	.4356	.4368	.4380	.4392	.4404	.4416	.4429	.4441	.4453	.4465
.420	.4477	.4489	.4501	.4513	.4526	.4538	.4550	.4562	.4574	.4587
.430	.4599	.4611	.4623	.4636	.4648	.4660	.4673	.4685	.4697	.4710
.440	.4722	.4735	.4747	.4760	.4772	.4784	.4797	.4809	.4822	.4835
.450	4847	.4860	.4872	.4885	.4897	.4910	.4923	.4935	.4948	.4961
.460	.4973	.4986	.4999	.5011	.5024	.5037	.5049	.5062	.5075	.5088
.470	.5101	.5114	.5126	.5139	.5152	.5165	.5178	.5191	.5204	.5217
.480	.5230	.5243	.5256	.5279	.5282	.5295	.5308	.5321	.5334	.5347
.490	.5361	.5374	.5387	.5400	.5413	.5427	.5440	.5453	.5466	.5480

TABLE A11. VALUES OF *z* FOR GIVEN VALUES OF *r*.—(*Continued*)

r	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.500	.5493	.5506	.5520	.5533	.5547	.5560	.5573	.5587	.5600	.5614
.510	.5627	.5641	.5654	.5668	.5681	.5695	.5709	.5722	.5736	.5750
.520	.5763	.5777	.5791	.5805	.5818	.5832	.5846	.5860	.5874	.5888
.530	.5901	.5915	.5929	.5943	.5957	.5971	.5985	.5999	.6013	.6027
.540	.6042	.6056	.6070	.6084	.6098	.6112	.6127	.6141	.6155	.6170
.550	.6184	.6198	.6213	.6227	.6241	.6256	.6270	.6285	.6299	.6314
.560	.6328	.6343	.6358	.6372	.6387	.6401	.6416	.6431	.6446	.6460
.570	.6475	.6490	.6505	.6520	.6535	.6550	.6565	.6579	.6594	.6610
.580	.6625	.6640	.6655	.6670	.6685	.6700	.6715	.6731	.6746	.6761
590	.6777	.6792	.6807	.6823	.6838	.6854	.6869	.6885	.6900	.6916
600	.6931	.6947	.6963	.6978	.6994	.7010	.7026	.7042	.7057	.7073
.610	7089	.7105	.7121	.7137	.7153	.7169	.7185	.7201	.7218	.7234
.620	7250	.7266	.7283	.7299	.7315	.7332	.7348	.7364	.7381	.7398
.630	.7414	.7431	.7447	.7464	.7481	.7497	.7514	.7531	.7548	.7565
.640	.7582	.7599	.7616	.7633	.7650	.7667	.7684	.7701	.7718	.7736
.650	.7753	.7770	.7788	.7805	.7823	.7840	.7858	.7875	.7893	.7910
.660	.7928	.7946	.7964	.7981	.7999	.8017	.8035	.8053	.8071	.8089
.670	.8107	.8126	.8144	.8162	.8180	.8199	.8217	.8236	.8254	.8273
.680	.8291	.8310	.8328	.8347	.8366	.8385	.8404	.8423	.8442	.8461
.690	8480	.8499	.8518	.8537	.8556	.8576	.8595	.8614	.8634	.8653
700	8673	.8693	.8712	.8732	.8752	.8772	.8792	.8812	.8832	.8852
710	.8872	.8892	.8912	.8933	.8953	.8973	.8994	.9014	.9035	.9056
.720	.9076	.9097	.9118	.9139	.9160	.9181	.9202	.9223	.9245	.9266
730	.9287	.9309	.9330	.9352	.9373	.9395	.9417	.9439	.9461	.9483
740	.9505	.9527	.9549	.9571	.9594	.9616	.9639	.9661	.9684	.9707
.750	.9730	.9752	.9775	.9799	.9822	.9845	.9868	.9892	.9915	.9939
.760	.9962	.9986	1.0010	1.0034	1 0058	1.0082	1.0106	1.0130	1 0154	1.0179
.770	1 0203	1.0228	1.0253	1.0277	1 0302	1.0327	1.0352	1.0378	1.0403	1.0428
.780	1.0454	1.0479	1.0505	1 0531	1.0557	1 0583	1.0609	1 0635	1.0661	1.0688
790	1 0714	1.0741	1.0768	1.0795	1.0822	1 0849	1.0876	1.0903	1.0931	1.0958
.800	1.0986	1.1014	1.1041	1.1070	1.1098	1.1127	1 1155	1.1184	1.1212	1.1241
810	1 1270	1.1299	1.1329	1.1358	1.1388	1.1417	1.1447	1 1477	1.1507	1.1538
820	1.1568	1.1599	1.1630	1.1660	1.1692	1.1723	1.1754	1.1786	1.1817	1.1849
830	1 1870	1.1913	1.1946	1.1979	1.2011	1.2044	1.2077	1.2111	1.2144	1.2178
.840	1 2212	1.2246	1.2280	1.2315	1.2349	1.2384	1.2419	1.2454	1.2490	1.2526
850	1.2561	1 2598	1 2634	1.2670	1 2708	1 2744	1.2782	1.2819	1.2857	1.2895
860	1.2934	1.2972	1.3011	1 3050	1.3089	1.3129	1.3168	1.3209	1.3249	1.3290
870	1.3331	1.3372	1 3414	1.3456	1.3498	1 3540	1 3583	1.3626	1.3670	1.3714
.880	1 3758	1 3802	1.3847	1.3892	1.3938	1.3984	1.4030	1.4077	1.4124	1.4171
.890	1.4219	1.4268	1.4316	1.4366	1.4415	1.4465	1.4516	1.4566	1.4618	1 4670
.900	1 4722	1.4775	1.4828	1.4883	1.4937	1.4992	1.5047	1.5103	1.5160	1.5217
.910	1 5275	1.5334	1.5393	1.5453	1 5513	1.5574	1.5636	1.5698	1.5762	1.5825
.920	1.5890	1.5956	1.6022	1.6089	1.6157	1.6226	1.6296	1.6366	1.6438	1.6510
.930	1.6584	1.6659	1.6734	1.6811	1.6888	1.6967	1.7047	1.7129	1.7211	1.7295
.940	1.7380	1.7467	1 7555	1.7645	1.7736	1.7828	1.7923	1.8019	1.8117	1.8216
.950	1.8318	1.8421	1.8527	1.8635	1.8745	1.8857	1.8972	1.9090	1.9210	1.9333
.960	1.9459	1.9588	1.9721	1 9857	1.9996	2.0140	2.0287	2.0439	2.0595	2.0756
970	2 0923	2.1095	2.1273	2 1457	2.1649	2.1847	2.2054	2.2269	2.2494	2.2729
980	2 2976	2.3223	2.3507	2 3796	2.4101	2.4426	2.4774	2.5147	2.5550	2.5988
.990	2.6467	2.6996	2.7587	2.8257	2.9031	2.9945	3.1063	3.2504	3.4534	3.8002

r	z
.9999	4.95172
.99999	6.10303

TABLE A12. VALUES OF r FOR VARIOUS VALUES OF z FROM 1 TO 3[1]
Each figure in the body of the table is preceded by a decimal point.
Example: When z has a value of 1.23, r has a value of 0.8426

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0000	0100	0200	0300	0400	0500	0599	0699	0798	0898
0.1	0997	1096	1194	1293	1391	1489	1586	1684	1781	1877
0.2	1974	2070	2165	2260	2355	2449	2543	2636	2729	2821
0.3	2913	3004	3095	3185	3275	3364	3452	3540	3627	3714
0.4	3800	3885	3969	4053	4136	4219	4301	4382	4462	4542
0.5	4621	4699	4777	4854	4930	5005	5080	5154	5227	5299
0.6	5370	5441	5511	5580	5649	5717	5784	5850	5915	5980
0.7	6044	6107	6169	6231	6291	6351	6411	6469	6528	6584
0.8	6640	6696	6751	6805	6858	6911	6963	7014	7064	7114
0.9	7163	7211	7259	7306	7352	7398	7443	7487	7531	7574
1.0	7616	7658	7699	7739	7779	7818	7857	7895	7932	7969
1.1	8005	8041	8076	8110	8144	8178	8210	8243	8275	8306
1.2	8337	8367	8397	8426	8455	8483	8511	8538	8565	8591
1.3	8617	8643	8668	8692	8717	8741	8764	8787	8810	8832
1.4	8854	8875	8896	8917	8937	8957	8977	8996	9015	9033
1.5	9051	9069	9087	9104	9121	9138	9154	9170	9186	9201
1.6	9217	9232	9246	9261	9275	9289	9302	9316	9329	9341
1.7	9354	9366	9379	9391	9402	9414	9425	9436	9447	9458
1.8	94681	94783	94884	94983	95080	95175	95268	95359	95449	95537
1.9	95624	95709	95792	95873	95953	96032	96109	96185	96259	96331
2.0	96404	96473	96541	96609	96675	96739	96803	96865	96926	96986
2.1	97045	97103	97159	97215	97269	97323	97375	97426	97477	97526
2.2	97574	97622	97668	97714	97759	97803	97846	97888	97929	97970
2.3	98010	98049	98087	98124	98161	98197	98233	98267	98301	98335
2.4	98367	98399	98431	98462	98492	98522	98551	98579	98607	98635
2.5	98661	98688	98714	98739	98764	98788	98812	98835	98858	98881
2.6	98903	98924	98945	98966	98987	99007	99026	99045	99064	99083
2.7	99101	99118	99136	99153	99170	99186	99202	99218	99233	99248
2.8	99263	99278	99292	99306	99320	99333	99346	99359	99372	99384
2.9	99396	99408	99420	99431	99443	99454	99464	99475	99485	99495
3.0	99505									
3.5	998178									
4.0	999329									
4.5	999753									
5.0	999909									
5.5	9999666									
6.0	9999877									
6.5	99999548									

For greater accuracy, or for values beyond the table:

$$r = \frac{e^{2z} - 1}{e^{2z} + 1} \qquad z = 1.15129254 \log_{10} \frac{1 + r}{1 - r}$$

$$\sigma_z = \frac{1}{\sqrt{n - 3}}$$

[1] Taken by permission, with minor additions, from R. A. Fisher, "Statistical Methods for Research Workers," Oliver & Boyd, London, 1932.

TABLE A13. VALUES OF $\sqrt{pq}$ WHEN $p + q = 1$
Each figure in the table is to be preceded by a decimal point

	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009	
.000	0000	0316	0447	0547	0631	0705	0772	0834	0891	0944	.990
.010	0995	1043	1089	1133	1175	1216	1255	1293	1330	1365	.980
020	1400	1434	1467	1499	1530	1561	1591	1621	1650	1678	.970
030	1706	1733	1760	1786	1812	1838	1863	1888	1912	1936	.960
.040	1960	1983	2006	2029	2051	2073	2095	2116	2138	2159	.950
.050	2179	2200	2220	2240	2260	2280	2299	2318	2337	2356	.940
060	2375	2393	2412	2430	2448	2465	2483	2500	2517	2535	.930
070	2551	2568	2585	2601	2618	2634	2650	2666	2682	2697	.920
.080	2713	2728	2744	2759	2774	2789	2804	2818	2833	2847	.910
.090	2862	2876	2890	2904	2918	2932	2946	2960	2973	2987	.900
.100	3000	3013	3026	3040	3053	3066	3078	3091	3104	3116	.890
.110	3129	3141	3154	3166	3178	3190	3202	3214	3226	3238	.880
.120	3250	3261	3273	3284	3296	3307	3318	3330	3341	3352	.870
.130	3363	3374	3385	3396	3407	3417	3428	3438	3449	3459	860
.140	3470	3480	3491	3501	3511	3521	3531	3541	3551	3561	.850
.150	3571	3580	3590	3600	3609	3619	3628	3638	3647	3657	840
.160	3666	3675	3685	3694	3703	3712	3721	3730	3739	3748	.830
.170	3756	3765	3774	3782	3791	3800	3808	3817	3825	3834	.820
.180	3842	3850	3858	3867	3875	3883	3891	3899	3907	3915	.810
.190	3923	3931	3939	3947	3954	3962	3970	3977	3985	3993	.800
.200	4000	4007	4015	4022	4030	4037	4044	4052	4059	4066	.790
.210	4073	4080	4087	4094	4101	4108	4115	4122	4129	4136	780
220	4142	4149	4156	4163	4169	4176	4182	4189	4195	4202	.770
.230	4208	4215	4221	4227	4234	4240	4246	4252	4259	4265	.760
.240	4271	4277	4283	4289	4295	4301	4307	4313	4319	4324	.750
.250	4330	4336	4342	4347	4353	4359	4364	4370	4375	4381	.740
.260	4386	4392	4397	4404	4408	4413	4419	4424	4429	4434	.730
270	4440	4445	4450	4455	4460	4465	4470	4475	4480	4485	.720
.280	4490	4495	4500	4505	4509	4514	4519	4524	4528	4533	.710
290	4538	4542	4547	4551	4556	4560	4565	4569	4574	4578	.700
300	4583	4587	4591	4596	4600	4604	4608	4612	4617	4621	.690
.310	4625	4629	4633	4637	4641	4645	4649	4653	4657	4661	680
.320	4665	4669	4672	4676	4680	4684	4687	4691	4695	4698	.670
.330	4702	4706	4709	4713	4716	4720	4723	4727	4730	4734	.660
.340	4737	4740	4744	4747	4750	4754	4757	4760	4763	4767	.650
.350	4770	4773	4776	4779	4782	4785	4788	4791	4794	4797	.640
.360	4800	4803	4806	4809	4811	4814	4817	4820	4823	4825	.630
.370	4828	4831	4833	4836	4839	4841	4844	4846	4849	4851	.620
.380	4854	4856	4859	4861	4864	4866	4868	4871	4873	4875	.610
.390	4877	4880	4882	4884	4886	4889	4891	4893	4895	4897	.600
.400	4899	4901	4903	4905	4907	4909	4911	4913	4915	4916	.590
.410	4918	4920	4922	4924	4925	4927	4929	4931	4932	4934	.580
.420	4936	4937	4939	4940	4942	4943	4945	4946	4948	4949	.570
.430	4951	4952	4954	4955	4956	4958	4959	4960	4961	4963	.560
.440	4964	4965	4966	4967	4969	4970	4971	4972	4973	4974	.550
.450	4975	4976	4977	4978	4979	4980	4981	4981	4982	4983	.540
.460	4984	4985	4986	4986	4987	4988	4988	4989	4990	4990	.530
.470	4991	4992	4992	4993	4993	4994	4994	4995	4995	4996	.520
.480	4996	4996	4997	4997	4997	4998	4998	4998	4999	4999	.510
.490	4999	4999	4999	5000	5000	5000	5000	5000	5000	5000	.500
	.010	.009	.008	.007	.006	.005	.004	.003	.002	.001	

TABLE A14. FOUR-PLACE COMMON LOGARITHMS OF NUMBERS*

	0	1	2	3	4	5	6	7	8	9	10	Tenths of the Tabular Difference 1 2 3 4 5
1.0	0.0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	0414	To avoid interpolation in the first ten lines, use the special table on pages 44–45.
1.1	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	0792	
1.2	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	1139	
1.3	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	1461	
1.4	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	1761	
1.5	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	2041	
1.6	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	2304	
1.7	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2553	
1.8	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2788	
1.9	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	3010	
2.0	0.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	3222	2 4 6 8 11
2.1	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	3424	2 4 6 8 10
2.2	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	3617	2 4 6 8 10
2.3	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	3802	2 4 5 7 9
2.4	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	3979	2 4 5 7 9
2.5	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	4150	2 3 5 7 9
2.6	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	4314	2 3 5 7 8
2.7	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	4472	2 3 5 6 8
2.8	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	4624	2 3 5 6 8
2.9	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	4771	1 3 4 6 7
3.0	0.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	4914	1 3 4 6 7
3.1	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	5051	1 3 4 6 7
3.2	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	5185	1 3 4 5 7
3.3	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	5315	1 3 4 5 6
3.4	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	5441	1 3 4 5 6
3.5	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	5563	1 2 4 5 6
3.6	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	5682	1 2 4 5 6
3.7	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	5798	1 2 3 5 6
3.8	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	5911	1 2 3 5 6
3.9	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	6021	1 2 3 4 6
4.0	0.6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	6128	1 2 3 4 5
4.1	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	6232	1 2 3 4 5
4.2	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	6335	1 2 3 4 5
4.3	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	6435	1 2 3 4 5
4.4	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	6532	1 2 3 4 5
4.5	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	6628	1 2 3 4 5
4.6	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	6721	1 2 3 4 5
4.7	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	6812	1 2 3 4 5
4.8	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	6902	1 2 3 4 4
4.9	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	6990	1 2 3 4 4
5.0	0.6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	7076	1 2 3 3 4
5.1	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	7160	1 2 3 3 4
5.2	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	7243	1 2 2 3 4
5.3	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	7324	1 2 2 3 4
5.4	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	7404	1 2 2 3 4

* Taken, with permission, from E. V. Huntington's *Four Place Tables of Logarithms and Trigonometric Functions*, Harvard Cooperative Society, Inc., 1907.

TABLE A14. FOUR-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

	0	1	2	3	4	5	6	7	8	9	10	Tenths of the Tabular Difference				
												1	2	3	4	5
5 5	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	7482	1	2	2	3	4
5.6	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	7559	1	2	2	3	4
5.7	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	7634	1	2	2	3	4
5.8	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	7709	1	1	2	3	4
5.9	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7782	1	1	2	3	4
6.0	0.7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7853	1	1	2	3	4
6.1	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7924	1	1	2	3	4
6.2	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7993	1	1	2	3	3
6 3	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	8062	1	1	2	3	3
6.4	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	8129	1	1	2	3	3
6 5	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	8195	1	1	2	3	3
6 6	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	8261	1	1	2	3	3
6.7	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	8325	1	1	2	3	3
6.8	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	8388	1	1	2	3	3
6.9	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	8451	1	1	2	3	3
7.0	0 8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	8513	1	1	2	2	3
7.1	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	8573	1	1	2	2	3
7.2	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	8633	1	1	2	2	3
7.3	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	8692	1	1	2	2	3
7.4	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	8751	1	1	2	2	3
7.5	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	8808	1	1	2	2	3
7.6	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	8865	1	1	2	2	3
7.7	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	8921	1	1	2	2	3
7.8	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	8976	1	1	2	2	3
7.9	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	9031	1	1	2	2	3
8 0	0 9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	9085	1	1	2	2	3
8.1	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	9138	1	1	2	2	3
8.2	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	9191	1	1	2	2	3
8 3	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	9243	1	1	2	2	3
8.4	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	9294	1	1	2	2	3
8.5	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	9345	1	1	2	2	3
8.6	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	9395	1	1	2	2	3
8 7	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	9445	0	1	1	2	2
8 8	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	9494	0	1	1	2	2
8.9	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	9542	0	1	1	2	2
9.0	0.9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	9590	0	1	1	2	2
9.1	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	9638	0	1	1	2	2
9.2	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	9685	0	1	1	2	2
9.3	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	9731	0	1	1	2	2
9.4	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	9777	0	1	1	2	2
9 5	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	9823	0	1	1	2	2
9.6	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	9868	0	1	1	2	2
9 7	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	9912	0	1	1	2	2
9.8	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	9956	0	1	1	2	2
9 9	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996		0	1	1	2	2

TABLE A14. FOUR-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

	0	1	2	3	4	5	6	7	8	9	10
1.00	0.0000	0004	0009	0013	0017	0022	0026	0030	0035	0039	0043
1.01	0043	0048	0052	0056	0060	0065	0069	0073	0077	0082	0086
1.02	0086	0090	0095	0099	0103	0107	0111	0116	0120	0124	0128
1.03	0128	0133	0137	0141	0145	0149	0154	0158	0162	0166	0170
1.04	0170	0175	0179	0183	0187	0191	0195	0199	0204	0208	0212
1.05	0212	0216	0220	0224	0228	0233	0237	0241	0245	0249	0253
1.06	0253	0257	0261	0265	0269	0273	0278	0282	0286	0290	0294
1.07	0294	0298	0302	0306	0310	0314	0318	0322	0326	0330	0334
1.08	0334	0338	0342	0346	0350	0354	0358	0362	0366	0370	0374
1.09	0374	0378	0382	0386	0390	0394	0398	0402	0406	0410	0414
1.10	0.0414	0418	0422	0426	0430	0434	0438	0441	0445	0449	0453
1.11	0453	0457	0461	0465	0469	0473	0477	0481	0484	0488	0492
1.12	0492	0496	0500	0504	0508	0512	0515	0519	0523	0527	0531
1.13	0531	0535	0538	0542	0546	0550	0554	0558	0561	0565	0569
1.14	0569	0573	0577	0580	0584	0588	0592	0596	0599	0603	0607
1.15	0607	0611	0615	0618	0622	0626	0630	0633	0637	0641	0645
1.16	0645	0648	0652	0656	0660	0663	0667	0671	0674	0678	0682
1.17	0682	0686	0689	0693	0697	0700	0704	0708	0711	0715	0719
1.18	0719	0722	0726	0730	0734	0737	0741	0745	0748	0752	0755
1.19	0755	0759	0763	0766	0770	0774	0777	0781	0785	0788	0792
1.20	0.0792	0795	0799	0803	0806	0810	0813	0817	0821	0824	0828
1.21	0828	0831	0835	0839	0842	0846	0849	0853	0856	0860	0864
1.22	0864	0867	0871	0874	0878	0881	0885	0888	0892	0896	0899
1.23	0899	0903	0906	0910	0913	0917	0920	0924	0927	0931	0934
1.24	0934	0938	0941	0945	0948	0952	0955	0959	0962	0966	0969
1.25	0969	0973	0976	0980	0983	0986	0990	0993	0997	1000	1004
1.26	1004	1007	1011	1014	1017	1021	1024	1028	1031	1035	1038
1.27	1038	1041	1045	1048	1052	1055	1059	1062	1065	1069	1072
1.28	1072	1075	1079	1082	1086	1089	1092	1096	1099	1103	1106
1.29	1106	1109	1113	1116	1119	1123	1126	1129	1133	1136	1139
1.30	0.1139	1143	1146	1149	1153	1156	1159	1163	1166	1169	1173
1.31	1173	1176	1179	1183	1186	1189	1193	1196	1199	1202	1206
1.32	1206	1209	1212	1216	1219	1222	1225	1229	1232	1235	1239
1.33	1239	1242	1245	1248	1252	1255	1258	1261	1265	1268	1271
1.34	1271	1274	1278	1281	1284	1287	1290	1294	1297	1300	1303
1.35	1303	1307	1310	1313	1316	1319	1323	1326	1329	1332	1335
1.36	1335	1339	1342	1345	1348	1351	1355	1358	1361	1364	1367
1.37	1367	1370	1374	1377	1380	1383	1386	1389	1392	1396	1399
1.38	1399	1402	1405	1408	1411	1414	1418	1421	1424	1427	1430
1.39	1430	1433	1436	1440	1443	1446	1449	1452	1455	1458	1461
1.40	0.1461	1464	1467	1471	1474	1477	1480	1483	1486	1489	1492
1.41	1492	1495	1498	1501	1504	1508	1511	1514	1517	1520	1523
1.42	1523	1526	1529	1532	1535	1538	1541	1544	1547	1550	1553
1.43	1553	1556	1559	1562	1565	1569	1572	1575	1578	1581	1584
1.44	1584	1587	1590	1593	1596	1599	1602	1605	1608	1611	1614
1.45	1614	1617	1620	1623	1626	1629	1632	1635	1638	1641	1644
1.46	1644	1647	1649	1652	1655	1658	1661	1664	1667	1670	1673
1.47	1673	1676	1679	1682	1685	1688	1691	1694	1697	1700	1703
1.48	1703	1706	1708	1711	1714	1717	1720	1723	1726	1729	1732
1.49	1732	1735	1738	1741	1744	1746	1749	1752	1755	1758	1761

TABLE A14. FOUR-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

	0	1	2	3	4	5	6	7	8	9	10
1.50	0.1761	1764	1767	1770	1772	1775	1778	1781	1784	1787	1790
1.51	1790	1793	1796	1798	1801	1804	1807	1810	1813	1816	1818
1.52	1818	1821	1824	1827	1830	1833	1836	1838	1841	1844	1847
1.53	1847	1850	1853	1855	1858	1861	1864	1867	1870	1872	1875
1.54	1875	1878	1881	1884	1886	1889	1892	1895	1898	1901	1903
1.55	1903	1906	1909	1912	1915	1917	1920	1923	1926	1928	1931
1.56	1931	1934	1937	1940	1942	1945	1948	1951	1953	1956	1959
1.57	1959	1962	1965	1967	1970	1973	1976	1978	1981	1984	1987
1.58	1987	1989	1992	1995	1998	2000	2003	2006	2009	2011	2014
1.59	2014	2017	2019	2022	2025	2028	2030	2033	2036	2038	2041
1.60	0.2041	2044	2047	2049	2052	2055	2057	2060	2063	2066	2068
1.61	2068	2071	2074	2076	2079	2082	2084	2087	2090	2092	2095
1.62	2095	2098	2101	2103	2106	2109	2111	2114	2117	2119	2122
1.63	2122	2125	2127	2130	2133	2135	2138	2140	2143	2146	2148
1.64	2148	2151	2154	2156	2159	2162	2164	2167	2170	2172	2175
1.65	2175	2177	2180	2183	2185	2188	2191	2193	2196	2198	2201
1.66	2201	2204	2206	2209	2212	2214	2217	2219	2222	2225	2227
1.67	2227	2230	2232	2235	2238	2240	2243	2245	2243	2251	2253
1.68	2253	2256	2258	2261	2263	2266	2269	2271	2274	2276	2279
1.69	2279	2281	2284	2287	2289	2292	2294	2297	2299	2302	2304
1.70	0.2304	2307	2310	2312	2315	2317	2320	2322	2325	2327	2330
1.71	2330	2333	2335	2338	2340	2343	2345	2348	2350	2353	2355
1.72	2355	2358	2360	2363	2365	2368	2370	2373	2375	2378	2380
1.73	2380	2383	2385	2388	2390	2393	2395	2398	2400	2403	2405
1.74	2405	2408	2410	2413	2415	2418	2420	2423	2425	2428	2430
1.75	2430	2433	2435	2438	2440	2443	2445	2448	2450	2453	2455
1.76	2455	2458	2460	2463	2465	2467	2470	2472	2475	2477	2480
1.77	2480	2482	2485	2487	2490	2492	2494	2497	2499	2502	2504
1.78	2504	2507	2509	2512	2514	2516	2519	2521	2524	2526	2529
1.79	2529	2531	2533	2536	2538	2541	2543	2545	2548	2550	2553
1.80	0.2553	2555	2558	2560	2562	2565	2567	2570	2572	2574	2577
1.81	2577	2579	2582	2584	2586	2589	2591	2594	2596	2598	2601
1.82	2601	2603	2605	2608	2610	2613	2615	2617	2620	2622	2625
1.83	2625	2627	2629	2632	2634	2636	2639	2641	2643	2646	2648
1.84	2648	2651	2653	2655	2658	2660	2662	2665	2667	2669	2672
1.85	2672	2674	2676	2679	2681	2683	2686	2688	2690	2693	2695
1.86	2695	2697	2700	2702	2704	2707	2709	2711	2714	2716	2718
1.87	2718	2721	2723	2725	2728	2730	2732	2735	2737	2739	2742
1.88	2742	2744	2746	2749	2751	2753	2755	2758	2760	2762	2765
1.89	2765	2767	2769	2772	2774	2776	2778	2781	2783	2785	2788
1.90	0.2788	2790	2792	2794	2797	2799	2801	2804	2806	2808	2810
1.91	2810	2813	2815	2817	2819	2822	2824	2826	2828	2831	2833
1.92	2833	2835	2838	2840	2842	2844	2847	2849	2851	2853	2856
1.93	2856	2858	2860	2862	2865	2867	2869	2871	2874	2876	2878
1.94	2878	2880	2882	2885	2887	2889	2891	2894	2896	2898	2900
1.95	2900	2903	2905	2907	2909	2911	2914	2916	2918	2920	2923
1.96	2923	2925	2927	2929	2931	2934	2936	2938	2940	2942	2945
1.97	2945	2947	2949	2951	2953	2956	2958	2960	2962	2964	2967
1.98	2967	2969	2971	2973	2975	2978	2980	2982	2984	2986	2989
1.99	2989	2991	2993	2995	2997	2999	3002	3004	3006	3008	3010

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS[1]

100–149

No.	0	1	2	3	4	5	6	7	8	9
100	00 000	00 043	00 087	00 130	00 173	00 217	00 260	00 303	00 346	00 389
101	00 432	00 475	00 518	00 561	00 604	00 647	00 689	00 732	00 775	00 817
102	00 860	00 903	00 945	00 988	01 030	01 072	01 115	01 157	01 199	01 242
103	01 284	01 326	01 368	01 410	01 452	01 494	01 536	01 578	01 620	01 662
104	01 703	01 745	01 787	01 828	01 870	01 912	01 953	01 995	02 036	02 078
105	02 119	02 160	02 202	02 243	02 284	02 325	02 366	02 407	02 449	02 490
106	02 531	02 572	02 612	02 653	02 694	02 735	02 776	02 816	02 857	02 898
107	02 938	02 979	03 019	03 060	03 100	03 141	03 181	03 222	03 262	03 302
108	03 342	03 383	03 423	03 463	03 503	03 543	03 583	03 623	03 663	03 703
109	03 743	03 782	03 822	03 862	03 902	03 941	03 981	04 021	04 060	04 100
110	04 139	04 179	04 218	04 258	04 297	04 336	04 376	04 415	04 454	04 493
111	04 532	04 571	04 610	04 650	04 689	04 727	04 766	04 805	04 844	04 883
112	04 922	04 961	04 999	05 038	05 077	05 115	05 154	05 192	05 231	05 269
113	05 308	05 346	05 385	05 423	05 461	05 500	05 538	05 576	05 614	05 652
114	05 690	05 729	05 767	05 805	05 843	05 881	05 918	05 956	05 994	06 032
115	06 070	06 108	06 145	06 183	06 221	06 258	06 296	06 333	06 371	06 408
116	06 446	06 483	06 521	06 558	06 595	06 633	06 670	06 707	06 744	06 781
117	06 819	06 856	06 893	06 930	06 967	07 004	07 041	07 078	07 115	07 151
118	07 188	07 225	07 262	07 298	07 335	07 372	07 408	07 445	07 482	07 518
119	07 555	07 591	07 628	07 664	07 700	07 737	07 773	07 809	07 846	07 882
120	07 918	07 954	07 990	08 027	08 063	08 099	08 135	08 171	08 207	08 243
121	08 279	08 314	08 350	08 386	08 422	08 458	08 493	08 529	08 565	08 600
122	08 636	08 672	08 707	08 743	08 778	08 814	08 849	08 884	08 920	08 955
123	08 991	09 026	09 061	09 096	09 132	09 167	09 202	09 237	09 272	09 307
124	09 342	09 377	09 412	09 447	09 482	09 517	09 552	09 587	09 621	09 656
125	09 691	09 726	09 760	09 795	09 830	09 864	09 899	09 934	09 968	10 003
126	10 037	10 072	10 106	10 140	10 175	10 209	10 243	10 278	10 312	10 346
127	10 380	10 415	10 449	10 483	10 517	10 551	10 585	10 619	10 653	10 687
128	10 721	10 755	10 789	10 823	10 857	10 890	10 924	10 958	10 992	11 025
129	11 059	11 093	11 126	11 160	11 193	11 227	11 261	11 294	11 327	11 361
130	11 394	11 428	11 461	11 494	11 528	11 561	11 594	11 628	11 661	11 694
131	11 727	11 760	11 793	11 826	11 860	11 893	11 926	11 959	11 992	12 024
132	12 057	12 090	12 123	12 156	12 189	12 222	12 254	12 287	12 320	12 352
133	12 385	12 418	12 450	12 483	12 516	12 548	12 581	12 613	12 646	12 678
134	12 710	12 743	12 775	12 808	12 840	12 872	12 905	12 937	12 969	13 001
135	13 033	13 066	13 098	13 130	13 162	13 194	13 226	13 258	13 290	13 322
136	13 354	13 386	13 418	13 450	13 481	13 513	13 545	13 577	13 609	13 640
137	13 672	13 704	13 735	13 767	13 799	13 830	13 862	13 893	13 925	13 956
138	13 988	14 019	14 051	14 082	14 114	14 145	14 176	14 208	14 239	14 270
139	14 301	14 333	14 364	14 395	14 426	14 457	14 489	14 520	14 551	14 582
140	14 613	14 644	14 675	14 706	14 737	14 768	14 799	14 829	14 860	14 891
141	14 922	14 953	14 983	15 014	15 045	15 076	15 106	15 137	15 168	15 198
142	15 229	15 259	15 290	15 320	15 351	15 381	15 412	15 442	15 473	15 503
143	15 534	15 564	15 594	15 625	15 655	15 685	15 715	15 746	15 776	15 806
144	15 836	15 866	15 897	15 927	15 957	15 987	16 017	16 047	16 077	16 107
145	16 137	16 167	16 197	16 227	16 256	16 286	16 316	16 346	16 376	16 406
146	16 435	16 465	16 495	16 524	16 554	16 584	16 613	16 643	16 673	16 702
147	16 732	16 761	16 791	16 820	16 850	16 879	16 909	16 938	16 967	16 997
148	17 026	17 056	17 085	17 114	17 143	17 173	17 202	17 231	17 260	17 289
149	17 319	17 348	17 377	17 406	17 435	17 464	17 493	17 522	17 551	17 580
No.	0	1	2	3	4	5	6	7	8	9

100–149

[1] This table taken with permission from Riggleman and Frisbee, "Business Statistics," McGraw-Hill Book Company, Inc., New York, 1932.

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

150–199

No.	0	1	2	3	4	5	6	7	8	9
150	17 609	17 638	17 667	17 696	17 725	17 754	17 782	17 811	17 840	17 869
151	17 898	17 926	17 955	17 984	18 013	18 041	18 070	18 099	18 127	18 156
152	18 184	18 213	18 241	18 270	18 298	18 327	18 355	18 384	18 412	18 441
153	18 469	18 498	18 526	18 554	18 583	18 611	18 639	18 667	18 696	18 724
154	18 752	18 780	18 808	18 837	18 865	18 893	18 921	18 949	18 977	19 005
155	19 033	19 061	19 089	19 117	19 145	19 173	19 201	19 229	19 257	19 285
156	19 312	19 340	19 368	19 396	19 424	19 451	19 479	19 507	19 535	19 562
157	19 590	19 618	19 645	19 673	19 700	19 728	19 756	19 783	19 811	19 838
158	19 866	19 893	19 921	19 948	19 976	20 003	20 030	20 058	20 085	20 112
159	20 140	20 167	20 194	20 222	20 249	20 276	20 303	20 330	20 358	20 385
160	20 412	20 439	20 466	20 493	20 520	20 548	20 575	20 602	20 629	20 656
161	20 683	20 710	20 737	20 763	20 790	20 817	20 844	20 871	20 898	20 925
162	20 952	20 978	21 005	21 032	21 059	21 085	21 112	21 139	21 165	21 192
163	21 219	21 245	21 272	21 299	21 325	21 352	21 378	21 405	21 431	21 458
164	21 484	21 511	21 537	21 564	21 590	21 617	21 643	21 669	21 696	21 722
165	21 748	21 775	21 801	21 827	21 854	21 880	21 906	21 932	21 958	21 985
166	22 011	22 037	22 063	22 089	22 115	22 141	22 167	22 194	22 220	22 246
167	22 272	22 298	22 324	22 350	22 376	22 401	22 427	22 453	22 479	22 505
168	22 531	22 557	22 583	22 608	22 634	22 660	22 686	22 712	22 737	22 763
169	22 789	22 814	22 840	22 866	22 891	22 917	22 943	22 968	22 994	23 019
170	23 045	23 070	23 096	23 121	23 147	23 172	23 198	23 223	23 249	23 274
171	23 300	23 325	23 350	23 376	23 401	23 426	23 452	23 477	23 502	23 528
172	23 553	23 578	23 603	23 629	23 654	23 679	23 704	23 729	23 754	23 779
173	23 805	23 830	23 855	23 880	23 905	23 930	23 955	23 980	24 005	24 030
174	24 055	24 080	24 105	24 130	24 155	24 180	24 204	24 229	24 254	24 279
175	24 304	24 329	24 353	24 378	24 403	24 428	24 452	24 477	24 502	24 527
176	24 551	24 576	24 601	24 625	24 650	24 674	24 699	24 724	24 748	24 773
177	24 797	24 822	24 846	24 871	24 895	24 920	24 944	24 969	24 993	25 018
178	25 042	25 066	25 091	25 115	25 139	25 164	25 188	25 212	25 237	25 261
179	25 285	25 310	25 334	25 358	25 382	25 406	25 431	25 455	25 479	25 503
180	25 527	25 551	25 575	25 600	25 624	25 648	25 672	25 696	25 720	25 744
181	25 768	25 792	25 816	25 840	25 864	25 888	25 912	25 935	25 959	25 983
182	26 007	26 031	26 055	26 079	26 102	26 126	26 150	26 174	26 198	26 221
183	26 245	26 269	26 293	26 316	26 340	26 364	26 387	26 411	26 435	26 458
184	26 482	26 505	26 529	26 553	26 576	26 600	26 623	26 647	26 670	26 694
185	26 717	26 741	26 764	26 788	26 811	26 834	26 858	26 881	26 905	26 928
186	26 951	26 975	26 998	27 021	27 045	27 068	27 091	27 114	27 138	27 161
187	27 184	27 207	27 231	27 254	27 277	27 300	27 323	27 346	27 370	27 393
188	27 416	27 439	27 462	27 485	27 508	27 531	27 554	27 577	27 600	27 623
189	27 646	27 669	27 692	27 715	27 738	27 761	27 784	27 807	27 830	27 852
190	27 875	27 898	27 921	27 944	27 967	27 989	28 012	28 035	28 058	28 081
191	28 103	28 126	28 149	28 171	28 194	28 217	28 240	28 262	28 285	28 307
192	28 330	28 353	28 375	28 398	28 421	28 443	28 466	28 488	28 511	28 533
193	28 556	28 578	28 601	28 623	28 646	28 668	28 691	28 713	28 735	28 758
194	28 780	28 803	28 825	28 847	28 870	28 892	28 914	28 937	28 959	28 981
195	29 003	29 026	29 048	29 070	29 092	29 115	29 137	29 159	29 181	29 203
196	29 226	29 248	29 270	29 292	29 314	29 336	29 358	29 380	29 403	29 425
197	29 447	29 469	29 491	29 513	29 535	29 557	29 579	29 601	29 623	29 645
198	29 667	29 688	29 710	29 732	29 754	29 776	29 798	29 820	29 842	29 863
199	29 885	29 907	29 929	29 951	29 973	29 994	30 016	30 038	30 060	30 081
No.	0	1	2	3	4	5	6	7	8	9

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

200–249

No.	0	1	2	3	4	5	6	7	8	9
200	30 103	30 125	30 146	30 168	30 190	30 211	30 233	30 255	30 276	30 298
201	30 320	30 341	30 363	30 384	30 406	30 428	30 449	30 471	30 492	30 514
202	30 535	30 557	30 578	30 600	30 621	30 643	30 664	30 685	30 707	30 728
203	30 750	30 771	30 792	30 814	30 835	30 856	30 878	30 899	30 920	30 942
204	30 963	30 984	31 006	31 027	31 048	31 069	31 091	31 112	31 133	31 154
205	31 175	31 197	31 218	31 239	31 260	31 281	31 302	31 323	31 345	31 366
206	31 387	31 408	31 429	31 450	31 471	31 492	31 513	31 534	31 555	31 576
207	31 597	31 618	31 639	31 660	31 681	31 702	31 723	31 744	31 765	31 785
208	31 806	31 827	31 848	31 869	31 890	31 911	31 931	31 952	31 973	31 994
209	32 015	32 035	32 056	32 077	32 098	32 118	32 139	32 160	32 181	32 201
210	32 222	32 243	32 263	32 284	32 305	32 325	32 346	32 366	32 387	32 408
211	32 428	32 449	32 469	32 490	32 510	32 531	32 552	32 572	32 593	32 613
212	32 634	32 654	32 675	32 695	32 715	32 736	32 756	32 777	32 797	32 818
213	32 838	32 858	32 879	32 899	32 919	32 940	32 960	32 980	33 001	33 021
214	33 041	33 062	33 082	33 102	33 122	33 143	33 163	33 183	33 203	33 224
215	33 244	33 264	33 284	33 304	33 325	33 345	33 365	33 385	33 405	33 425
216	33 445	33 465	33 486	33 506	33 526	33 546	33 566	33 586	33 606	33 626
217	33 646	33 666	33 686	33 706	33 726	33 746	33 766	33 786	33 806	33 826
218	33 846	33 866	33 885	33 905	33 925	33 945	33 965	33 985	34 005	34 025
219	34 044	34 064	34 084	34 104	34 124	34 143	34 163	34 183	34 203	34 223
220	34 242	34 262	34 282	34 301	34 321	34 341	34 361	34 380	34 400	34 420
221	34 439	34 459	34 479	34 498	34 518	34 537	34 557	34 577	34 596	34 616
222	34 635	34 655	34 674	34 694	34 713	34 733	34 753	34 772	34 792	34 811
223	34 830	34 850	34 869	34 889	34 908	34 928	34 947	34 967	34 986	35 005
224	35 025	35 044	35 064	35 083	35 102	35 122	35 141	35 160	35 180	35 199
225	35 218	35 238	35 257	35 276	35 295	35 315	35 334	35 353	35 372	35 392
226	35 411	35 430	35 449	35 468	35 488	35 507	35 526	35 545	35 564	35 583
227	35 603	35 622	35 641	35 660	35 679	35 698	35 717	35 736	35 755	35 774
228	35 793	35 813	35 832	35 851	35 870	35 889	35 908	35 927	35 946	35 965
229	35 984	36 003	36 021	36 040	36 059	36 078	36 097	36 116	36 135	36 154
230	36 173	36 192	36 211	36 229	36 248	36 267	36 286	36 305	36 324	36 342
231	36 361	36 380	36 399	36 418	36 436	36 455	36 474	36 493	36 511	36 530
232	36 549	36 568	36 586	36 605	36 624	36 642	36 661	36 680	36 698	36 717
233	36 736	36 754	36 773	36 791	36 810	36 829	36 847	36 866	36 884	36 903
234	36 922	36 940	36 959	36 977	36 996	37 014	37 033	37 051	37 070	37 088
235	37 107	37 125	37 144	37 162	37 181	37 199	37 218	37 236	37 254	37 273
236	37 291	37 310	37 328	37 346	37 365	37 383	37 401	37 420	37 438	37 457
237	37 475	37 493	37 511	37 530	37 548	37 566	37 585	37 603	37 621	37 639
238	37 658	37 676	37 694	37 712	37 731	37 749	37 767	37 785	37 803	37 822
239	37 840	37 858	37 876	37 894	37 912	37 931	37 949	37 967	37 985	38 003
240	38 021	38 039	38 057	38 075	38 093	38 112	38 130	38 148	38 166	38 184
241	38 202	38 220	38 238	38 256	38 274	38 292	38 310	38 328	38 346	38 364
242	38 382	38 399	38 417	38 435	38 453	38 471	38 489	38 507	38 525	38 543
243	38 561	38 578	38 596	38 614	38 632	38 650	38 668	38 686	38 703	38 721
244	38 739	38 757	38 775	38 792	38 810	38 828	38 846	38 863	38 881	38 899
245	38 917	38 934	38 952	38 970	38 987	39 005	39 023	39 041	39 058	39 076
246	39 094	39 111	39 129	39 146	39 164	39 182	39 199	39 217	39 235	39 252
247	39 270	39 287	39 305	39 322	39 340	39 358	39 375	39 393	39 410	39 428
248	39 445	39 463	39 480	39 498	39 515	39 533	39 550	39 568	39 585	39 602
249	39 620	39 637	39 655	39 672	39 690	39 707	39 724	39 742	39 759	39 777
No.	0	1	2	3	4	5	6	7	8	9

Table A15. Five-place Common Logarithms of Numbers.—(*Continued*)

250–299

No.	0	1	2	3	4	5	6	7	8	9
250	39 794	39 811	39 829	39 846	39 863	39 881	39 898	39 915	39 933	39 950
251	39 967	39 985	40 002	40 019	40 037	40 054	40 071	40 088	40 106	40 123
252	40 140	40 157	40 175	40 192	40 209	40 226	40 243	40 261	40 278	40 295
253	40 312	40 329	40 346	40 364	40 381	40 398	40 415	40 432	40 449	40 466
254	40 483	40 500	40 518	40 535	40 552	40 569	40 586	40 603	40 620	40 637
255	40 654	40 671	40 688	40 705	40 722	40 739	40 756	40 773	40 790	40 807
256	40 824	40 841	40 858	40 875	40 892	40 909	40 926	40 943	40 960	40 976
257	40 993	41 010	41 027	41 044	41 061	41 078	41 095	41 111	41 128	41 145
258	41 162	41 179	41 196	41 212	41 229	41 246	41 263	41 280	41 296	41 313
259	41 330	41 347	41 363	41 380	41 397	41 414	41 430	41 447	41 464	41 481
260	41 497	41 514	41 531	41 547	41 564	41 581	41 597	41 614	41 631	41 647
261	41 664	41 681	41 697	41 714	41 731	41 747	41 764	41 780	41 797	41 814
262	41 830	41 847	41 863	41 880	41 896	41 913	41 929	41 946	41 963	41 979
263	41 996	42 012	42 029	42 045	42 062	42 078	42 095	42 111	42 127	42 144
264	42 160	42 177	42 193	42 210	42 226	42 243	42 259	42 275	42 292	42 308
265	42 325	42 341	42 357	42 374	42 390	42 406	42 423	42 439	42 455	42 472
266	42 488	42 504	42 521	42 537	42 553	42 570	42 586	42 602	42 619	42 635
267	42 651	42 667	42 684	42 700	42 716	42 732	42 749	42 765	42 781	42 797
268	42 813	42 830	42 846	42 862	42 878	42 894	42 911	42 927	42 943	42 959
269	42 975	42 991	43 008	43 024	43 040	43 056	43 072	43 088	43 104	43 120
270	43 136	43 152	43 169	43 185	43 201	43 217	43 233	43 249	43 265	43 281
271	43 297	43 313	43 329	43 345	43 361	43 377	43 393	43 409	43 425	43 441
272	43 457	43 473	43 489	43 505	43 521	43 537	43 553	43 569	43 584	43 600
273	43 616	43 632	43 648	43 664	43 680	43 696	43 712	43 727	43 743	43 759
274	43 775	43 791	43 807	43 823	43 838	43 854	43 870	43 886	43 902	43 917
275	43 933	43 949	43 965	43 981	43 996	44 012	44 028	44 044	44 059	44 075
276	44 091	44 107	43 122	44 138	44 154	44 170	44 185	44 201	44 217	44 232
277	44 248	44 264	44 279	44 295	44 311	44 326	44 342	44 358	44 373	44 389
278	44 404	44 420	44 436	44 451	44 467	44 483	44 498	44 514	44 529	44 545
279	44 560	44 576	44 592	44 607	44 623	44 638	44 654	44 669	44 685	44 700
280	44 716	44 731	44 747	44 762	44 778	44 793	44 809	44 824	44 840	44 855
281	44 871	44 886	44 902	44 917	44 932	44 948	44 963	44 979	44 994	45 010
282	45 025	45 040	45 056	45 071	45 086	45 102	45 117	45 133	45 148	45 163
283	45 179	45 194	45 209	45 225	45 240	45 255	45 271	45 286	45 301	45 317
284	45 332	45 347	45 362	45 378	45 393	45 408	45 423	45 439	45 454	45 469
285	45 484	45 500	45 515	45 530	45 545	45 561	45 576	45 591	45 606	45 621
286	45 637	45 652	45 667	45 682	45 697	45 712	45 728	45 743	45 758	45 773
287	45 788	45 803	45 818	45 834	45 849	45 864	45 879	45 894	45 909	45 924
288	45 939	45 954	45 969	45 984	46 000	46 015	46 030	46 045	46 060	46 075
289	46 090	46 105	46 120	46 135	46 150	46 165	46 180	46 195	46 210	46 225
290	46 240	46 255	46 270	46 285	46 300	46 315	46 330	46 345	46 359	46 374
291	46 389	46 404	46 419	46 434	46 449	46 464	46 479	46 494	46 509	46 523
292	46 538	46 553	46 568	46 583	46 598	46 613	46 627	46 642	46 657	46 672
293	46 687	46 702	46 716	46 731	46 746	46 761	46 776	46 790	46 805	46 820
294	46 835	46 850	46 864	46 879	46 894	46 909	46 923	46 938	46 953	46 967
295	46 982	46 997	47 012	47 026	47 041	47 056	47 070	47 085	47 100	47 114
296	47 129	47 144	47 159	47 173	47 188	47 202	47 217	47 232	47 246	47 261
297	47 276	47 290	47 305	47 319	47 334	47 349	47 363	47 378	47 392	47 407
298	47 422	47 436	47 451	47 465	47 480	47 494	47 509	47 524	47 538	47 553
299	47 567	47 582	47 596	47 611	47 625	47 640	47 654	47 669	47 683	47 698
No.	0	1	2	3	4	5	6	7	8	9

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

300–349

No.	0	1	2	3	4	5	6	7	8	9
300	47 712	47 727	47 741	47 756	47 770	47 784	47 799	47 813	47 828	47 842
301	47 857	47 871	47 885	47 900	47 914	47 929	47 943	47 958	47 972	47 986
302	48 001	48 015	48 029	48 044	48 058	48 073	48 087	48 101	48 116	48 130
303	48 144	48 159	48 173	48 187	48 202	48 216	48 230	48 244	48 259	48 273
304	48 287	48 302	48 316	48 330	48 344	48 359	48 373	48 387	48 401	48 416
305	48 430	48 444	48 458	48 473	48 487	48 501	48 515	48 530	48 544	48 558
306	48 572	48 586	48 601	48 615	48 629	48 643	48 657	48 671	48 686	48 700
307	48 714	48 728	48 742	48 756	48 770	48 785	48 799	48 813	48 827	48 841
308	48 855	48 869	48 883	48 897	48 911	48 926	48 940	48 954	48 968	48 982
309	48 996	49 010	49 024	49 038	49 052	49 066	49 080	49 094	49 108	49 122
310	49 136	49 150	49 164	49 178	49 192	49 206	49 220	49 234	49 248	49 262
311	49 276	49 290	49 304	49 318	49 332	49 346	49 360	49 374	49 388	49 402
312	49 415	49 429	49 443	49 457	49 471	49 485	49 499	49 513	49 527	49 541
313	49 554	49 568	49 582	49 596	49 610	49 624	49 638	49 651	49 665	49 679
314	49 693	49 707	49 721	49 734	49 748	49 762	49 776	49 790	49 803	49 817
315	49 831	49 845	49 859	49 872	49 886	49 900	49 914	49 927	49 941	49 955
316	49 969	49 982	49 996	50 010	50 024	50 037	50 051	50 065	50 079	50 092
317	50 106	50 120	50 133	50 147	50 161	50 174	50 188	50 202	50 215	50 229
318	50 243	50 256	50 270	50 284	50 297	50 311	50 325	50 338	50 352	50 365
319	50 379	50 393	50 406	50 420	50 433	50 447	50 461	50 474	50 488	50 501
320	50 515	50 529	50 542	50 556	50 569	50 583	50 596	50 610	50 623	50 637
321	50 651	50 664	50 678	50 691	50 705	50 718	50 732	50 745	50 759	50 772
322	50 786	50 799	50 813	50 826	50 840	50 853	50 866	50 880	50 893	50 907
323	50 920	50 934	50 947	50 961	50 974	50 987	51 001	51 014	51 028	51 041
324	51 055	51 068	51 081	51 095	51 108	51 121	51 135	51 148	51 162	51 175
325	51 188	51 202	51 215	51 228	51 242	51 255	51 268	51 282	51 295	51 308
326	51 322	51 335	51 348	51 362	51 375	51 388	51 402	51 415	51 428	51 441
327	51 455	51 468	51 481	51 495	51 508	51 521	51 534	51 548	51 561	51 574
328	51 587	51 601	51 614	51 627	51 640	51 654	51 667	51 680	51 693	51 706
329	51 720	51 733	51 746	51 759	51 772	51 786	51 799	51 812	51 825	51 838
330	51 851	51 865	51 878	51 891	51 904	51 917	51 930	51 943	51 957	51 970
331	51 983	51 996	52 009	52 022	52 035	52 048	52 061	52 075	52 088	52 101
332	52 114	52 127	52 140	52 153	52 166	52 179	52 192	52 205	52 218	52 231
333	52 244	52 257	52 270	52 284	52 297	52 310	52 323	52 336	52 349	52 362
334	52 375	52 388	52 401	52 414	52 427	52 440	52 453	52 466	52 479	52 492
335	52 504	52 517	52 530	52 543	52 556	52 569	52 582	52 595	52 608	52 621
336	52 634	52 647	52 660	52 673	52 686	52 699	52 711	52 724	52 737	52 750
337	52 763	52 776	52 789	52 802	52 815	52 827	52 840	52 853	52 866	52 879
338	52 892	52 905	52 917	52 930	52 943	52 956	52 969	52 982	52 994	53 007
339	53 020	53 033	53 046	53 058	53 071	53 084	53 097	53 110	53 122	53 135
340	53 148	53 161	53 173	53 186	53 199	53 212	53 224	53 237	53 250	53 263
341	53 275	53 288	53 301	53 314	53 326	53 339	53 352	53 364	53 377	53 390
342	53 403	53 415	53 428	53 441	53 453	53 466	53 479	53 491	53 504	53 517
343	53 529	53 542	53 555	53 567	53 580	53 593	53 605	53 618	53 631	53 643
344	53 656	53 668	53 681	53 694	53 706	53 719	53 732	53 744	53 757	53 769
345	53 782	53 794	53 807	53 820	53 832	53 845	53 857	53 870	53 882	53 895
346	53 908	53 920	53 933	53 945	53 958	53 970	53 983	53 995	54 008	54 020
347	54 033	54 045	54 058	54 070	54 083	54 095	54 108	54 120	54 133	54 145
348	54 158	54 170	54 183	54 195	54 208	54 220	54 233	54 245	54 258	54 270
349	54 283	54 295	54 307	54 320	54 332	54 345	54 357	54 370	54 382	54 394
No.	0	1	2	3	4	5	6	7	8	9

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

350–399

No.	0	1	2	3	4	5	6	7	8	9
350	54 407	54 419	54 432	54 444	54 456	54 469	54 481	54 494	54 506	54 518
351	54 531	54 543	54 555	54 568	54 580	54 593	54 605	54 617	54 630	54 642
352	54 654	54 667	54 679	54 691	54 704	54 716	54 728	54 741	54 753	54 765
353	54 777	54 790	54 802	54 814	54 827	54 839	54 851	54 864	54 876	54 888
354	54 900	54 913	54 925	54 937	54 949	54 962	54 974	54 986	54 998	55 011
355	55 023	55 035	55 047	55 060	55 072	55 084	55 096	55 108	55 121	55 133
356	55 145	55 157	55 169	55 182	55 194	55 206	55 218	55 230	55 242	55 255
357	55 267	55 279	55 291	55 303	55 315	55 328	55 340	55 352	55 364	55 376
358	55 388	55 400	55 413	55 425	55 437	55 449	55 461	55 473	55 485	55 497
359	55 509	55 522	55 534	55 546	55 558	55 570	55 582	55 594	55 606	55 618
360	55 630	55 642	55 654	55 666	55 678	55 691	55 703	55 715	55 727	55 739
361	55 751	55 763	55 775	55 787	55 799	55 811	55 823	55 835	55 847	55 859
362	55 871	55 883	55 895	55 907	55 919	55 931	55 943	55 955	55 967	55 979
363	55 991	56 003	56 015	56 027	56 038	56 050	56 062	56 074	56 086	56 098
364	56 110	56 122	56 134	56 146	56 158	56 170	56 182	56 194	56 205	56 217
365	56 229	56 241	56 253	56 265	56 277	56 289	56 301	56 312	56 324	56 336
366	56 348	56 360	56 372	56 384	56 396	56 407	56 419	56 431	56 443	56 455
367	56 467	56 478	56 490	56 502	56 514	56 526	56 538	56 549	56 561	56 573
368	56 585	56 597	56 608	56 620	56 632	56 644	56 656	56 667	56 679	56 691
369	56 703	56 714	56 726	56 738	56 750	56 761	56 773	56 785	56 797	56 808
370	56 820	56 832	56 844	56 855	56 867	56 879	56 891	56 902	56 914	56 926
371	56 937	56 949	56 961	56 972	56 984	56 996	57 008	57 019	57 031	57 043
372	57 054	57 066	57 078	57 089	57 101	57 113	57 124	57 136	57 148	57 159
373	57 171	57 183	57 194	57 206	57 217	57 229	57 241	57 252	57 264	57 276
374	57 287	57 299	57 310	57 322	57 334	57 345	57 357	57 368	57 380	57 392
375	57 403	57 415	57 426	57 438	57 449	57 461	57 473	57 484	57 496	57 507
376	57 519	57 530	57 542	57 553	57 565	57 576	57 588	57 600	57 611	57 623
377	57 634	57 646	57 657	57 669	57 680	57 692	57 703	57 715	57 726	57 738
378	57 749	57 761	57 772	57 784	57 795	57 807	57 818	57 830	57 841	57 852
379	57 864	57 875	57 887	57 898	57 910	57 921	57 933	57 944	57 955	57 967
380	57 978	57 990	58 001	58 013	58 024	58 035	58 047	58 058	58 070	58 081
381	58 092	58 104	58 115	58 127	58 138	58 149	58 161	58 172	58 184	58 195
382	58 206	58 218	58 229	58 240	58 252	58 263	58 274	58 286	58 297	58 309
383	58 320	58 331	58 343	58 354	58 365	58 377	58 388	58 399	58 410	58 422
384	58 433	58 444	58 456	58 467	58 478	58 490	58 501	58 512	58 524	58 535
385	58 546	58 557	58 569	58 580	58 591	58 602	58 614	58 625	58 636	58 647
386	58 659	58 670	58 681	58 692	58 704	58 715	58 726	58 737	58 749	58 760
387	58 771	58 782	58 794	58 805	58 816	58 827	58 838	58 850	58 861	58 872
388	58 883	58 894	58 906	58 917	58 928	58 939	58 950	58 961	58 973	58 984
389	58 995	59 006	59 017	59 028	59 040	59 051	59 062	59 073	59 084	59 095
390	59 106	59 118	59 129	59 140	59 151	59 162	59 173	59 184	59 195	59 207
391	59 218	59 229	59 240	59 251	59 262	59 273	59 284	59 295	59 306	59 318
392	59 329	59 340	59 351	59 362	59 373	59 384	59 395	59 406	59 417	59 428
393	59 439	59 450	59 461	59 472	59 483	59 494	59 506	59 517	59 528	59 539
394	59 550	59 561	59 572	59 583	59 594	59 605	59 616	59 627	59 638	59 649
395	59 660	59 671	59 682	59 693	59 704	59 715	59 726	59 737	59 748	59 759
396	59 770	59 780	59 791	59 802	59 813	59 824	59 835	59 846	59 857	59 868
397	59 879	59 890	59 901	59 912	59 923	59 934	59 945	59 956	59 966	59 977
398	59 988	59 999	60 010	60 021	60 032	60 043	60 054	60 065	60 076	60 086
399	60 097	60 108	60 119	60 130	60 141	60 152	60 163	60 173	60 184	60 195
No.	0	1	2	3	4	5	6	7	8	9

Table A15. Five-place Common Logarithms of Numbers.—(*Continued*)

400–449

No.	0	1	2	3	4	5	6	7	8	9
400	60 206	60 217	60 228	60 239	60 249	60 260	60 271	60 282	60 293	60 304
401	60 314	60 325	60 336	60 347	60 358	60 369	60 379	60 390	60 401	60 412
402	60 423	60 433	60 444	60 455	60 466	60 477	60 487	60 498	60 509	60 520
403	60 531	60 541	60 552	60 563	60 574	60 584	60 595	60 606	60 617	60 627
404	60 638	60 649	60 660	60 670	60 681	60 692	60 703	60 713	60 724	60 735
405	60 746	60 756	60 767	60 778	60 788	60 799	60 810	60 821	60 831	60 842
406	60 853	60 863	60 874	60 885	60 895	60 906	60 917	60 927	60 938	60 949
407	60 959	60 970	60 981	60 991	61 002	61 013	61 023	61 034	61 045	61 055
408	61 066	61 077	61 087	61 098	61 109	61 119	61 130	61 140	61 151	61 162
409	61 172	61 183	61 194	61 204	61 215	61 225	61 236	61 247	61 257	61 268
410	61 278	61 289	61 300	61 310	61 321	61 331	61 342	61 352	61 363	61 374
411	61 384	61 395	61 405	61 416	61 426	61 437	61 448	61 458	61 469	61 479
412	61 490	61 500	61 511	61 521	61 532	61 542	61 553	61 563	61 574	61 584
413	61 595	61 606	61 616	61 627	61 637	61 648	61 658	61 669	61 679	61 690
414	61 700	61 711	61 721	61 731	61 742	61 752	61 763	61 773	61 784	61 794
415	61 805	61 815	61 826	61 836	61 847	61 857	61 868	61 878	61 888	61 899
416	61 909	61 920	61 930	61 941	61 951	61 962	61 972	61 982	61 993	62 003
417	62 014	62 024	62 034	62 045	62 055	62 066	62 076	62 086	62 097	62 107
418	62 118	62 128	62 138	62 149	62 159	62 170	62 180	62 190	62 201	62 211
419	62 221	62 232	62 242	62 252	62 263	62 273	62 284	62 294	62 304	62 315
420	62 325	62 335	62 346	62 356	62 366	62 377	62 387	62 397	62 408	62 418
421	62 428	62 439	62 449	62 459	62 469	62 480	62 490	62 500	62 511	62 521
422	62 531	62 542	62 552	62 562	62 572	62 583	62 593	62 603	62 613	62 624
423	62 634	62 644	62 655	62 665	62 675	62 685	62 696	62 706	62 716	62 726
424	62 737	62 747	62 757	62 767	62 778	62 788	62 798	62 808	62 818	62 829
425	62 839	62 849	62 859	62 870	62 880	62 890	62 900	62 910	62 921	62 931
426	62 941	62 951	62 961	62 972	62 982	62 992	63 002	63 012	63 022	63 033
427	63 043	63 053	63 063	63 073	63 083	63 094	63 104	63 114	63 124	63 134
428	63 144	63 155	63 165	63 175	63 185	63 195	63 205	63 215	63 225	63 236
429	63 246	63 256	63 266	63 276	63 286	63 296	63 306	63 317	63 327	63 337
430	63 347	63 357	63 367	63 377	63 387	63 397	63 407	63 417	63 428	63 438
431	63 448	63 458	63 468	63 478	63 488	63 498	63 508	63 518	63 528	63 538
432	63 548	63 558	63 568	63 579	63 589	63 599	63 609	63 619	63 629	63 639
433	63 649	63 659	63 669	63 679	63 689	63 699	63 709	63 719	63 729	63 739
434	63 749	63 759	63 769	63 779	63 789	63 799	63 809	63 819	63 829	63 839
435	63 849	63 859	63 869	63 879	63 889	63 899	63 909	63 919	63 929	63 939
436	63 949	63 959	63 969	63 979	63 988	63 998	64 008	64 018	64 028	64 038
437	64 048	64 058	64 068	64 078	64 088	64 098	64 108	64 118	64 128	64 137
438	64 147	64 157	64 167	64 177	64 187	64 197	64 207	64 217	64 227	64 237
439	64 246	64 256	64 266	64 276	64 286	64 296	64 306	64 316	64 326	64 335
440	64 345	64 355	64 365	64 375	64 385	64 395	64 404	64 414	64 424	64 434
441	64 444	64 454	64 464	64 473	64 483	64 493	64 503	64 513	64 523	64 532
442	64 542	64 552	64 562	64 572	64 582	64 591	64 601	64 611	64 621	64 631
443	64 640	64 650	64 660	64 670	64 680	64 689	64 699	64 709	64 719	64 729
444	64 738	64 748	64 758	64 768	64 777	64 787	64 797	64 807	64 816	64 826
445	64 836	64 846	64 856	64 865	64 875	64 885	64 895	64 904	64 914	64 924
446	64 933	64 943	64 953	64 963	64 972	64 982	64 992	65 002	65 011	65 021
447	65 031	65 040	65 050	65 060	65 070	65 079	65 089	65 099	65 108	65 118
448	65 128	65 137	65 147	65 157	65 167	65 176	65 186	65 196	65 205	65 215
449	65 225	65 234	65 244	65 254	65 263	65 273	65 283	65 292	65 302	65 312
No.	0	1	2	3	4	5	6	7	8	9

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

450–499

No.	0	1	2	3	4	5	6	7	8	9
450	65 321	65 331	65 341	65 350	65 360	65 369	65 379	65 389	65 398	65 408
451	65 418	65 427	65 437	65 447	65 456	65 466	65 475	65 485	65 495	65 504
452	65 514	65 523	65 533	65 543	65 552	65 562	65 571	65 581	65 591	65 600
453	65 610	65 619	65 629	65 639	65 648	65 658	65 667	65 677	65 686	65 696
454	65 706	65 715	65 725	65 734	65 744	65 753	65 763	65 772	65 782	65 792
455	65 801	65 811	65 820	65 830	65 839	65 849	65 858	65 868	65 877	65 887
456	65 896	65 906	65 916	65 925	65 935	65 944	65 954	65 963	65 973	65 982
457	65 992	66 001	66 011	66 020	66 030	66 039	66 049	66 058	66 068	66 077
458	66 087	66 096	66 106	66 115	66 124	66 134	66 143	66 153	66 162	66 172
459	66 181	66 191	66 200	66 210	66 219	66 229	66 238	66 247	66 257	66 266
460	66 276	66 285	66 295	66 304	66 314	66 323	66 332	66 342	66 351	66 361
461	66 370	66 380	66 389	66 398	66 408	66 417	66 427	66 436	66 445	66 455
462	66 464	66 474	66 483	66 492	66 502	66 511	66 521	66 530	66 539	66 549
463	66 558	66 567	66 577	66 586	66 596	66 605	66 614	66 624	66 633	66 642
464	66 652	66 661	66 671	66 680	66 689	66 699	66 708	66 717	66 727	66 736
465	66 745	66 755	66 764	66 773	66 783	66 792	66 801	66 811	66 820	66 829
466	66 839	66 848	66 857	66 867	66 876	66 885	66 894	66 904	66 913	66 922
467	66 932	66 941	66 950	66 960	66 969	66 978	66 987	66 997	67 006	67 015
468	67 025	67 034	67 043	67 052	67 062	67 071	67 080	67 089	67 099	67 108
469	67 117	67 127	67 136	67 145	67 154	67 164	67 173	67 182	67 191	67 201
470	67 210	67 219	67 228	67 237	67 247	67 256	67 265	67 274	67 284	67 293
471	67 302	67 311	67 321	67 330	67 339	67 348	67 357	67 367	67 376	67 385
472	67 394	67 403	67 413	67 422	67 431	67 440	67 449	67 459	67 468	67 477
473	67 486	67 495	67 504	67 514	67 523	67 532	67 541	67 550	67 560	67 569
474	67 578	67 587	67 596	67 605	67 614	67 624	67 633	67 642	67 651	67 660
475	67 669	67 679	67 688	67 697	67 706	67 715	67 724	67 733	67 742	67 752
476	67 761	67 770	67 779	67 788	67 797	67 806	67 815	67 825	67 834	67 843
477	67 852	67 861	67 870	67 879	67 888	67 897	67 906	67 916	67 925	67 934
478	67 943	67 952	67 961	67 970	67 979	67 988	67 997	68 006	68 015	68 024
479	68 034	68 043	68 052	68 061	68 070	68 079	68 088	68 097	68 106	68 115
480	68 124	68 133	68 142	68 151	68 160	68 169	68 178	68 187	68 196	68 205
481	68 215	68 224	68 233	68 242	68 251	68 260	68 269	68 278	68 287	68 296
482	68 305	68 314	68 323	68 332	68 341	68 350	68 359	68 368	68 377	68 386
483	68 395	68 404	68 413	68 422	68 431	68 440	68 449	68 458	68 467	68 476
484	68 485	68 494	68 502	68 511	68 520	68 529	68 538	68 547	68 556	68 565
485	68 574	68 583	68 592	68 601	68 610	68 619	68 628	68 637	68 646	68 655
486	68 664	68 673	68 681	68 690	68 699	68 708	68 717	68 726	68 735	68 744
487	68 753	68 762	68 771	68 780	68 789	68 797	68 806	68 815	68 824	68 833
488	68 842	68 851	68 860	68 869	68 878	68 886	68 895	68 904	68 913	68 922
489	68 931	68 940	68 949	68 958	68 966	68 975	68 984	68 993	69 002	69 011
490	69 020	69 028	69 037	69 046	69 055	69 064	69 073	69 082	69 090	69 099
491	69 108	69 117	69 126	69 135	69 144	69 152	69 161	69 170	69 179	69 188
492	69 197	69 205	69 214	69 223	69 232	69 241	69 249	69 258	69 267	69 276
493	69 285	69 294	69 302	69 311	69 320	69 329	69 338	69 346	69 355	69 364
494	69 373	69 381	69 390	69 399	69 408	69 417	69 425	69 434	69 443	69 452
495	69 461	69 460	69 478	69 487	69 496	69 504	69 513	69 522	69 531	69 539
496	69 548	69 557	69 566	69 574	69 583	69 592	69 601	69 609	69 618	69 627
497	69 636	69 644	69 653	69 662	69 671	69 679	69 688	69 697	69 705	69 714
498	69 723	69 732	69 740	69 749	69 758	69 767	69 775	69 784	69 793	69 801
499	69 810	69 819	69 827	69 836	69 845	69 854	69 862	69 871	69 880	69 888
No.	0	1	2	3	4	5	6	7	8	9

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

500–549

No.	0	1	2	3	4	5	6	7	8	9
500	69 897	69 906	69 914	69 923	69 932	69 940	69 949	69 958	69 966	69 975
501	69 984	69 992	70 001	70 010	70 018	70 027	70 036	70 044	70 053	70 062
502	70 070	70 079	70 088	70 096	70 105	70 114	70 122	70 131	70 140	70 148
503	70 157	70 165	70 174	70 183	70 191	70 200	70 209	70 217	70 226	70 234
504	70 243	70 252	70 260	70 269	70 278	70 286	70 295	70 303	70 312	70 321
505	70 329	70 338	70 346	70 355	70 364	70 372	70 381	70 389	70 398	70 406
506	70 415	70 424	70 432	70 441	70 449	70 458	70 467	70 475	70 484	70 492
507	70 501	70 509	70 518	70 526	70 535	70 544	70 552	70 561	70 569	70 578
508	70 586	70 595	70 603	70 612	70 621	70 629	70 638	70 646	70 655	70 663
509	70 672	70 680	70 689	70 697	70 706	70 714	70 723	70 731	70 740	70 749
510	70 757	70 766	70 774	70 783	70 791	70 800	70 808	70 817	70 825	70 834
511	70 842	70 851	70 859	70 868	70 876	70 885	70 893	70 902	70 910	70 919
512	70 927	70 935	70 944	70 952	70 961	70 969	70 978	70 986	70 995	71 003
513	71 012	71 020	71 029	71 037	71 046	71 054	71 063	71 071	71 079	71 088
514	71 096	71 105	71 113	71 122	71 130	71 139	71 147	71 155	71 164	71 172
515	71 181	71 189	71 198	71 206	71 214	71 223	71 231	71 240	71 248	71 257
516	71 265	71 273	71 282	71 290	71 299	71 307	71 315	71 324	71 332	71 341
517	71 349	71 357	71 366	71 374	71 383	71 391	71 399	71 408	71 416	71 425
518	71 433	71 441	71 450	71 458	71 466	71 475	71 483	71 492	71 500	71 508
519	71 517	71 525	71 533	71 542	71 550	71 559	71 567	71 575	71 584	71 592
520	71 600	71 609	71 617	71 625	71 634	71 642	71 650	71 659	71 667	71 675
521	71 684	71 692	71 700	71 709	71 717	71 725	71 734	71 742	71 750	71 759
522	71 767	71 775	71 784	71 792	71 800	71 809	71 817	71 825	71 834	71 842
523	71 850	71 858	71 867	71 875	71 883	71 892	71 900	71 908	71 917	71 925
524	71 933	71 941	71 950	71 958	71 966	71 975	71 983	71 991	71 999	72 008
525	72 016	72 024	72 032	72 041	72 049	72 057	72 066	72 074	72 082	72 090
526	72 099	72 107	72 115	72 123	72 132	72 140	72 148	72 156	72 165	72 173
527	72 181	72 189	72 198	72 206	72 214	72 222	72 230	72 239	72 247	72 255
528	72 263	72 272	72 280	72 288	72 296	72 304	72 313	72 321	72 329	72 337
529	72 346	72 354	72 362	72 370	72 378	72 387	72 395	72 403	72 411	72 419
530	72 428	72 436	72 444	72 452	72 460	72 469	72 477	72 485	72 493	72 501
531	72 509	72 518	72 526	72 534	72 542	72 550	72 558	72 567	72 575	72 583
532	72 591	72 599	72 607	72 616	72 624	72 632	72 640	72 648	72 656	72 665
533	72 673	72 681	72 689	72 697	72 705	72 713	72 722	72 730	72 738	72 746
534	72 754	72 762	72 770	72 779	72 787	72 795	72 803	72 811	72 819	72 827
535	72 835	72 843	72 852	72 860	72 868	72 876	72 884	72 892	72 900	72 908
536	72 916	72 925	72 933	72 941	72 949	72 957	72 965	72 973	72 981	72 989
537	72 997	73 006	73 014	73 022	73 030	73 038	73 046	73 054	73 062	73 070
538	73 078	73 086	73 094	73 102	73 111	73 119	73 127	73 135	73 143	73 151
539	73 159	73 167	73 175	73 183	73 191	73 199	73 207	73 215	73 223	73 231
540	73 239	73 247	73 255	73 263	73 272	73 280	73 288	73 296	73 304	73 312
541	73 320	73 328	73 336	73 344	73 352	73 360	73 368	73 376	73 384	73 392
542	73 400	73 408	73 416	73 424	73 432	73 440	73 448	73 456	73 464	73 472
543	73 480	73 488	73 496	73 504	73 512	73 520	73 528	73 536	73 544	73 552
544	73 560	73 568	73 576	73 584	73 592	73 600	73 608	73 616	73 624	73 632
545	73 640	73 648	73 656	73 664	73 672	73 679	73 687	73 695	73 703	73 711
546	73 719	73 727	73 735	73 743	73 751	73 759	73 767	73 775	73 783	73 791
547	73 799	73 807	73 815	73 823	73 830	73 838	73 846	73 854	73 862	73 870
548	73 878	73 886	73 894	73 902	73 910	73 918	73 926	73 933	73 941	73 949
549	73 957	73 965	73 973	73 981	73 989	73 997	74 005	74 013	74 020	74 028

No. 0 1 2 3 4 5 6 7 8 9

MMON LOGARITHMS OF NUMBERS.—(*Continued*)

550–599

No.	0	1	2	3	4	5	6	7	8	9
550	74 036	74 044	74 052	74 060	74 068	74 076	74 084	74 092	74 099	74 107
551	74 115	74 123	74 131	74 139	74 147	74 155	74 162	74 170	74 178	74 186
552	74 194	74 202	74 210	74 218	74 225	74 233	74 241	74 249	74 257	74 265
553	74 273	74 280	74 288	74 296	74 304	74 312	74 320	74 327	74 335	74 343
554	74 351	74 359	74 367	74 374	74 382	74 390	74 398	74 406	74 414	74 421
555	74 429	74 437	74 445	74 453	74 461	74 468	74 476	74 484	74 492	74 500
556	74 507	74 515	74 523	74 531	74 539	74 547	74 554	74 562	74 570	74 578
557	74 586	74 593	74 601	74 609	74 617	74 624	74 632	74 640	74 648	74 656
558	74 663	74 671	74 679	74 687	74 695	74 702	74 710	74 718	74 726	74 733
559	74 741	74 749	74 757	74 764	74 772	74 780	74 788	74 796	74 803	74 811
560	74 819	74 827	74 834	74 842	74 850	74 858	74 865	74 873	74 881	74 889
561	74 896	74 904	74 912	74 920	74 927	74 935	74 943	74 950	74 958	74 966
562	74 974	74 981	74 989	74 997	75 005	75 012	75 020	75 028	75 035	75 043
563	75 051	75 059	75 066	75 074	75 082	75 089	75 097	75 105	75 113	75 120
564	75 128	75 136	75 143	75 151	75 159	75 166	75 174	75 182	75 189	75 197
565	75 205	75 213	75 220	75 228	75 236	75 243	75 251	75 259	75 266	75 274
566	75 282	75 289	75 297	75 305	75 312	75 320	75 328	75 335	75 343	75 351
567	75 358	75 366	75 374	75 381	75 389	75 397	75 404	75 412	75 420	75 427
568	75 435	75 442	75 450	75 458	75 465	75 473	75 481	75 488	75 496	75 504
569	75 511	75 519	75 526	75 534	75 542	75 549	75 557	75 565	75 572	75 580
570	75 587	75 595	75 603	75 610	75 618	75 626	75 633	75 641	75 648	75 656
571	75 664	75 671	75 679	75 686	75 694	75 702	75 709	75 717	75 724	75 732
572	75 740	75 747	75 755	75 762	75 770	75 778	75 785	75 793	75 800	75 808
573	75 815	75 823	75 831	75 838	75 846	75 853	75 861	75 868	75 876	75 884
574	75 891	75 899	75 906	75 914	75 921	75 929	75 937	75 944	75 952	75 959
575	75 967	75 974	75 982	75 989	75 997	76 005	76 012	76 020	76 027	76 035
576	76 042	76 050	76 057	76 065	76 072	76 080	76 087	76 095	76 103	76 110
577	76 118	76 125	76 133	76 140	76 148	76 155	76 163	76 170	76 178	76 185
578	76 193	76 200	76 208	76 215	76 223	76 230	76 238	76 245	76 253	76 260
579	76 268	76 275	76 283	76 290	76 298	76 305	76 313	76 320	76 328	76 335
580	76 343	76 350	76 358	76 365	76 373	76 380	76 388	76 395	76 403	76 410
581	76 418	76 425	76 433	76 440	76 448	76 455	76 462	76 470	76 477	76 485
582	76 492	76 500	76 507	76 515	76 522	76 530	76 537	76 545	76 552	76 559
583	76 567	76 574	76 582	76 589	76 597	76 604	76 612	76 619	76 626	76 634
584	76 641	76 649	76 656	76 664	76 671	76 678	76 686	76 693	76 701	76 708
585	76 716	76 723	76 730	76 738	76 745	76 753	76 760	76 768	76 775	76 782
586	76 790	76 797	76 805	76 812	76 819	76 827	76 834	76 842	76 849	76 856
587	76 864	76 871	76 879	76 886	76 893	76 901	76 908	76 916	76 923	76 930
588	76 938	76 945	76 953	76 960	76 967	76 975	76 982	76 989	76 997	77 004
589	77 012	77 019	77 026	77 034	77 041	77 048	77 056	77 063	77 070	77 078
590	77 085	77 093	77 100	77 107	77 115	77 122	77 129	77 137	77 144	77 151
591	77 159	77 166	77 173	77 181	77 188	77 195	77 203	77 210	77 217	77 225
592	77 232	77 240	77 247	77 254	77 262	77 269	77 276	77 283	77 291	77 298
593	77 305	77 313	77 320	77 327	77 335	77 342	77 349	77 357	77 364	77 371
594	77 379	77 386	77 393	77 401	77 408	77 415	77 422	77 430	77 437	77 444
595	77 452	77 459	77 466	77 474	77 481	77 488	77 495	77 503	77 510	77 517
596	77 525	77 532	77 539	77 546	77 554	77 561	77 568	77 576	77 583	77 590
597	77 597	77 605	77 612	77 619	77 627	77 634	77 641	77 648	77 656	77 663
598	77 670	77 677	77 685	77 692	77 699	77 706	77 714	77 721	77 728	77 735
599	77 743	77 750	77 757	77 764	77 772	77 779	77 786	77 793	77 801	77 808
No.	0	1	2	3	4	5	6	7	8	9

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

600–649

No.	0	1	2	3	4	5	6	7	8	9
600	77 815	77 822	77 830	77 837	77 844	77 851	77 859	77 866	77 873	77 880
601	77 887	77 895	77 902	77 909	77 916	77 924	77 931	77 938	77 945	77 952
602	77 960	77 967	77 974	77 981	77 988	77 996	78 003	78 010	78 017	78 025
603	78 032	78 039	78 046	78 053	78 061	78 068	78 075	78 082	78 089	78 097
604	78 104	78 111	78 118	78 125	78 132	78 140	78 147	78 154	78 161	78 168
605	78 176	78 183	78 190	78 197	78 204	78 211	78 219	78 226	78 233	78 240
606	78 247	78 254	78 262	78 269	78 276	78 283	78 290	78 297	78 305	78 312
607	78 319	78 326	78 333	78 340	78 347	78 355	78 362	78 369	78 376	78 383
608	78 390	78 398	78 405	78 412	78 419	78 426	78 433	78 440	78 447	78 455
609	78 462	78 469	78 476	78 483	78 490	78 497	78 504	78 512	78 519	78 526
610	78 533	78 540	78 547	78 554	78 561	78 569	78 576	78 583	78 590	78 597
611	78 604	78 611	78 618	78 625	78 633	78 640	78 647	78 654	78 661	78 668
612	78 675	78 682	78 689	78 696	78 704	78 711	78 718	78 725	78 732	78 739
613	78 746	78 753	78 760	78 767	78 774	78 781	78 789	78 796	78 803	78 810
614	78 817	78 824	78 831	78 838	78 845	78 852	78 859	78 866	78 873	78 880
615	78 888	78 895	78 902	78 909	78 916	78 923	78 930	78 937	78 944	78 951
616	78 958	78 965	78 972	78 979	78 986	78 993	79 000	79 007	79 014	79 021
617	79 029	79 036	79 043	79 050	79 057	79 064	79 071	79 078	79 085	79 092
618	79 099	79 106	79 113	79 120	79 127	79 134	79 141	79 148	79 155	79 162
619	79 169	79 176	79 183	79 190	79 197	79 204	79 211	79 218	79 225	79 232
620	79 239	79 246	79 253	79 260	79 267	79 274	79 281	79 288	79 295	79 302
621	79 309	79 316	79 323	79 330	79 337	79 344	79 351	78 358	79 365	79 372
622	79 379	79 386	79 393	79 400	79 407	79 414	79 421	79 428	79 435	79 442
623	79 449	79 456	79 463	79 470	79 477	79 484	79 491	79 498	79 505	79 511
624	79 518	79 525	79 532	79 539	79 546	79 553	79 560	79 567	79 574	79 581
625	79 588	79 595	79 602	79 609	79 616	79 623	79 630	79 637	79 644	79 650
626	79 657	79 664	79 671	79 678	79 685	79 692	79 699	79 706	79 713	79 720
627	79 727	79 734	79 741	79 748	79 754	79 761	79 768	79 775	79 782	79 789
628	79 796	79 803	79 810	79 817	79 824	79 831	79 837	79 844	79 851	79 858
629	79 865	79 872	79 879	79 886	79 893	79 900	79 906	79 913	79 920	79 927
630	79 934	79 941	79 948	79 955	79 962	79 969	79 975	79 982	79 989	79 996
631	80 003	80 010	80 017	80 024	80 030	80 037	80 044	80 051	80 058	80 065
632	80 072	80 079	80 085	80 092	80 099	80 106	80 113	80 120	80 127	80 134
633	80 140	80 147	80 154	80 161	80 168	80 175	80 182	80 188	80 195	80 202
634	80 209	80 216	80 223	80 229	80 236	80 243	80 250	80 257	80 264	80 271
635	80 277	80 284	80 291	80 298	80 305	80 312	80 318	80 325	80 332	80 339
636	80 346	80 353	80 359	80 366	80 373	80 380	80 387	80 393	80 400	80 407
637	80 414	80 421	80 428	80 434	80 441	80 448	80 455	80 462	80 468	80 475
638	80 482	80 489	80 496	80 502	80 509	80 516	80 523	80 530	80 536	80 543
639	80 550	80 557	80 564	80 570	80 577	80 584	80 591	80 598	80 604	80 611
640	80 618	80 625	80 632	80 638	80 645	80 652	80 659	80 665	80 672	80 679
641	80 686	80 693	80 699	80 706	80 713	80 720	80 726	80 733	80 740	80 747
642	80 754	80 760	80 767	80 774	80 781	80 787	80 794	80 801	80 808	80 814
643	80 821	80 828	80 835	80 841	80 848	80 855	80 862	80 868	80 875	80 882
644	80 889	80 895	80 902	80 909	80 916	80 922	80 929	80 936	80 943	80 949
645	80 956	80 963	80 969	80 976	80 983	80 990	80 996	81 003	81 010	81 017
646	81 023	81 030	81 037	81 043	81 050	81 057	81 064	81 070	81 077	81 084
647	81 090	81 097	81 104	81 111	81 117	81 124	81 131	81 137	81 144	81 151
648	81 158	81 164	81 171	81 178	81 184	81 191	81 198	81 204	81 211	81 218
649	81 224	81 231	81 238	81 245	81 251	81 258	81 265	81 271	81 278	81 285
No.	0	1	2	3	4	5	6	7	8	9

Table A15. Five-place Common Logarithms of Numbers.—(*Continued*)

650–699

No.	0	1	2	3	4	5	6	7	8	9
650	81 291	81 298	81 305	81 311	81 318	81 325	81 331	81 338	81 345	81 351
651	81 358	81 365	81 371	81 378	81 385	81 391	81 398	81 405	81 411	81 418
652	81 425	81 431	81 438	81 445	81 451	81 458	81 465	81 471	81 478	81 485
653	81 491	81 498	81 505	81 511	81 518	81 525	81 531	81 538	81 544	81 551
654	81 558	81 564	81 571	81 578	81 584	81 591	81 598	81 604	81 611	81 617
655	81 624	81 631	81 637	81 644	81 651	81 657	81 664	81 671	81 677	81 684
656	81 690	81 697	81 704	81 710	81 717	81 723	81 730	81 737	81 743	81 750
657	81 757	81 763	81 770	81 776	81 783	81 790	81 796	81 803	81 809	81 816
658	81 823	81 829	81 836	81 842	81 849	81 856	81 862	81 869	81 875	81 882
659	81 889	81 895	81 902	81 908	81 915	81 921	81 928	81 935	81 941	81 948
660	81 954	81 961	81 968	81 974	81 981	81 987	81 994	82 000	82 007	82 014
661	82 020	82 027	82 033	82 040	82 046	82 053	82 060	82 066	82 073	82 079
662	82 086	82 092	82 099	82 105	82 112	82 119	82 125	82 132	82 138	82 145
663	82 151	82 158	82 164	82 171	82 178	82 184	82 191	82 197	82 204	82 210
664	82 217	82 223	82 230	82 236	82 243	82 249	82 256	82 263	82 269	82 276
665	82 282	82 289	82 295	82 302	82 308	82 315	82 321	82 328	82 334	82 341
666	82 347	82 354	82 360	82 367	82 373	82 380	82 387	82 393	82 400	82 406
667	82 413	82 419	82 426	82 432	82 439	82 445	82 452	82 458	82 465	82 471
668	82 478	82 484	82 491	82 497	82 504	82 510	82 517	82 523	82 530	82 536
669	82 543	82 549	82 556	82 562	82 569	82 575	82 582	82 588	82 595	82 601
670	82 607	82 614	82 620	82 627	82 633	82 640	82 646	82 653	82 659	82 666
671	82 672	82 679	82 685	82 692	82 698	82 705	82 711	82 718	82 724	82 730
672	82 737	82 743	82 750	82 756	82 763	82 769	82 776	82 782	82 789	82 795
673	82 802	82 808	82 814	82 821	82 827	82 834	82 840	82 847	82 853	82 860
674	82 866	82 872	82 879	82 885	82 892	82 898	82 905	82 911	82 918	82 924
675	82 930	82 937	82 943	82 950	82 956	82 963	82 969	82 975	82 982	82 988
676	82 995	83 001	83 008	83 014	83 020	83 027	83 033	83 040	83 046	83 052
677	83 059	83 065	83 072	83 078	83 085	83 091	83 097	83 104	83 110	83 117
678	83 123	83 129	83 136	83 142	83 149	83 155	83 161	83 168	83 174	83 181
679	83 187	83 193	83 200	83 206	83 213	83 219	83 225	83 232	83 238	83 245
680	83 251	83 257	83 264	83 270	83 276	83 283	83 289	83 296	83 302	83 308
681	83 315	83 321	83 327	83 334	83 340	83 347	83 353	83 359	83 366	83 372
682	83 378	83 385	83 391	83 398	83 404	83 410	83 417	83 423	83 429	83 436
683	83 442	83 448	83 455	83 461	83 467	83 474	83 480	83 487	83 493	83 499
684	83 506	83 512	83 518	83 525	83 531	83 537	83 544	83 550	83 556	83 563
685	83 569	83 575	83 582	83 588	83 594	83 601	83 607	83 613	83 620	83 626
686	83 632	83 639	83 645	83 651	83 658	83 664	83 670	83 677	83 683	83 689
687	83 696	83 702	83 708	83 715	83 721	83 727	83 734	83 740	83 746	83 753
688	83 759	83 765	83 771	83 778	83 784	83 790	83 797	83 803	83 809	83 816
689	83 822	83 828	83 835	83 841	83 847	83 853	83 860	83 866	83 872	83 879
690	83 885	83 891	83 897	83 904	83 910	83 916	83 923	83 929	83 935	83 942
691	83 948	83 954	83 960	83 967	83 973	83 979	83 985	83 992	83 998	84 004
692	84 011	84 017	84 023	84 029	84 036	84 042	84 048	84 055	84 061	84 067
693	84 073	84 080	84 086	84 092	84 098	84 105	84 111	84 117	84 123	84 130
694	84 136	84 142	84 148	84 155	84 161	84 167	84 173	84 180	84 186	84 192
695	84 198	84 205	84 211	84 217	84 223	84 230	84 236	84 242	84 248	84 255
696	84 261	84 267	84 273	84 280	84 286	84 292	84 298	84 305	84 311	84 317
697	84 323	84 330	84 336	84 342	84 348	84 354	84 361	84 367	84 373	84 379
698	84 386	84 392	84 398	84 404	84 410	84 417	84 423	84 429	84 435	84 442
699	84 448	84 454	84 460	84 466	84 473	84 479	84 485	84 491	84 497	84 504
No.	0	1	2	3	4	5	6	7	8	9

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

700–749

No.	0	1	2	3	4	5	6	7	8	9
700	84 510	84 516	84 522	84 528	84 535	84 541	84 547	84 553	84 559	84 566
701	84 572	84 578	84 584	84 590	84 597	84 603	84 609	84 615	84 621	84 628
702	84 634	84 640	84 646	84 652	84 658	84 665	84 671	84 677	84 683	84 689
703	84 696	84 702	84 708	84 714	84 720	84 726	84 733	84 739	84 745	84 751
704	84 757	84 763	84 770	84 776	84 782	84 788	84 794	84 800	84 807	84 813
705	84 819	84 825	84 831	84 837	84 844	84 850	84 856	84 862	84 868	84 874
706	84 880	84 887	84 893	84 899	84 905	84 911	84 917	84 924	84 930	84 936
707	84 942	84 948	84 954	84 960	84 967	84 973	84 979	84 985	84 991	84 997
708	85 003	85 009	85 016	85 022	85 028	85 034	85 040	85 046	85 052	85 058
709	85 065	85 071	85 077	85 083	85 089	85 095	85 101	85 107	85 114	85 120
710	85 126	85 132	85 138	85 144	85 150	85 156	85 163	85 169	85 175	85 181
711	85 187	85 193	85 199	85 205	85 211	85 217	85 224	85 230	85 236	85 242
712	85 248	85 254	85 260	85 266	85 272	85 278	85 285	85 291	85 297	85 303
713	85 309	85 315	85 321	85 327	85 333	85 339	85 345	85 352	85 358	85 364
714	85 370	85 376	85 382	85 388	85 394	85 400	85 406	85 412	85 418	85 425
715	85 431	85 437	85 443	85 449	85 455	85 461	85 467	85 473	85 479	85 485
716	85 491	85 497	85 503	85 509	85 516	85 522	85 528	85 534	85 540	85 546
717	85 552	85 558	85 564	85 570	85 576	85 582	85 588	85 594	85 600	85 606
718	85 612	85 618	85 625	85 631	85 637	85 643	85 649	85 655	85 661	85 667
719	85 673	85 679	85 685	85 691	85 697	85 703	85 709	85 715	85 721	85 727
720	85 733	85 739	85 745	85 751	85 757	85 763	85 769	85 775	85 781	85 788
721	85 794	85 800	85 806	85 812	85 818	85 824	85 830	85 836	85 842	85 848
722	85 854	85 860	85 866	85 872	85 878	85 884	85 890	85 896	85 902	85 908
723	85 914	85 920	85 926	85 932	85 938	85 944	85 950	85 956	85 962	85 968
724	85 974	85 980	85 986	85 992	85 998	86 004	86 010	86 016	86 022	86 028
725	86 034	86 040	86 046	86 052	86 058	86 064	86 070	86 076	86 082	86 088
726	86 094	86 100	86 106	86 112	86 118	86 124	86 130	86 136	86 141	86 147
727	86 153	86 159	86 165	86 171	86 177	86 183	86 189	86 195	86 201	86 207
728	86 213	86 219	86 225	86 231	86 237	86 243	86 249	86 255	86 261	86 267
729	86 273	86 279	86 285	86 291	86 297	86 303	86 308	86 314	86 320	86 326
730	86 332	86 338	86 344	86 350	86 356	86 362	86 368	86 374	86 380	86 386
731	86 392	86 398	86 404	86 410	86 415	86 421	86 427	86 433	86 439	86 445
732	86 451	86 457	86 463	86 469	86 475	86 481	86 487	86 493	86 499	86 504
733	86 510	86 516	86 522	86 528	86 534	86 540	86 546	86 552	86 558	86 564
734	86 570	86 576	86 581	86 587	86 593	86 599	86 605	86 611	86 617	86 623
735	86 629	86 635	86 641	86 646	86 652	86 658	86 664	86 670	86 676	86 682
736	86 688	86 694	86 700	86 705	86 711	86 717	86 723	86 729	86 735	86 741
737	86 747	86 753	86 759	86 764	86 770	86 776	86 782	86 788	86 794	86 800
738	86 806	86 812	86 817	86 823	86 829	86 835	86 841	86 847	86 853	86 859
739	86 864	86 870	86 876	86 882	86 888	86 894	86 900	86 906	86 911	86 917
740	86 923	86 929	86 935	86 941	86 947	86 953	86 958	86 964	86 970	86 976
741	86 982	86 988	86 994	86 999	87 005	87 011	87 017	87 023	87 029	87 035
742	87 040	87 046	87 052	87 058	87 064	87 070	87 075	87 081	87 087	87 093
743	87 099	87 105	87 111	87 116	87 122	87 128	87 134	87 140	87 146	87 151
744	87 157	87 163	87 169	87 175	87 181	87 186	87 192	87 198	87 204	87 210
745	87 216	87 221	87 227	87 233	87 239	87 245	87 251	87 256	87 262	87 268
746	87 274	87 280	87 286	87 291	87 297	87 303	87 309	87 315	87 320	87 326
747	87 332	87 338	87 344	87 349	87 355	87 361	87 367	87 373	87 379	87 384
748	87 390	87 396	87 402	87 408	87 413	87 419	87 425	87 431	87 437	87 442
749	87 448	87 454	87 460	87 466	87 471	87 477	87 483	87 489	87 495	87 500
No.	0	1	2	3	4	5	6	7	8	9

700–749

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

750–799

No.	0	1	2	3	4	5	6	7	8	9
750	87 506	87 512	87 518	87 523	87 529	87 535	87 541	87 547	87 552	87 558
751	87 564	87 570	87 576	87 581	87 587	87 593	87 599	87 604	87 610	87 616
752	87 622	87 628	87 633	87 639	87 645	87 651	87 656	87 662	87 668	87 674
753	87 679	87 685	87 691	87 697	87 703	87 708	87 714	87 720	87 726	87 731
754	87 737	87 743	87 749	87 754	87 760	87 766	87 772	87 777	87 783	87 789
755	87 795	87 800	87 806	87 812	87 818	87 823	87 829	87 835	87 841	87 846
756	87 852	87 858	87 864	87 869	87 875	87 881	87 887	87 892	87 898	87 904
757	87 910	87 915	87 921	87 927	87 933	87 938	87 944	87 950	87 955	87 961
758	87 967	87 973	87 978	87 984	87 990	87 996	88 001	88 007	88 013	88 018
759	88 024	88 030	88 036	88 041	88 047	88 053	88 058	88 064	88 070	88 076
760	88 081	88 087	88 093	88 098	88 104	88 110	88 116	88 121	88 127	88 133
761	88 138	88 144	88 150	88 156	88 161	88 167	88 173	88 178	88 184	88 190
762	88 195	88 201	88 207	88 213	88 218	88 224	88 230	88 235	88 241	88 247
763	88 252	88 258	88 264	88 270	88 275	88 281	88 287	88 292	88 298	88 304
764	88 309	88 315	88 321	88 326	88 332	88 338	88 343	88 349	88 355	88 360
765	88 366	88 372	88 377	88 383	88 389	88 395	88 400	88 406	88 412	88 417
766	88 423	88 429	88 434	88 440	88 446	88 451	88 457	88 463	88 468	88 474
767	88 480	88 485	88 491	88 497	88 502	88 508	88 513	88 519	88 525	88 530
768	88 536	88 542	88 547	88 553	88 559	88 564	88 570	88 576	88 581	88 587
769	88 593	88 598	88 604	88 610	88 615	88 621	88 627	88 632	88 638	88 643
770	88 649	88 655	88 660	88 666	88 672	88 677	88 683	88 689	88 694	88 700
771	88 705	88 711	88 717	88 722	88 728	88 734	88 739	88 745	88 750	88 756
772	88 762	88 767	88 773	88 779	88 784	88 790	88 795	88 801	88 807	88 812
773	88 818	88 824	88 829	88 835	88 840	88 846	88 852	88 857	88 863	88 868
774	88 874	88 880	88 885	88 891	88 897	88 902	88 908	88 913	88 919	88 925
775	88 930	88 936	88 941	88 947	88 953	88 958	88 964	88 969	88 975	88 981
776	88 986	88 992	88 997	89 003	89 009	89 014	89 020	89 025	89 031	89 037
777	89 042	89 048	89 053	89 059	89 064	89 070	89 076	89 081	89 087	89 092
778	89 098	89 104	89 109	89 115	89 120	89 126	89 131	89 137	89 143	89 148
779	89 154	89 159	89 165	89 170	89 176	89 182	89 187	80 193	89 198	89 204
780	89 209	89 215	89 221	89 226	89 232	89 237	89 243	89 248	89 254	89 260
781	89 265	89 271	89 276	89 282	89 287	89 293	89 298	89 304	89 310	89 315
782	89 321	89 326	89 332	89 337	89 343	89 348	89 354	89 360	89 365	89 371
783	89 376	89 382	89 387	89 393	89 398	89 404	89 409	89 415	89 421	89 426
784	89 432	89 437	89 443	89 448	89 454	89 459	89 465	89 470	89 476	89 481
785	89 487	89 492	89 498	89 504	89 509	89 515	89 520	89 526	89 531	89 537
786	89 542	89 548	89 553	89 559	89 564	89 570	89 575	89 581	89 586	89 592
787	89 597	89 603	89 609	89 614	89 620	89 625	89 631	89 636	89 642	89 647
788	89 653	89 658	89 664	89 669	89 675	89 680	89 686	89 691	89 697	89 702
789	89 708	89 713	89 719	89 724	89 730	89 735	89 741	89 746	89 752	89 757
790	89 763	89 768	89 774	89 779	89 785	89 790	89 796	89 801	89 807	89 812
791	89 818	89 823	89 829	89 834	89 840	89 845	89 851	89 856	89 862	89 867
792	89 873	89 878	89 883	89 889	89 894	89 900	89 905	89 911	89 916	89 922
793	89 927	89 933	89 938	89 944	89 949	89 955	89 960	89 966	89 971	89 977
794	89 982	89 988	89 993	89 998	90 004	90 009	90 015	90 020	90 026	90 031
795	90 037	90 042	90 048	90 053	90 059	90 064	90 069	90 075	90 080	90 086
796	90 091	90 097	90 102	90 108	90 113	90 119	90 124	90 129	90 135	90 140
797	90 146	90 151	90 157	90 162	90 168	90 173	90 179	90 184	90 189	90 195
798	90 200	90 206	90 211	90 217	90 222	90 227	90 233	90 238	90 244	90 249
799	90 255	90 260	90 266	90 271	90 276	90 282	90 287	90 293	90 298	90 304
No.	0	1	2	3	4	5	6	7	8	9

Table A15. Five-place Common Logarithms of Numbers.—(*Continued*)

800–849

No.	0	1	2	3	4	5	6	7	8	9
800	90 309	90 314	90 320	90 325	90 331	90 336	90 342	90 347	90 352	90 358
801	90 363	90 369	90 374	90 380	90 385	90 390	90 396	90 401	90 407	90 412
802	90 417	90 423	90 428	90 434	90 439	90 445	90 450	90 455	90 461	90 466
803	90 472	90 477	90 482	90 488	90 493	90 499	90 504	90 509	90 515	90 520
804	90 526	90 531	90 536	90 542	90 547	90 553	90 558	90 563	90 569	90 574
805	90 580	90 585	90 590	90 596	90 601	90 607	90 612	90 617	90 623	90 628
806	90 634	90 639	90 644	90 650	90 655	90 660	90 666	90 671	90 677	90 682
807	90 687	90 693	90 698	90 703	90 709	90 714	90 720	90 725	90 730	90 736
808	90 741	90 747	90 752	90 757	90 763	90 768	90 773	90 779	90 784	90 789
809	90 795	90 800	90 806	90 811	90 816	90 822	90 827	90 832	90 838	90 843
810	90 849	90 854	90 859	90 865	90 870	90 875	90 881	90 886	90 891	90 897
811	90 902	90 907	90 913	90 918	90 924	90 929	90 934	90 940	90 945	90 950
812	90 956	90 961	90 966	90 972	90 977	90 982	90 988	90 993	90 998	91 004
813	91 009	91 014	91 020	91 025	91 030	91 036	91 041	91 046	91 052	91 057
814	91 062	91 068	91 073	91 078	91 084	91 089	91 094	91 100	91 105	91 110
815	91 116	91 121	91 126	91 132	91 137	91 142	91 148	91 153	91 158	91 164
816	91 169	91 174	91 180	91 185	91 190	91 196	91 201	91 206	91 212	91 217
817	91 222	91 228	91 233	91 238	91 243	91 249	91 254	91 259	91 265	91 270
818	91 275	91 281	91 286	91 291	91 297	91 302	91 307	91 312	91 318	91 323
819	91 328	91 334	91 339	91 344	91 350	91 355	91 360	91 365	91 371	91 376
820	91 381	91 387	91 392	91 397	91 403	91 408	91 413	91 418	91 424	91 429
821	91 434	91 440	91 445	91 450	91 455	91 461	91 466	91 471	91 477	91 482
822	91 487	91 492	91 498	91 503	91 508	91 514	91 519	91 524	91 529	91 535
823	91 540	91 545	91 551	91 556	91 561	91 566	91 572	91 577	91 582	91 587
824	91 593	91 598	91 603	91 609	91 614	91 619	91 624	91 630	91 635	91 640
825	91 645	91 651	91 656	91 661	91 666	91 672	91 677	91 682	91 687	91 693
826	91 698	91 703	91 709	91 714	91 719	91 724	91 730	91 735	91 740	91 745
827	91 751	91 756	91 761	91 766	91 772	91 777	91 782	91 787	91 793	91 798
828	91 803	91 808	91 814	91 819	91 824	91 829	91 834	91 840	91 845	91 850
829	91 855	91 861	91 866	91 871	91 876	91 882	91 887	91 892	91 897	91 903
830	91 908	91 913	91 918	91 924	91 929	91 934	91 939	91 944	91 950	91 955
831	91 960	91 965	91 971	91 976	91 981	91 986	91 991	91 997	92 002	92 007
832	92 012	92 018	92 023	92 028	92 033	92 038	92 044	92 049	92 054	92 059
833	92 065	92 070	92 075	92 080	92 085	92 091	92 096	92 101	92 106	92 111
834	92 117	92 122	92 127	92 132	92 137	92 143	92 148	92 153	92 158	92 163
835	92 169	92 174	92 179	92 184	92 189	92 195	92 200	92 205	92 210	92 215
836	92 221	92 226	92 231	92 236	92 241	92 247	92 252	92 257	92 262	92 267
837	92 273	92 278	92 283	92 288	92 293	92 298	92 304	92 309	92 314	92 319
838	92 324	92 330	92 335	92 340	92 345	92 350	92 355	92 361	92 366	92 371
839	92 376	92 381	92 387	92 392	92 397	92 402	92 407	92 412	92 418	92 423
840	92 428	92 433	92 438	92 443	92 449	92 454	92 459	92 464	92 469	92 474
841	92 480	92 485	92 490	92 495	92 500	92 505	92 511	92 516	92 521	92 526
842	92 531	92 536	92 542	92 547	92 552	92 557	92 562	92 567	92 572	92 578
843	92 583	92 588	92 593	92 598	92 603	92 609	92 614	92 619	92 624	92 629
844	92 634	92 639	92 645	92 650	92 655	92 660	92 665	92 670	92 675	92 681
845	92 686	92 691	92 696	92 701	92 706	92 711	92 716	92 722	92 727	92 732
846	92 737	92 742	92 747	92 752	92 758	92 763	92 768	92 773	92 778	92 783
847	92 788	92 793	92 799	92 804	92 809	92 814	92 819	92 824	92 829	92 834
848	92 840	92 845	92 850	92 855	92 860	92 865	92 870	92 875	92 881	92 886
849	92 891	92 896	92 901	92 906	92 911	92 916	92 921	92 927	92 932	92 937
No.	0	1	2	3	4	5	6	7	8	9

800–849

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

850–899

No.	0	1	2	3	4	5	6	7	8	9
850	92 942	92 947	92 952	92 957	92 962	92 967	92 973	92 978	92 983	92 988
851	92 993	92 998	93 003	93 008	93 013	93 018	93 024	93 029	93 034	93 039
852	93 044	93 049	93 054	93 059	93 064	93 069	93 075	93 080	93 085	93 090
853	93 095	93 100	93 105	93 110	93 115	93 120	93 125	93 131	93 136	93 141
854	93 146	93 151	93 156	93 161	93 166	93 171	93 176	93 181	93 186	93 192
855	93 197	93 202	93 207	93 212	93 217	93 222	93 227	93 232	93 237	93 242
856	93 247	93 252	93 258	93 263	93 268	93 273	93 278	93 283	93 288	93 293
857	93 298	93 303	93 308	93 313	93 318	93 323	93 328	93 334	93 339	93 344
858	93 349	93 354	93 359	93 364	93 369	93 374	93 379	93 384	93 389	93 394
859	93 399	93 404	93 409	93 414	93 420	93 425	93 430	93 435	93 440	93 445
860	93 450	93 455	93 460	93 465	93 470	93 475	93 480	93 485	93 490	93 495
861	93 500	93 505	93 510	94 515	93 520	93 526	93 531	93 536	93 541	93 546
862	93 551	93 556	93 561	93 566	93 571	93 576	93 581	93 586	93 591	93 596
863	93 601	93 606	93 611	93 616	93 621	93 626	93 631	93 636	93 641	93 646
864	93 651	93 656	93 661	93 666	93 671	93 676	93 682	93 687	93 692	93 697
865	93 702	93 707	93 712	93 717	93 722	93 727	93 732	93 737	93 742	93 747
866	93 752	93 757	93 762	93 767	93 772	93 777	93 782	93 787	93 792	93 797
867	93 802	93 807	93 812	93 817	93 822	93 827	93 832	93 837	93 842	93 847
868	93 852	93 857	93 862	93 867	93 872	93 877	93 882	93 887	93 892	93 897
869	93 902	93 907	93 912	93 917	93 922	93 927	93 932	93 937	93 942	93 947
870	93 952	93 957	93 962	93 967	93 972	93 977	93 982	93 987	93 992	93 997
871	94 002	94 007	94 012	94 017	94 022	94 027	94 032	94 037	94 042	94 047
872	94 052	94 057	94 062	94 067	94 072	94 077	94 082	94 086	94 091	94 096
873	94 101	94 106	94 111	94 116	94 121	94 126	94 131	94 136	94 141	94 146
874	94 151	94 156	94 161	94 166	94 171	94 176	94 181	94 186	94 191	94 196
875	94 201	94 206	94 211	94 216	94 221	94 226	94 231	94 236	94 240	94 245
876	94 250	94 255	94 260	94 265	94 270	94 275	94 280	94 285	94 290	94 295
877	94 300	94 305	94 310	94 315	94 320	94 325	94 330	94 335	94 340	94 345
878	94 349	94 354	94 359	94 364	94 369	94 374	94 379	94 384	94 389	94 394
879	94 399	94 404	94 409	94 414	94 419	94 424	94 429	94 433	94 438	94 443
880	94 448	94 453	94 458	94 463	94 468	94 473	94 478	94 483	94 488	94 493
881	94 498	94 503	94 507	94 512	94 517	94 522	94 527	94 532	94 537	94 542
882	94 547	94 552	94 557	94 562	94 567	94 571	94 576	94 581	94 586	94 591
883	94 596	94 601	94 606	94 611	94 616	94 621	94 626	94 630	94 635	94 640
884	94 645	94 650	94 655	94 660	94 665	94 670	94 675	94 680	94 685	94 689
885	94 694	94 699	94 704	94 709	94 714	94 719	94 724	94 729	94 734	94 738
886	94 743	94 748	94 753	94 758	94 763	94 768	94 773	94 778	94 783	94 787
887	94 792	94 797	94 802	94 807	94 812	94 817	94 822	94 827	94 832	94 836
888	94 841	94 846	94 851	94 856	94 861	94 866	94 871	94 876	94 880	94 885
889	94 890	94 895	94 900	94 905	94 910	94 915	94 919	94 924	94 929	94 934
890	94 939	94 944	94 949	94 954	94 959	94 963	94 968	94 973	94 978	94 983
891	94 988	94 993	94 998	95 002	95 007	95 012	95 017	95 022	95 027	95 032
892	95 036	95 041	95 046	95 051	95 056	95 061	95 066	95 071	95 075	95 080
893	95 085	95 090	95 095	95 100	95 105	95 109	95 114	95 119	95 124	95 129
894	95 134	95 139	95 143	95 148	95 153	95 158	95 163	95 168	95 173	95 177
895	95 182	95 187	95 192	95 197	95 202	95 207	95 211	95 216	95 221	95 226
896	95 231	95 236	95 240	95 245	95 250	95 255	95 260	95 265	95 270	95 274
897	95 279	95 284	95 289	95 294	95 299	95 303	95 308	95 313	95 318	95 323
898	95 328	95 332	95 337	95 342	95 347	95 352	95 357	95 361	95 366	95 371
899	95 376	95 381	95 386	95 390	95 395	95 400	95 405	95 410	95 415	95 419
No.	0	1	2	3	4	5	6	7	8	9

850–899

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

900–949

No.	0	1	2	3	4	5	6	7	8	9
900	95 424	95 429	95 434	95 439	95 444	95 448	95 453	95 458	95 463	95 468
901	95 472	95 477	95 482	95 487	95 492	95 497	95 501	95 506	95 511	95 516
902	95 521	95 525	95 530	95 535	95 540	95 545	95 550	95 554	95 559	95 564
903	95 569	95 574	95 578	95 583	95 588	95 593	95 598	95 602	95 607	95 612
904	95 617	95 622	95 626	95 631	95 636	95 641	95 646	95 650	95 655	95 660
905	95 665	95 670	95 674	95 679	95 684	95 689	95 694	95 698	95 703	95 708
906	95 713	95 718	95 722	95 727	95 732	95 737	95 742	95 746	95 751	95 756
907	95 761	95 766	95 770	95 775	95 780	95 785	95 789	95 794	95 799	95 804
908	95 809	95 813	95 818	95 823	95 828	95 832	95 837	95 842	95 847	95 852
909	95 856	95 861	95 866	95 871	95 875	95 880	95 885	95 890	95 895	95 899
910	95 904	95 909	95 914	95 918	95 923	95 928	95 933	95 938	95 942	95 947
911	95 952	95 957	95 961	95 966	95 971	95 976	95 980	95 985	95 990	95 995
912	95 999	96 004	96 009	96 014	96 019	96 023	96 028	96 033	96 038	96 042
913	96 047	96 052	96 057	96 061	96 066	96 071	96 076	96 080	96 085	96 090
914	96 095	96 099	96 104	96 109	96 114	96 118	96 123	96 128	96 133	96 137
915	96 142	96 147	96 152	96 156	96 161	96 166	96 171	96 175	96 180	96 185
916	96 190	96 194	96 199	96 204	96 209	96 213	96 218	96 223	96 227	96 232
917	96 237	96 242	96 246	96 251	96 256	96 261	96 265	96 270	96 275	96 280
918	96 284	96 289	96 294	96 298	96 303	96 308	96 313	96 317	96 322	96 327
919	96 332	96 336	96 341	96 346	96 350	96 355	96 360	96 365	96 369	96 374
920	96 379	96 384	96 388	96 393	96 398	96 402	96 407	96 412	96 417	96 421
921	96 426	96 431	96 435	96 440	96 445	96 450	96 454	96 459	96 464	96 468
922	96 473	96 478	96 483	96 487	96 492	96 497	96 501	96 506	96 511	96 515
923	96 520	96 525	96 530	96 534	96 539	96 544	96 548	96 553	96 558	96 562
924	96 567	96 572	96 577	96 581	96 586	96 591	96 595	96 600	96 605	96 609
925	96 614	96 619	96 624	96 628	96 633	96 638	96 642	96 647	96 652	96 656
926	96 661	96 666	96 670	96 675	96 680	96 685	96 689	96 694	96 699	96 703
927	96 708	96 713	96 717	96 722	96 727	96 731	96 736	96 741	96 745	96 750
928	96 755	96 759	96 764	96 769	96 774	96 778	96 783	96 788	96 792	96 797
929	96 802	96 806	96 811	96 816	96 820	96 825	96 830	96 834	96 839	96 844
930	96 848	96 853	96 858	96 862	96 867	96 872	96 876	96 881	96 886	96 890
931	96 895	96 900	96 904	96 909	96 914	96 918	96 923	96 928	96 932	96 937
932	96 942	96 946	96 951	96 956	96 960	96 965	96 970	96 974	96 979	96 984
933	96 988	96 993	96 997	97 002	97 007	97 011	97 016	97 021	97 025	97 030
934	97 035	97 039	97 044	97 049	97 053	97 058	97 063	97 067	97 072	97 077
935	97 081	97 086	97 090	97 095	97 100	97 104	97 109	97 114	97 118	97 123
936	97 128	97 132	97 137	97 142	97 146	97 151	97 155	97 160	97 165	97 169
937	97 174	97 179	97 183	97 188	97 192	97 197	97 202	97 206	97 211	97 216
938	97 220	97 225	97 230	97 234	97 239	97 243	97 248	97 253	97 257	97 262
939	97 267	97 271	97 276	97 280	97 285	97 290	97 294	97 299	97 304	97 308
940	97 313	97 317	97 322	97 327	97 331	97 336	97 340	97 345	97 350	97 354
941	97 359	97 364	97 368	97 373	97 377	97 382	97 387	97 391	97 396	97 400
942	97 405	97 410	97 414	97 419	97 424	97 428	97 433	97 437	97 442	97 447
943	97 451	97 456	97 460	97 465	97 470	97 474	97 479	97 483	97 488	97 493
944	97 497	97 502	97 506	97 511	97 516	97 520	97 525	97 529	97 534	97 539
945	97 543	97 548	97 552	97 557	97 562	97 566	97 571	97 575	97 580	97 585
946	97 589	97 594	97 598	97 603	97 607	97 612	97 617	97 621	97 626	97 630
947	97 635	97 640	97 644	97 649	97 653	97 658	97 663	97 667	97 672	97 676
948	97 681	97 685	97 690	97 695	97 699	97 704	97 708	97 713	97 717	97 722
949	97 727	97 731	97 736	97 740	97 745	97 749	97 754	97 759	97 763	97 768
No.	0	1	2	3	4	5	6	7	8	9

TABLE A15. FIVE-PLACE COMMON LOGARITHMS OF NUMBERS.—(*Continued*)

950–1000

No.	0	1	2	3	4	5	6	7	8	9
950	97 772	97 777	97 782	97 786	97 791	97 795	97 800	97 804	97 809	97 813
951	97 818	97 823	97 827	97 832	97 836	97 841	97 845	97 850	97 855	97 859
952	97 864	97 868	97 873	97 877	97 882	97 886	97 891	97 896	97 900	97 905
953	97 909	97 914	97 918	97 923	97 928	97 932	97 937	97 941	97 946	97 950
954	97 955	97 959	97 964	97 968	97 973	97 978	97 982	97 987	97 991	97 996
955	98 000	98 005	98 009	98 014	98 019	98 023	98 028	98 032	98 037	98 041
956	98 046	98 050	98 055	98 059	98 064	98 068	98 073	98 078	98 082	98 087
957	98 091	98 096	98 100	98 105	98 109	98 114	98 118	98 123	98 127	98 132
958	98 137	98 141	98 146	98 150	98 155	98 159	98 164	98 168	98 173	98 177
959	98 182	98 186	98 191	98 195	98 200	98 204	98 209	98 214	98 218	98 223
960	98 227	98 232	98 236	98 241	98 245	98 250	98 254	98 259	98 263	98 268
961	98 272	98 277	98 281	98 286	98 290	98 295	98 299	98 304	98 308	98 313
962	98 318	98 322	98 327	98 331	98 336	98 340	98 345	98 349	98 354	98 358
963	98 363	98 367	98 372	98 376	98 381	98 385	98 390	98 394	98 399	98 403
964	98 408	98 412	98 417	98 421	98 426	98 430	98 435	98 439	98 444	98 448
965	98 453	98 457	98 462	98 466	98 471	98 475	98 480	98 484	98 489	98 493
966	98 498	98 502	98 507	98 511	98 516	98 520	98 525	98 529	98 534	98 538
967	98 543	98 547	98 552	98 556	98 561	98 565	98 570	98 574	98 579	98 583
968	98 588	98 592	98 597	98 601	98 605	98 610	98 614	98 619	98 623	98 628
969	98 632	98 637	98 641	98 646	98 650	98 655	98 659	98 664	98 668	98 673
970	98 677	98 682	98 686	98 691	98 695	98 700	98 704	98 709	98 713	98 717
971	98 722	98 726	98 731	98 735	98 740	98 744	98 749	98 753	98 758	98 762
972	98 767	98 771	98 776	98 780	98 784	98 789	98 793	98 798	98 802	98 807
973	98 811	98 816	98 820	98 825	98 829	98 834	98 838	98 843	98 847	98 851
974	98 856	98 860	98 865	98 869	98 874	98 878	98 883	98 887	98 892	98 896
975	98 900	98 905	98 909	98 914	98 918	98 923	98 927	98 932	98 936	98 941
976	98 945	98 949	98 954	98 958	98 963	98 967	98 972	98 976	98 981	98 985
977	98 989	98 994	98 998	99 003	99 007	99 012	99 016	99 021	99 025	99 029
978	99 034	99 038	99 043	99 047	99 052	99 056	99 061	99 065	99 069	99 074
979	99 078	99 083	99 087	99 092	99 096	99 100	99 105	99 109	99 114	99 118
980	99 123	99 127	99 131	99 136	99 140	99 145	99 149	99 154	99 158	99 162
981	99 167	99 171	99 176	99 180	99 185	99 189	99 193	99 198	99 202	99 207
982	99 211	99 216	99 220	99 224	99 229	99 233	99 238	99 242	99 247	99 251
983	99 255	99 260	99 264	99 269	99 273	99 277	99 282	99 286	99 291	99 295
984	99 300	99 304	99 308	99 313	99 317	99 322	99 326	99 330	99 335	99 339
985	99 344	99 348	99 352	99 357	99 361	99 366	99 370	99 374	99 379	99 383
986	99 388	99 392	99 396	99 401	99 405	99 410	99 414	99 419	99 423	99 427
987	99 432	99 436	99 441	99 445	99 449	99 454	99 458	99 463	99 467	99 471
988	99 476	99 480	99 484	99 489	99 493	99 498	99 502	99 506	99 511	99 515
989	99 520	99 524	99 528	99 533	99 537	99 542	99 546	99 550	99 555	99 559
990	99 564	99 568	99 572	99 577	99 581	99 585	99 590	99 594	99 599	99 603
991	99 607	99 612	99 616	99 621	99 625	99 629	99 634	99 638	99 642	99 647
992	99 651	99 656	99 660	99 664	99 669	99 673	99 677	99 682	99 686	99 691
993	99 695	99 699	99 704	99 708	99 712	99 717	99 721	99 726	99 730	99 734
994	99 739	99 743	99 747	99 752	99 756	99 760	99 765	99 769	99 774	99 778
995	99 782	99 787	99 791	99 795	99 800	99 804	99 808	99 813	99 817	99 822
996	99 826	99 830	99 835	99 839	99 843	99 848	99 852	99 856	99 861	99 865
997	99 870	99 874	99 878	99 883	99 887	99 891	99 896	99 900	99 904	99 909
998	99 913	99 917	99 922	99 926	99 930	99 935	99 939	99 944	99 948	99 952
999	99 957	99 961	99 965	99 970	99 974	99 978	99 983	99 987	99 991	99 996
1000	00 000	00 004	00 009	00 013	00 017	00 022	00 026	00 030	00 035	00 039
No.	0	1	2	3	4	5	6	7	8	9

TABLE A16. NATURAL OR NAPIERIAN LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean differences								
											1	2	3	4	5	6	7	8	9
1.0	0.0000	0099	0198	0296	0392	0488	0583	0677	0770	0862	10	19	29	38	48	57	67	76	86
1.1	.0953	1044	1133	1222	1310	1398	1484	1570	1655	1740	9	17	26	35	44	52	61	70	78
1.2	.1823	1906	1989	2070	2151	2231	2311	2390	2469	2546	8	16	24	32	40	48	56	64	72
1.3	.2624	2700	2776	2852	2927	3001	3075	3148	3221	3293	7	15	22	30	37	44	52	59	67
1.4	.3365	3436	3507	3577	3646	3716	3784	3853	3920	3988	7	14	21	28	35	41	48	55	62
1.5	.4055	4121	4187	4253	4318	4383	4447	4511	4574	4637	6	13	19	26	32	39	45	52	58
1.6	.4700	4762	4824	4886	4947	5008	5068	5128	5188	5247	6	12	18	24	30	36	42	48	55
1.7	.5306	5365	5423	5481	5539	5596	5653	5710	5766	5822	6	11	17	24	29	34	40	46	51
1.8	.5878	5933	5988	6043	6098	6152	6206	6259	6313	6366	5	11	16	22	27	32	38	43	49
1.9	.6419	6471	6523	6575	6627	6678	6729	6780	6831	6881	5	10	15	20	26	31	36	41	46
2.0	.6931	6981	7031	7080	7129	7178	7227	7275	7324	7372	5	10	15	20	24	29	34	39	44
2.1	.7419	7467	7514	7561	7608	7655	7701	7747	7793	7839	5	9	14	19	23	28	33	37	42
2.2	.7885	7930	7975	8020	8065	8109	8154	8198	8242	8286	4	9	13	18	22	27	31	36	40
2.3	.8329	8372	8416	8459	8502	8544	8587	8629	8671	8713	4	9	13	17	21	26	30	34	38
2.4	.8755	8796	8838	8879	8920	8961	9002	9042	9083	9123	4	8	12	16	20	24	29	33	37
2.5	.9163	9203	9243	9282	9322	9361	9400	9439	9478	9517	4	8	12	16	20	24	27	31	35
2.6	.9555	9594	9632	9670	9708	9746	9783	9821	9858	9895	4	8	11	15	19	23	26	30	34
2.7	.9933	9969	1.0006	0043	0080	0116	0152	0188	0225	0260	4	7	11	15	18	22	25	29	33
2.8	1.0296	0332	0367	0403	0438	0473	0508	0543	0578	0613	4	7	11	14	18	21	25	28	32
2.9	1.0647	0682	0716	0750	0784	0818	0852	0886	0919	0953	3	7	10	14	17	20	24	27	31
3.0	1.0986	1019	1053	1086	1119	1151	1184	1217	1249	1282	3	7	10	13	16	20	23	26	30
3.1	1.1314	1346	1378	1410	1442	1474	1506	1537	1569	1600	3	6	10	13	16	19	22	25	29
3.2	1.1632	1663	1694	1725	1756	1787	1817	1848	1878	1909	3	6	9	12	15	18	22	25	28
3.3	1.1939	1969	1.2000	2030	2060	2090	2119	2149	2179	2208	3	6	9	12	15	18	21	24	27
3.4	1.2238	2267	2296	2326	2355	2384	2413	2442	2470	2499	3	6	9	12	15	17	20	23	26
3.5	1.2528	2556	2585	2613	2641	2669	2698	2726	2754	2782	3	6	8	11	14	17	20	23	25
3.6	1.2809	2837	2865	2892	2920	2947	2975	3002	3029	3056	3	5	8	11	14	16	19	22	25
3.7	1.3083	3110	3137	3164	3191	3218	3244	3271	3297	3324	3	5	8	11	13	16	19	21	24
3.8	1.3350	3376	3403	3429	3455	3481	3507	3533	3558	3584	3	5	8	10	13	16	18	21	23
3.9	1.3610	3635	3661	3686	3712	3737	3762	3788	3813	3838	3	5	8	10	13	15	18	20	23
4.0	1.3863	3888	3913	3938	3962	3987	4012	4036	4061	4085	2	5	7	10	12	15	17	20	22
4.1	1.4110	4134	4159	4183	4207	4231	4255	4279	4303	4327	2	5	7	10	12	14	17	19	22
4.2	1.4351	4375	4398	4422	4446	4469	4493	4516	4540	4563	2	5	7	9	12	14	16	19	21
4.3	1.4586	4609	4633	4656	4679	4702	4725	4748	4770	4793	2	5	7	9	12	14	16	18	21
4.4	1.4816	4839	4861	4884	4907	4929	4951	4974	4996	5019	2	5	7	9	11	14	16	18	20
4.5	1.5041	5063	5085	5107	5129	5151	5173	5195	5217	5239	2	4	7	9	11	13	15	18	20
4.6	1.5261	5282	5304	5326	5347	5369	5390	5412	5433	5454	2	4	6	9	11	13	15	17	19
4.7	1.5476	5497	5518	5539	5560	5581	5602	5623	5644	5665	2	4	6	8	11	13	15	17	19
4.8	1.5686	5707	5728	5748	5769	5790	5810	5831	5851	5872	2	4	6	8	10	12	14	16	19
4.9	1.5892	5913	5933	5953	5974	5994	6014	6034	6054	6074	2	4	6	8	10	12	14	16	18
5.0	1.6094	6114	6134	6154	6174	6194	6214	6233	6253	6273	2	4	6	8	10	12	14	16	18
5.1	1.6292	6312	6332	6351	6371	6390	6409	6429	6448	6467	2	4	6	8	10	12	14	16	18
5.2	1.6487	6506	6525	6544	6563	6582	6601	6620	6639	6658	2	4	6	8	10	11	13	15	17
5.3	1.6677	6696	6715	6734	6752	6771	6790	6808	6827	6845	2	4	6	7	9	11	13	15	17
5.4	1.6864	6882	6901	6919	6938	6956	6974	6993	7011	7029	2	4	5	7	9	11	13	15	17

TABLE A16. NATURAL OR NAPIERIAN LOGARITHMS.—(*Continued*)

	0	1	2	3	4	5	6	7	8	9	Mean differences								
											1	2	3	4	5	6	7	8	9
5.5	1.7047	7066	7084	7102	7120	7138	7156	7174	7192	7210	2	4	5	7	9	11	13	14	16
5.6	1.7228	7246	7263	7281	7299	7317	7334	7352	7370	7387	2	4	5	7	9	11	12	14	16
5.7	1.7405	7422	7440	7457	7475	7492	7509	7527	7544	7561	2	3	5	7	9	10	12	14	16
5.8	1.7579	7596	7613	7630	7647	7664	7681	7699	7716	7733	2	3	5	7	9	10	12	14	15
5.9	1.7750	7766	7783	7800	7817	7834	7851	7867	7884	7901	2	3	5	7	8	10	12	13	15
6.0	1.7918	7934	7951	7967	7984	8001	8017	8034	8050	8066	2	3	5	7	8	10	12	13	15
6.1	1.8083	8099	8116	8132	8148	8165	8181	8197	8213	8229	2	3	5	6	8	10	11	13	15
6.2	1.8245	8262	8278	8294	8310	8326	8342	8358	8374	8390	2	3	5	6	8	10	11	13	14
6.3	1.8405	8421	8437	8453	8469	8485	8500	8516	8532	8547	2	3	5	6	8	9	11	13	14
6.4	1.8563	8579	8594	8610	8625	8641	8656	8672	8687	8703	2	3	5	6	8	9	11	12	14
6.5	1.8718	8733	8749	8764	8779	8795	8810	8825	8840	8856	2	3	5	6	8	9	11	12	14
6.6	1.8871	1.8886	1.8901	8916	8931	8946	8961	8976	8991	9006	2	3	5	6	8	9	11	12	14
6.7	1.9021	9036	9051	9066	9081	9095	9110	9125	9140	9155	1	3	4	6	7	9	10	12	13
6.8	1.9169	9184	9199	9213	9228	9242	9257	9272	9286	9301	1	3	4	6	7	9	10	12	13
6.9	1.9315	9330	9344	9359	9373	9387	9402	9416	9430	9445	1	3	4	6	7	9	10	12	13
7.0	1.9459	9473	9488	9502	9516	9530	9544	9559	1.9573	9587	1	3	4	6	7	9	10	11	13
7.1	1.9601	9615	9629	9643	9657	9671	9685	9699	9713	9727	1	3	4	6	7	8	10	11	13
7.2	1.9741	9755	9769	9782	9796	9810	9824	9838	9851	9865	1	3	4	6	7	8	10	11	12
7.3	1.9879	9892	9906	9920	9933	9947	9961	9974	9988	2.0001	1	3	4	5	7	8	10	11	12
7.4	2.0015	0028	0042	0055	0069	0082	0096	0109	0122	0136	1	3	4	5	7	8	9	11	12
7.5	2.0149	0162	0176	0189	0202	0215	0229	0242	0255	0268	1	3	4	5	7	8	9	11	12
7.6	2.0281	0295	0308	0321	0334	0347	0360	0373	0386	0399	1	3	4	5	7	8	9	10	12
7.7	2.0412	0425	0438	0451	0464	0477	0490	0503	0516	0528	1	3	4	5	6	8	9	10	12
7.8	2.0541	0554	0567	0580	0592	0605	0618	0631	0643	0656	1	3	4	5	6	8	9	10	11
7.9	2.0669	0681	0694	0707	0719	0732	0744	0757	0769	0782	1	3	4	5	6	8	9	10	11
8.0	2.0794	0807	0819	0832	0844	0857	0869	0882	0894	0906	1	3	4	5	6	7	9	10	11
8.1	2.0919	0931	0943	0956	0968	0980	0992	1005	1017	1029	1	2	4	5	6	7	9	10	11
8.2	2.1041	1054	1066	1078	1090	1102	1114	1126	1138	1150	1	2	4	5	6	7	9	10	11
8.3	2.1163	1175	1187	1199	1211	1223	1235	1247	1258	1270	1	2	4	5	6	7	8	10	11
8.4	2.1282	1294	1306	1318	1330	1342	1353	1365	1377	1389	1	2	4	5	6	7	8	9	11
8.5	2.1401	1412	1424	1436	1448	1459	1471	1483	1494	1506	1	2	4	5	6	7	8	9	11
8.6	2.1518	1529	1541	1552	1564	1576	1587	1599	1610	1622	1	2	3	5	6	7	8	9	10
8.7	2.1633	1645	1656	1668	1679	1691	1702	1713	1725	1736	1	2	3	5	6	7	8	9	10
8.8	2.1748	1759	1770	1782	1793	1804	1815	1827	1838	1849	1	2	3	5	6	7	8	9	10
8.9	2.1861	1872	1883	1894	1905	1917	1928	1939	1950	1961	1	2	3	4	6	7	8	9	10
9.0	2.1972	1983	1994	2006	2017	2028	2039	2050	2061	2072	1	2	3	4	6	7	8	9	10
9.1	2.2083	2094	2105	2116	2127	2138	2148	2159	2170	2181	1	2	3	4	5	7	8	9	10
9.2	2.2192	2203	2214	2225	2235	2246	2257	2268	2279	2289	1	2	3	4	5	6	8	9	10
9.3	2.2300	2311	2322	2332	2343	2354	2364	2375	2386	2396	1	2	3	4	5	6	7	9	10
9.4	2.2407	2418	2428	2439	2450	2460	2471	2481	2492	2502	1	2	3	4	5	6	7	8	10
9.5	2.2513	2523	2534	2544	2555	2565	2576	2586	2597	2607	1	2	3	4	5	6	7	8	9
9.6	2.2618	2628	2638	2649	2659	2670	2680	2690	2701	2711	1	2	3	4	5	6	7	8	9
9.7	2.2721	2732	2742	2752	2762	2773	2783	2793	2803	2814	1	2	3	4	5	6	7	8	9
9.8	2.2824	2834	2844	2854	2865	2875	2885	2895	2905	2915	1	2	3	4	5	6	7	8	9
9.9	2.2925	2935	2946	2956	2966	2976	2986	2996	3006	3016	1	2	3	4	5	6	7	8	9
10.0	2.3026																		

NAPIERIAN LOGARITHMS OF 10^{+n}

n	1	2	3	4	5	6	7	8	9
$\log_e 10^n$	2.3026	4.6052	6.9078	9.2103	11.5129	13.8155	16.1181	18.4207	20.7233

TABLE A17. THE SUMS OF THE FIRST SIX POWERS OF THE NATURAL NUMBERS FROM 1 TO n

n	$\Sigma(n)$	$\Sigma(n^2)$	$\Sigma(n^3)$	$\Sigma(n^4)$	$\Sigma(n^5)$	$\Sigma(n^6)$
1	1	1	1	1	1	1
2	3	5	9	17	33	65
3	6	14	36	98	276	794
4	10	30	100	354	1,300	4,890
5	15	55	225	979	4,425	20,515
6	21	91	441	2,275	12,201	67,171
7	28	140	784	4,676	29,008	184,820
8	36	204	1,296	8,772	61,776	446,964
9	45	285	2,025	15,333	120,825	978,405
10	55	385	3,025	25,333	220,825	1,978,405
11	66	506	4,356	39,974	381,876	3,749,966
12	78	650	6,084	60,710	630,708	6,735,950
13	91	819	8,281	89,271	1,002,001	11,562,759
14	105	1,015	11,025	127,687	1,539,825	19,092,295
15	120	1,240	14,400	178,312	2,299,200	30,482,920
16	136	1,496	18,496	234,848	3,347,776	47,260,136
17	153	1,785	23,409	327,369	4,767,633	71,397,705
18	171	2,109	29,241	432,345	6,657,201	105,409,929
19	190	2,470	36,100	562,666	9,133,300	152,455,810
20	210	2,870	44,100	722,666	12,333,300	216,455,810
21	231	3,311	53,361	917,147	16,417,401	302,221,931
22	253	3,795	64,009	1,151,403	21,571,033	415,601,835
23	276	4,324	76,176	1,431,244	28,007,376	563,637,724
24	300	4,900	90,000	1,763,020	35,970,000	754,740,700
25	325	5,525	105,625	2,153,645	45,735,625	998,881,325
26	351	6,201	123,201	2,610,621	57,617,001	1,307,797,101
27	378	6,930	142,884	3,142,062	71,965,908	1,695,217,590
28	406	7,714	164,836	3,756,718	89,176,276	2,177,107,894
29	435	8,555	189,225	4,463,999	109,687,425	2,771,931,215
30	465	9,455	216,225	5,273,999	133,987,425	3,500,931,215
31	496	10,416	246,016	6,197,520	162,616,576	4,388,434,896
32	528	11,440	278,784	7,246,096	196,171,008	5,462,176,720
33	561	12,529	314,721	8,432,017	235,306,401	6,753,644,689
34	595	13,685	354,025	9,768,353	280,741,825	8,298,449,105
35	630	14,910	396,900	11,268,978	333,263,700	10,136,714,730
36	666	16,206	443,556	12,948,594	393,729,876	12,313,497,066
37	703	17,575	494,209	14,822,755	463,073,833	14,879,223,475
38	741	19,019	549,081	16,907,891	542,309,001	17,890,159,859
39	780	20,540	608,400	19,221,332	632,533,200	21,408,903,620
40	820	22,140	672,400	21,781,332	734,933,200	25,504,903,620

TABLE A18. THE SUMS OF THE FIRST SIX POWERS OF THE NATURAL ODD NUMBERS FROM 1 TO o

o	$\Sigma(o)$	$\Sigma(o^2)$	$\Sigma(o^3)$	$\Sigma(o^4)$	$\Sigma(o^5)$	$\Sigma(o^6)$
1	1	1	1	1	1	1
3	4	10	28	82	244	730
5	9	35	153	707	3,369	16,355
7	16	84	496	3,108	20,176	134,004
9	25	165	1,225	9,669	79,225	665,445
11	36	286	2,556	24,310	240,276	2,437,006
13	49	455	4,753	52,871	611,569	7,263,815
15	64	680	8,128	103,496	1,370,944	18,654,440
17	81	969	13,041	187,017	2,790,801	42,792,009
19	100	1,330	19,900	317,338	5,266,900	89,837,890
21	121	1,771	29,161	511,819	9,351,001	175,604,011
23	144	2,300	41,328	791,660	15,787,344	323,639,900
25	169	2,925	56,953	1,182,285	25,552,969	567,780,525
27	196	3,654	76,636	1,713,726	39,901,876	955,201,014
29	225	4,495	101,025	2,421,007	60,413,025	1,550,024,335
31	256	5,456	130,816	3,344,528	89,042,176	2,437,528,016
33	289	6,545	166,753	4,530,449	128,177,569	3,728,995,985
35	324	7,770	209,628	6,031,074	180,699,444	5,567,261,610
37	361	9,139	260,281	7,905,235	250,043,401	8,132,988,019
39	400	10,660	319,600	10,218,676	340,267,600	11,651,731,780
41	441	12,341	388,521	13,044,437	456,123,801	16,401,836,021
43	484	14,190	468,028	16,463,238	603,132,244	22,723,199,070
45	529	16,215	559,153	20,563,863	787,660,369	31,026,964,695
47	576	18,424	662,976	25,443,544	1,017,005,376	41,806,180,024
49	625	20,825	780,625	31,208,345	1,299,480,625	55,647,467,225
51	676	23,426	913,276	37,973,546	1,644,505,876	73,243,755,026
53	729	26,235	1,062,153	45,864,027	2,062,701,369	95,408,116,155
55	784	29,260	1,228,528	55,014,652	2,565,985,744	123,088,756,780
57	841	32,509	1,413,721	65,570,653	3,167,677,801	157,385,204,029
59	900	35,990	1,619,100	77,688,014	3,882,602,100	199,565,737,670
61	961	39,711	1,846,081	91,533,855	4,727,198,401	251,086,112,031
63	1,024	43,680	2,096,128	107,286,816	5,719,634,944	313,609,614,240
65	1,089	47,905	2,370,753	125,137,441	6,879,925,569	389,028,504,865
67	1,156	52,394	2,671,516	145,288,562	8,230,050,676	479,486,887,034
69	1,225	57,155	3,000,025	167,955,683	9,794,082,025	587,405,050,115
71	1,296	62,196	3,357,936	193,367,364	11,598,311,376	715,505,334,036
73	1,369	67,525	3,746,953	221,765,605	13,671,382,969	866,839,560,325
75	1,444	73,150	4,168,828	253,406,230	16,044,429,844	1,044,818,075,950
77	1,521	79,079	4,625,361	288,559,271	18,751,214,001	1,253,240,456,039
79	1,600	85,320	5,118,400	327,509,352	21,828,270,400	1,496,327,911,560

TABLE A19. PERCENTAGE WHICH S_y IS OF σ_y FOR VARIOUS VALUES OF r

Pointing off two places in this table gives values of $\sqrt{1 - r^2}$, sometimes called the coefficient of alienation

r	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.000	100	100	100	100	100	100	100	100	100	100
.010	100	100	100	100	100	100	100	100	100	100
.020	100	100	100	100	100	100	100	100	100	100
.030	100	100	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9
.040	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9	99.9
.050	99.9	99.9	99.9	99.9	99.9	99.8	99.8	99.8	99.8	99.8
.060	99.8	99.8	99.8	99.8	99.8	99.8	99.8	99.8	99.8	99.8
.070	99.8	99.7	99.7	99.7	99.7	99.7	99.7	99.7	99.7	99.7
.080	99.7	99.7	99.7	99.7	99.6	99.6	99.6	99.6	99.6	99.6
.090	99.6	99.6	99.6	99.6	99.6	99.6	99.5	99.5	99.5	99.5
.100	99.5	99.5	99.5	99.5	99.5	99.4	99.4	99.4	99.4	99.4
.110	99.4	99.4	99.4	99.4	99.4	99.3	99.3	99.3	99.3	99.3
.120	99.3	99.3	99.2	99.2	99.2	99.2	99.2	99.2	99.2	99.2
.130	99.2	99.1	99.1	99.1	99.1	99.1	99.1	99.1	99.0	99.0
.140	99.0	99.0	99.0	99.0	99.0	98.9	98.9	98.9	98.9	98.9
.150	98.9	98.8	98.8	98.8	98.8	98.8	98.8	98.8	98.7	98.7
.160	98.7	98.7	98.7	98.7	98.6	98.6	98.6	98.6	98.6	98.6
.170	98.5	98.5	98.5	98.5	98.5	98.5	98.4	98.4	98.4	98 4
.180	98.4	98.4	98.3	98.3	98.3	98.3	98.2	98.2	98.2	98.2
.190	98.2	98.2	98.1	98.1	98.1	98.1	98.1	98.0	98.0	98.0
.200	98.0	98.0	97.9	97.9	97.9	97.9	97.9	97.8	97.8	97.8
.210	97.8	97.8	97.7	97.7	97.7	97.6	97.6	97.6	97.6	97.6
.220	97.6	97.5	97.5	97.5	97.5	97.4	97.4	97.4	97.4	97.3
.230	97.3	97.3	97.3	97.2	97.2	97.2	97.2	97.2	97.1	97.1
.240	97.1	97.0	97.0	97.0	97.0	97.0	96.9	96.9	96.9	96.8
.250	96.8	96.8	96.8	96.8	96.7	96.7	96.7	96.6	96.6	96.6
.260	96.6	96.5	96.5	96.5	96.4	96.4	96.4	96.4	96.3	96.3
.270	96.3	96.3	96.2	96.2	96.2	96.1	96.1	96.1	96.1	96.0
.280	96.0	96.0	95.9	95.9	95.9	95.8	95.8	95.8	95.8	95.7
.290	95.7	95.7	95.6	95.6	95.6	95.6	95.5	95.5	95.5	95.4
.300	95.4	95.4	95.3	95.3	95.3	95.2	95.2	95.2	95.1	95.1
.310	95.1	95.0	95.0	95.0	94.9	94.9	94.9	94.8	94.8	94.8
.320	94.7	94.7	94.7	94.6	94.6	94.6	94.5	94.5	94.5	94.4
.330	94.4	94.4	94.3	94.3	94.3	94.2	94.2	94.2	94.1	94.1
.340	94.0	94.0	94.0	93.9	93.9	93.9	93.8	93.8	93.8	93.7
.350	93.7	93.6	93.6	93.6	93.5	93.5	93.4	93.4	93.4	93.3
.360	93.3	93.3	93.2	93.2	93.1	93 1	93.1	93.0	93.0	92.9
.370	92.9	92.9	92.8	92.8	92.7	92.7	92.7	92.6	92.6	92.5
.380	92.5	92.5	92.4	92.4	92.3	92.3	92.2	92.2	92.2	92.1
.390	92.1	92.0	92.0	92.0	91.9	91.9	91.8	91.8	91.7	91.7
.400	91.6	91.6	91.6	91.5	91.5	91.4	91.4	91.3	91.3	91.2
.410	91.2	91.2	91.1	91.1	91.0	91.0	90.9	90.9	90.8	90.8
.420	90.8	90.7	90.7	90.6	90.6	90.5	90.5	90.4	90.4	90.3
.430	90.3	90.2	90.2	90.1	90.1	90.0	90.0	90.0	89.9	89.8
.440	89.8	89.8	89.7	89.6	89.6	89.6	89.5	89.4	89.4	89.4
.450	89.3	89.2	89.2	89.2	89.1	89.0	89.0	89.0	88.9	88.8
.460	88.8	88.7	88.7	88.6	88.6	88.5	88.5	88.4	88.4	88.3
.470	88.3	88.2	88.2	88.1	88.0	88.0	87.9	87.9	87.8	87.8
.480	87.7	87.7	87.6	87.6	87.5	87.4	87.4	87.3	87.3	87.2
.490	87.2	87.1	87.1	87.0	87.0	86.9	86.8	86.8	86.7	86.7

TABLE A19. PERCENTAGE WHICH S_y IS OF σ_y FOR VARIOUS VALUES OF r.—(*Continued*)

r	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
.500	86.6	86.5	86.5	86.4	86.4	86.3	86.2	86.2	86.1	86.1
.510	86.0	86.0	85.9	85.8	85.8	85.7	85.7	85.6	85.5	85.5
.520	85.4	85.4	85.3	85.2	85.2	85.1	85.0	85.0	84.9	84.9
.530	84.8	84.7	84.7	84.6	84.6	84.5	84.4	84.4	84.3	84.2
.540	84.2	84.1	84.0	84.0	83.9	83.8	83.8	83.7	83.6	83.6
.550	83.5	83.4	83.4	83.3	83.2	83.2	83.1	83.0	83.0	82.9
.560	82.8	82.8	82.7	82.6	82.6	82.5	82.4	82.4	82.3	82.2
.570	82.2	82.1	82.0	82.0	81.9	81.8	81.7	81.7	81.6	81.5
.580	81.5	81.4	81.3	81.2	81.2	81.1	81.0	81.0	80.9	80.8
.590	80.7	80.7	80.6	80.5	80.4	80.4	80.3	80.2	80.2	80.1
.600	80.0	79.9	79.8	79.8	79.7	79.6	79.6	79.5	79.4	79.3
.610	79.2	79.2	79.1	79.0	78.9	78.8	78.8	78.7	78.6	78.5
.620	78.5	78.4	78.3	78.2	78.1	78.1	78.0	77.9	77.8	77.7
.630	77.7	77.6	77.5	77.4	77.3	77.2	77.2	77.1	77.0	76.9
.640	76.8	76.8	76.7	76.6	76.5	76.4	76.3	76.2	76.2	76.1
.650	76.0	75.9	75.8	75.7	75.6	75.6	75.5	75.4	75.3	75.2
.660	75.1	75.0	75.0	74.9	74.8	74.7	74.6	74.5	74.4	74.3
.670	74.2	74.2	74.1	74.0	73.9	73.8	73.7	73.6	73.5	73.4
.680	73.3	73.2	73.1	73.0	73.0	72.8	72.8	72.7	72.6	72.5
.690	72.4	72.3	72.2	72.1	72.0	71.9	71.8	71.7	71.6	71.5
.700	71.4	71.3	71.2	71.1	71.0	70.9	70.8	70.7	70.6	70.5
.710	70.4	70.3	70.2	70.1	70.0	69.9	69.8	69.7	69.6	69.5
.720	69.4	69.3	69.2	69.1	69.0	68.9	68.8	68.7	68.6	68.4
.730	68.3	68.2	68.1	68.0	67.9	67.8	67.7	67.6	67.5	67.4
.740	67.3	67.2	67.0	66.9	66.8	66.7	66.6	66.5	66.4	66.3
.750	66.1	66.0	65.9	65.8	65.7	65.6	65.5	65.3	65.2	65.1
.760	65 0	64.9	64 8	64.6	64.5	64.4	64.3	64.2	64.0	63.9
.770	63.8	63.7	63.6	63.4	63 3	63.2	63.1	63.0	62 8	62 7
.780	62 6	62 4	62.3	62.2	62.1	62.0	61.8	61.7	61.6	61.4
.790	61.3	61.2	61.0	60.9	60.8	60.7	60.5	60.4	60.3	60.1
.800	60 0	59 9	59.7	59.6	59 5	59.3	59.2	59.1	58.9	58.8
.810	58.6	58.5	58.4	58.2	58.1	58.0	57.8	57.7	57.5	57.4
820	57.2	57.1	57.0	56.8	56.7	56.5	56.4	56.2	56.1	55.9
.830	55.8	55.6	55 5	55.3	55.2	55.0	54.9	54.7	54.6	54.4
.840	54.3	54.1	54.0	53.8	53.6	53.5	53.3	53.2	53.0	52.8
.850	52.7	52.5	52.4	52.2	52.0	51.9	51.7	51.5	51.4	51.2
.860	51.0	50.9	50.7	50.5	50.4	50.2	50.0	49.8	49.7	49.5
.870	49.3	49.1	49.0	48.8	48.6	48.4	48.2	48.0	47.9	47.7
.880	47.5	47.3	47.1	46.9	46.8	46.6	46.4	46.2	46.0	45.8
.890	45.6	45.4	45.2	45.0	44.8	44.6	44.4	44.2	44.0	43.8
.900	43 6	43.4	43.2	43.0	42.8	42.5	42.3	42.1	41.9	41.7
.910	41.5	41.2	41.0	40.8	40.6	40.4	40.1	39.9	39.7	39.4
.920	39.2	39.0	38.7	38.5	38.2	38.0	37.8	37.5	37.3	37.0
.930	36.8	36.5	36.2	36.0	35.7	35.5	35.2	34.9	34.7	34.4
.940	34.1	33.8	33.6	33.3	33.0	32.7	32.4	32.1	31.8	31.5
.950	31.2	30.9	30.6	30.3	30.0	29.7	29.3	29.0	28.7	28.3
.960	28 0	27.6	27.3	27.0	26.6	26.2	25.8	25.5	25.1	24.7
.970	24 3	23.9	23.5	23.1	22.6	22.2	21.8	21.3	20.9	20.4
.980	19 9	19.4	18.9	18.4	17.8	17.3	16.7	16.1	15.4	14.8
.990	14.1	13.4	12.6	11.8	10.9	10.0	8.9	7.7	6.3	4.5
1.000	0000									

TABLE A20. CHANCES OF DIFFERING FROM THE MEAN BY GIVEN NUMBERS OF STANDARD DEVIATIONS

Chances that an item chosen at random from a normal distribution will lie as far from the mean as the number of standard deviations stated.[1] *Example:* The chances are 0.0357 (that is, 357 chances out of 10,000) that an item chosen at random from a normal distribution will differ from the mean by as much as 2.1 standard deviations.

x/σ	Chances	x/σ	Chances
0.0	1.000	2.0	0.0454
0.1	0.920	2.1	0.0357
0.2	0.841	2.2	0.0277
0.3	0.764	2.3	0.0214
0.4	0.689	2.4	0.0163
0.5	0.617	2.5	0.0124
0.6	0.549	2.6	0.00932
0.7	0.484	2.7	0.00694
0.8	0.424	2.8	0.00512
0.9	0.368	2.9	0.00374
1.0	0.317	3.0	0.00270
1.1	0.271	3.5	0.000465
1.2	0.230	4.0	0.0000634
1.3	0.193	4.5	0.0000068
1.4	0.162	5.0	0.000000573
1.5	0.134		
1.6	0.109		
1.7	0.0891		
1.8	0.0719		
1.9	0.0574		

[1] The data in this table were computed from information in J. W. Glover's "Tables of Probability and Statistical Functions," Geo. Wahr, Ann Arbor, Michigan, 1930, except for the last four entries, which are based on H. O. Rugg, "Statistical Methods Applied to Education," Houghton Mifflin Company, Boston, 1917.

TABLE A21. CHANCES OF DIFFERING FROM THE MEAN IN A GIVEN DIRECTION BY MORE THAN GIVEN NUMBERS OF STANDARD DEVIATIONS

Chances that an item chosen at random from a normal distribution will be on the same side of the mean as the item chosen, and distant from the mean by as much as the stated number of standard deviations.[1] *Example:* The chances are 0.0179 (that is, 179 out of 10,000) that an item chosen at random from a normal distribution will be above the mean and removed from it by as much as 2.1 standard deviations. The chances that it will be this far below the mean are also 0.0179.

x/σ	Chances	x/σ	Chances
0.0	0.500	2.0	0.0227
0.1	0.460	2.1	0.0179
0.2	0.421	2.2	0.0139
0.3	0.382	2.3	0.0107
0.4	0.345	2.4	0.00820
0.5	0.309	2.5	0.00621
0.6	0.274	2.6	0.00547
0.7	0.242	2.7	0.00347
0.8	0.212	2.8	0.00256
0.9	0.184	2.9	0.00187
1.0	0.159	3.0	0.00135
1.1	0.136	3.5	0.000233
1.2	0.114	4.0	0.0000317
1.3	0.0968	4.5	0.0000034
1.4	0.0807	5.0	0.000000287
1.5	0.0668		
1.6	0.0548		
1.7	0.0446		
1.8	0.0359		
1.9	0.0287		

[1] Data from same sources as Table A20.

Table A22. Proportion of Cases in Class 0 of Poisson Distribution

A	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0		.9900	.9802	.9704	.9608	.9512	.9418	.9324	.9231	.9139
0.1	.9048	.8958	.8869	.8781	.8694	.8607	.8521	.8437	.8353	.8270
0.2	.8187	.8106	.8025	.7945	.7866	.7788	.7711	.7634	.7558	.7483
0.3	.7408	.7334	.7261	.7189	.7118	.7047	.6977	.6907	.6839	.6771
0.4	.6703	.6637	.6571	.6505	.6440	.6376	.6313	.6250	.6188	.6126
0.5	.6065	.6005	.5945	.5886	.5828	.5770	.5712	.5655	.5599	.5543
0.6	.5488	.5434	.5380	.5326	.5273	.5221	.5169	.5117	.5066	.5016
0.7	.4966	.4916	.4868	.4819	.4771	.4724	.4677	.4630	.4584	.4538
0.8	.4493	.4449	.4404	.4361	.4317	.4274	.4232	.4190	.4148	.4107
0.9	.4066	.4025	.3985	.3946	.3906	.3867	.3829	.3791	.3753	.3716
1.0	.3679	.3642	.3606	.3570	.3535	.3499	.3465	.3430	.3396	3362
1.1	.3329	.3296	.3263	.3230	.3198	.3166	.3135	.3104	.3073	.3042
1.2	.3012	.2982	.2952	.2923	.2894	.2865	.2837	.2808	.2780	.2753
1.3	.2725	.2698	.2671	.2645	.2619	.2592	.2567	.2541	.2516	.2491
1.4	.2466	.2441	.2417	.2393	.2369	.2346	.2322	.2299	.2276	.2254
1.5	.2231	.2209	.2187	.2165	.2144	.2123	.2101	.2081	.2060	.2039
1.6	.2019	.1999	.1979	.1959	.1940	.1921	.1901	.1882	.1864	.1845
1.7	.1827	.1809	.1791	.1773	.1755	.1738	.1721	.1703	.1686	.1670
1.8	.1653	1637	.1620	.1604	.1588	.1572	.1557	.1541	.1526	.1511
1.9	.1496	.1481	.1466	.1452	.1437	.1423	.1409	.1395	.1381	.1367
2.0	.1353	.1340	.1327	.1313	.1300	.1287	.1275	.1262	.1249	.1237
2.1	.1225	.1212	.1200	.1188	.1177	.1165	.1153	.1142	.1130	.1119
2.2	.1108	.1097	.1086	.1075	.1065	.1054	.1044	.1033	.1023	.1013
2.3	.1003	.0993	.0983	.0973	.0963	.0954	.0944	.0935	.0925	.0916
2.4	.0907	.0898	.0889	.0880	.0872	.0863	.0854	.0846	.0837	.0829
2.5	.0821	.0813	.0805	.0797	.0789	.0781	.0773	.0765	.0758	0750
2.6	.0743	.0735	.0728	.0721	.0714	.0707	.0699	.0693	.0686	0679
2.7	.0672	.0665	.0659	.0652	.0646	.0639	.0633	.0626	.0620	0614
2.8	0608	.0602	.0596	.0590	.0584	.0578	.0573	.0567	.0561	.0556
2.9	.0550	.0545	.0539	.0534	.0529	.0523	.0518	.0513	.0508	.0503
3.0	.0498	.0493	.0488	.0483	.0478	.0474	.0469	0464	.0460	.0455
3 1	.0450	.0446	.0442	.0437	.0433	.0429	.0424	0420	.0416	0412
3.2	.0408	.0404	.0400	.0396	.0392	.0388	.0384	.0380	.0376	.0373
3.3	.0369	.0365	.0362	.0358	.0354	.0351	.0347	.0344	.0340	.0337
3.4	.0334	.0330	.0327	.0324	.0321	.0317	.0314	.0311	.0308	.0305
3.5	.0302	.0299	.0296	.0293	.0290	.0287	.0284	.0282	.0279	.0276
3.6	.0273	.0271	.0268	.0265	.0263	.0260	.0257	.0255	.0252	.0250
3.7	.0247	.0245	.0242	.0240	.0238	.0235	.0233	.0231	.0228	.0226
3.8	.0224	.0221	.0219	.0217	.0215	.0213	.0211	.0209	.0207	0204
3.9	.0202	.0200	.0198	.0196	.0194	.0193	.0191	.0189	.0187	.0185
4.0	.0183	.0181	.0180	.0178	.0176	.0174	.0172	.0171	.0169	.0167
4.1	.0166	.0164	.0162	.0161	.0159	.0158	.0156	.0155	.0153	.0151
4.2	.0150	.0148	.0147	.0146	.0144	.0143	.0141	.0140	.0138	.0137
4.3	.0136	.0134	.0133	.0132	.0130	.0129	.0128	.0127	.0125	.0124
4.4	.0123	.0122	.0120	.0119	0118	.0117	.0116	.0114	.0113	.0112
4.5	.0111	.0110	.0109	.0108	.0107	.0106	.0105	.0104	.0103	.0102
4.6	.0101	.0100	.0099	.0098	.0097	.0096	0095	0094	.0093	.0092
4.7	.0091	.0090	.0089	.0088	.0087	.0087	.0086	.0085	.0084	.0083
4.8	.0082	.0081	.0081	.0080	.0079	.0078	.0078	.0077	.0076	.0075
4.9	.0074	.0074	.0073	.0072	.0072	.0071	.0070	.0069	.0069	.0068

TABLE A22. PROPORTION OF CASES IN CLASS 0 OF POISSON DISTRIBUTION.—(*Continued*)

Each of the following figures is preceded by two zeros

A	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
5	6738	6097	5517	4992	4517	4087	3698	3346	3028	2739
6	2479	2243	2029	1836	1662	1503	1360	1231	1114	1008

The following have 3 ciphers omitted

A	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
7	912	825	747	676	611	553	500	453	410	371
8	335	304	275	249	225	203	184	167	151	136
9	123	112	101							

The following have 4 ciphers omitted

A	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
9				914	827	749	677	613	555	502
10	454	411	372	336	304	275	249	225	204	185
11	167	151	137	124	112	101				

Example of reading above tables:

If A is 10.7, the proportion of cases in Class 0 (the first class in the Poisson Series) is .0000225.

$$y = \frac{A^x e^{-A}}{x!}$$

where A is the arithmetic mean with the classes numbered 0, 1, 2, 3, . . . n; x is the number of the class in question; and y is the proportion of the total population in the given class.

TABLE A23. TABLES OF P FOR THE CHI-SQUARE TEST OF GOODNESS OF FIT

χ^2	$n = 2$ $n' = 3$	$n = 3$ $n' = 4$	$n = 4$ $n' = 5$	$n = 5$ $n' = 6$	$n = 6$ $n' = 7$	$n = 7$ $n' = 8$	$n = 8$ $n' = 9$	$n = 9$ $n' = 10$	$n = 10$ $n' = 11$
1	.606531	.801253	.909796	.962566	.985612	.994829	.998249	.999438	.999828
2	.367879	.572407	.735759	.849146	.919699	.959840	.981012	.991468	.996340
3	.223130	.391625	.557825	.699986	.808847	.885002	.934357	.964295	.981424
4	.135335	.261464	.406006	.549416	.676676	.779778	.857123	.911413	.947347
5	.082085	.171797	.287298	.415880	.543813	.659963	.757576	.834308	.891178
6	.049787	.111610	.199148	.306219	.423190	.539750	.647232	.739919	.815263
7	.030197	.071897	.135888	.220640	.320847	.428880	.536632	.637119	.725444
8	.018316	.046012	.091578	.156236	.238103	.332594	.433470	.534146	.628837
9	.011109	.029291	.061099	.109064	.173578	.252656	.342296	.437274	.532104
10	.006738	.018566	.040428	.075235	.124652	.188573	.265026	.350485	.440493
11	.004087	.011726	.026564	.051380	.088376	.138619	.201699	.275709	.357518
12	.002479	.007383	.017351	.034787	.061969	.100558	.151204	.213308	.285057
13	.001503	.004637	.011276	.023379	.043036	.072109	.111850	.162607	.223672
14	.000912	.002905	.007295	.015609	.029636	.051181	.081765	.122325	.172992
15	.000553	.001817	.004701	.010363	.020256	.036000	.059145	.090937	.132061
16	.000335	.001134	.003019	.006844	.013754	.025116	.042380	.066881	.099632
17	.000203	.000707	.001933	.004500	.009283	.017396	.030109	.048716	.074364
18	.000123	.000440	.001234	.002947	.006232	.011970	.021226	.035174	.054964
19	.000075	.000273	.000786	.001922	.004164	.008187	.014860	.025193	.040263
20	.000045	.000170	.000499	.001250	.002769	.005570	.010336	.017913	.029253
21	.000028	.000105	.000317	.000810	.001835	.003770	.007147	.012650	.021093
22	.000017	.000065	.000200	.000524	.001211	.002541	.004916	.008880	.015105
23	.000010	.000040	.000127	.000338	.000796	.001705	.003364	.006197	.010747
24	.000006	.000025	.000080	.000217	.000522	.001139	.002292	.004301	.007600
25	.000004	.000016	.000050	.000139	.000341	.000759	.001554	.002971	.005345
26	.000002	.000010	.000032	.000090	.000223	.000504	.001050	.002043	.003740
27	.000001	.000006	.000020	.000057	.000145	.000333	.000707	.001399	.002604
28	.000001	.000004	.000012	.000037	.000094	.000220	.000474	.000954	.001805
29	.000001	.000002	.000008	.000023	.000061	.000145	.000317	.000648	.001246
30	.000000	.000001	.000005	.000015	.000039	.000095	.000211	.000439	.000857
40	.000000	.000000	.000000	.000000	.000001	.000001	.000003	.000008	.000017
50	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
60	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000
70	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000

NOTE: This is the Elderton table, taken from Pearson's "Tables for Statisticians and Biometricians" by arrangement with the publishers.

TABLE A23. TABLES OF P FOR THE CHI-SQUARE TEST OF GOODNESS OF FIT. (*Continued*)

χ^2	$n = 11$ $n' = 12$	$n = 12$ $n' = 13$	$n = 13$ $n' = 14$	$n = 14$ $n' = 15$	$n = 15$ $n' = 16$	$n = 16$ $n' = 17$	$n = 17$ $n' = 18$	$n = 18$ $n' = 19$	$n = 19$ $n' = 20$
1	.999950	.999986	.999997	.999999	1.	1.	1.	1.	1.
2	.998496	.999406	.999774	.999917	.999970	.999990	.999997	.999999	1.
3	.990726	.995544	.997934	.999074	.999598	.999830	.999931	.999972	.999989
4	.969917	.983436	.991191	.995466	.997737	.998903	.999483	.999763	.999894
5	.931167	.957979	.975193	.985813	.992127	.995754	.997771	.998860	.999431
6	.873365	.916082	.946153	.966491	.979749	.988095	.993187	.996197	.997929
7	.799073	.857613	.902151	.934711	.957650	.973260	.983549	.990125	.994213
8	.713340	.785131	.843601	.889327	.923783	.948867	.966547	.978637	.986671
9	.621892	.702931	.772943	.831051	.877517	.913414	.940261	.959743	.973479
10	.530387	.615960	.693934	.762183	.819739	.866628	.903610	.931906	.952946
11	.443263	.528919	.610817	.686036	.752594	.809485	.856564	.894357	.923839
12	.362642	.445680	.527643	.606303	.679028	.743980	.800136	.847327	.885624
13	.293326	.369041	.447812	.526524	.602298	.672758	.736186	.791573	.838571
14	.232993	.300708	.373844	.449711	.525529	.598714	.667102	.729091	.783691
15	.182498	.241436	.307354	.378154	.451418	.524638	.595482	.661967	.722598
16	.141130	.191236	.249129	.313374	.382051	.452961	.523834	.592547	.657277
17	.107876	.149597	.199304	.256178	.318864	.385597	.454366	.523105	.589868
18	.081581	.115691	.157520	.206781	.262666	.323897	.388841	.455653	.522438
19	.061094	.088529	.123104	.164949	.213734	.268663	.328532	.391823	.456836
20	.045341	.067086	.095210	.130141	.171932	.220220	.274229	.332819	.394578
21	.033371	.050380	.072929	.101632	.136830	.178510	.226291	.279413	.336801
22	.024374	.037520	.055362	.078614	.107804	.143191	.184719	.231985	.284256
23	.017676	.027726	.041677	.060270	.084140	.113735	.149251	.190590	.237342
24	.012733	.020341	.031130	.045822	.065093	.089504	.1[illegible]9435	.155028	.196152
25	.009117	.014822	.023084	.034566	.049943	.069824	.094710	.124915	.160542
26	.006490	.010734	.017001	.025887	.038023	.054028	.074461	.099758	.130189
27	.004595	.007727	.012441	.019254	.028736	.041483	.058068	.078995	.104653
28	.003238	.005532	.009050	.014228	.021569	.031620	.044938	.062055	.083428
29	.002270	.003940	.006546	.010450	.016085	.023936	.034526	.048379	.065985
30	.001585	.002792	.004710	.007632	.011921	.018002	.026345	.037446	.051798
40	.000036	.000072	.000138	.000255	.000453	.000778	.001294	.002087	.003272
50	.000001	.000001	.000003	.000006	.000012	.000023	.000042	.000075	.000131
60	.000000	.000000	.000000	.000000	.000000	.000001	.000001	.000002	.000004
70	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000	.000000

TABLE A23. TABLES OF P FOR THE CHI-SQUARE TEST OF GOODNESS OF FIT. (*Continued*)

χ^2	$n = 20$ $n' = 21$	$n = 21$ $n' = 22$	$n = 22$ $n' = 23$	$n = 23$ $n' = 24$	$n = 24$ $n' = 25$	$n = 25$ $n' = 26$	$n = 26$ $n' = 27$	$n = 27$ $n' = 28$	$n = 28$ $n' = 29$	$n = 29$ $n' = 30$
1	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
2	1.	1.	1.	1.	1.	1.	1.	1.	1.	1.
3	.999996	.999998	.999999	1.	1.	1.	1.	1.	1.	1.
4	.999954	.999980	.999992	.999997	.999999	1.	1.	1.	1.	1.
5	.999722	.999868	.999939	.999972	.999987	.999994	.999998	.999999	1.	1.
6	.998898	.999427	.999708	.999855	.999929	.999966	.999984	.999993	.999997	.999999
7	.996685	.998142	.998980	.999452	.999711	.999851	.999924	.999962	.999981	.999991
8	.991868	.995143	.997160	.998371	.999085	.999494	.999726	.999853	.999924	.999960
9	.982907	.989214	.993331	.995957	.997595	.998596	.999194	.999546	.999748	.999863
10	.968171	.978912	.986304	.991277	.994547	.996653	.997981	.998803	.999302	.999599
11	.946223	.962787	.974749	.983189	.989012	.992946	.995549	.997239	.998315	.998988
12	.916076	.939617	.957379	.970470	.979908	.986567	.991173	.994294	.996372	.997728
13	.877384	.908624	.933161	.951990	.966121	.976501	.983974	.989247	.992900	.995384
14	.830496	.869599	.901479	.926871	.946650	.961732	.973000	.981254	.987189	.991377
15	.776408	.822952	.862238	.894634	.920759	.941383	.957334	.969432	.978436	.985015
16	.716624	.769650	.815886	.855268	.888076	.914828	.936203	.952947	.965819	.975536
17	.652974	.711106	.763362	.809251	.848662	.881793	.909083	.931122	.948589	.962181
18	.587408	.649004	.705988	.757489	.803008	.842390	.875773	.903519	.926149	.944272
19	.521826	.585140	.645328	.701224	.751990	.797120	.836430	.870001	.898136	.921288
20	.457930	.521261	.583040	.641912	.696776	.746825	.791556	.830756	.864464	.892927
21	.397132	.458944	.520738	.581087	.638725	.692609	.741964	.786288	.825349	.859149
22	.340511	.399510	.459889	.520252	.579267	.635744	.688697	.737377	.781291	.820189
23	.288795	.343979	.401730	.460771	.519798	.577564	.632947	.685013	.733041	.776543
24	.242392	.293058	.347229	.403808	.461597	.519373	.575965	.630316	.681535	.728932
25	.201431	.247164	.297075	.350285	.405760	.462373	.518975	.574462	.627835	.678248
26	.165812	.206449	.251682	.300866	.353165	.407598	.463105	.518600	.573045	.625491
27	.135264	.170853	.211226	.255967	.304453	.355884	.409333	.463794	.518247	.571705
28	.109399	.140151	.175681	.215781	.260040	.307853	.358458	.410973	.464447	.517913
29	.087759	.114002	.144861	.180310	.220131	.263916	.311082	.360899	.412528	.465066
30	.069854	.091988	.118464	.149402	.184752	.224289	.267611	.314154	.363218	.414004
40	.004995	.007437	.010812	.015369	.021387	.029164	.039012	.051237	.066128	.083937
50	.000221	.000365	.000586	.000921	.001416	.002131	.003144	.004551	.006467	.009032
60	.000007	.000013	.000022	.000038	.000064	.000104	.000168	.000264	.000407	.000618
70	.000000	.000000	.000001	.000001	.000002	.000004	.000007	.000011	.000019	.000030

TABLE A24. TABLE OF CHI SQUARE[1]

n	P = .99	.98	.95	.90	.80	.70	.50	.30	.20	.10	.05	.02	.01
1	.000157	.000628	.00393	.0158	.0642	.148	.455	1.074	1.642	2.706	3.841	5.412	6.635
2	.0201	.0404	.103	.211	.446	.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210
3	.115	.185	.352	.584	1.005	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.341
4	.297	.429	.711	1.064	1.649	2.195	3.357	4.878	5.989	7.779	9.488	11.668	13.277
5	.554	.752	1.145	1.610	2.343	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086
6	.872	1.134	1.635	2.204	3.070	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475
8	1.646	2.032	2.733	3.490	4.594	5.527	7.344	9.524	11.030	13.362	15.507	18.168	20.090
9	2.088	2.532	3.325	4.168	5.380	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666
10	2.558	3.059	3.940	4.865	6.179	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209
11	3.053	3.609	4.575	5.578	6.989	8.148	10.341	12.899	14.631	17.275	19.675	22.618	24.725
12	3.571	4.178	5.226	6.304	7.807	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217
13	4.107	4.765	5.892	7.042	8.634	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688
14	4.660	5.368	6.571	7.790	9.467	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338	18.418	20.465	23.542	26.296	29.633	32.000
17	6.408	7.255	8.672	10.085	12.002	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409
18	7.015	7.906	9.390	10.865	12.857	14.440	17.338	20.601	22.760	25.989	28.869	32.346	34.805
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337	22.775	25.038	28.412	31.410	35.020	37.566
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337	23.858	26.171	29.615	32.671	36.343	38.932
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337	24.939	27.301	30.813	33.924	37.659	40.289
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638
24	10.856	11.992	13.848	15.659	18.062	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337	28.172	30.675	34.382	37.652	41.566	44.314
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336	29.246	31.795	35.563	38.885	42.856	45.642
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336	30.319	32.912	36.741	40.113	44.140	46.963
28	13.565	14.847	16.928	18.939	21.588	23.647	27.336	31.391	34.027	37.916	41.337	45.419	48.278
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336	32.461	35.139	39.087	42.557	46.693	49.588
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892

[1] Table A24 is abridged from Table IV of R. A. Fisher and F. Yates, "Statistical Tables for Biological, Agricultural and Medical Research," Oliver & Boyd, Ltd., Edinburgh, 1938, by permission of the authors and publishers.

TABLE A25.[1]

Values of n_2	Values of n_1, the number of degrees													
	1		2		3		4		5		6		7	
P =	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
1	161	4,052	200	4,999	216	5,403	225	5,625	230	5,764	234	5,859	237	5,928
2	18.51	98.49	19.00	99.01	19.16	99.17	19.25	99.25	19.30	99.30	19.33	99.33	19.36	99.34
3	10.13	34.12	9.55	30.81	9.28	29.46	9.12	28.71	9.01	28.24	8.94	27.91	8.88	27.67
4	7.71	21.20	6.94	18.00	6.59	16.69	6.39	15.98	6.26	15.52	6.16	15.21	6.09	14.98
5	6.61	16.26	5.79	13.27	5.41	12.06	5.19	11.39	5.05	10.97	4.95	10.67	4.88	10.45
6	5.99	13.74	5.14	10.92	4.76	9.78	4.53	9.15	4.39	8.75	4.28	8.47	4.21	8.26
7	5.59	12.25	4.74	9.55	4.35	8.45	4.12	7.85	3.97	7.46	3.87	7.19	3.79	7.00
8	5.32	11.26	4.46	8.65	4.07	7.59	3.84	7.01	3.69	6 63	3.58	6.37	3.50	6.19
9	5.12	10.56	4.26	8.02	3.86	6.99	3.63	6.42	3.48	6.06	3.37	5.80	3.29	5.62
10	4.96	10.04	4.10	7.56	3.71	6.55	3.48	5.99	3.33	5.64	3.22	5.39	3.14	5.21
11	4.84	9.65	3.98	7.20	3.59	6.22	3.36	5.67	3.20	5.32	3.09	5.07	3.01	4.88
12	4.75	9.33	3.88	6.93	3.49	5.95	3.26	5.41	3.11	5.06	3.00	4.82	2.92	4.65
13	4.67	9.07	3.80	6.70	3.41	5.74	3.18	5.20	3.02	4.86	2.92	4.62	2.84	4.44
14	4.60	8.86	3.74	6.51	3.34	5.56	3.11	5.03	2.96	4.69	2.85	4.46	2.77	4.28
15	4.54	8.68	3.68	6.36	3.29	5.42	3.06	4.89	2.90	4.56	2.79	4.32	2.70	4.14
16	4.49	8.53	3.63	6.23	3.24	5.29	3.01	4.77	2.85	4.44	2.74	4.20	2.66	4.03
17	4.45	8.40	3.59	6.11	3.20	5.18	2.96	4.67	2.81	4.34	2.70	4.10	2.62	3.93
18	4.41	8.28	3.55	6.01	3.16	5.09	2.93	4.58	2.77	4.25	2.66	4.01	2.58	3.85
19	4.38	8.18	3.52	5.93	3.13	5.01	2.90	4.50	2.74	4.17	2.63	3.94	2.55	3.77
20	4.35	8.10	3.49	5.85	3.10	4.94	2.87	4.43	2.71	4.10	2.60	3.87	2.52	3.71
21	4.32	8.02	3.47	5.78	3.07	4.87	2.84	4.37	2.68	4.04	2.57	3.81	2.49	3.65
22	4.30	7.94	3.44	5.72	3.05	4.82	2.82	4.31	2.66	3.99	2.55	3.76	2.47	3.59
23	4.28	7.88	3.42	5.66	3.03	4.76	2.80	4.26	2.64	3.94	2.53	3.71	2.45	3.54
24	4.26	7.82	3.40	5.61	3.01	4.72	2.78	4.22	2.62	3.90	2.51	3.67	2.43	3.50
25	4.24	7.77	3.38	5.57	2.99	4.68	2.76	4.18	2.60	3.86	2.49	3.63	2.41	3.46
26	4.22	7.72	3.37	5.53	2.98	4.64	2.74	4.14	2.59	3.82	2.47	3.59	2.39	3.42
27	4.21	7.68	3.35	5.49	2.96	4.60	2.73	4.11	2.57	3.79	2.46	3.56	2.37	3.39
28	4.20	7.64	3.34	5.45	2.95	4.57	2.71	4.07	2.56	3.76	2.44	3.53	2.36	3.36
29	4.18	7.60	3.33	5.42	2.93	4.54	2.70	4.04	2.54	3.73	2.43	3.50	2.35	3.33
30	4.17	7.56	3.32	5.39	2.92	4.51	2.69	4.02	2.53	3.70	2.42	3.47	2.34	3.30
32	4.15	7.50	3.30	5.34	2.90	4.46	2.67	3.97	2.51	3.66	2.40	3.42	2.32	3.25
34	4.13	7.44	3.28	5.29	2.88	4.42	2.65	3.93	2.49	3.61	2.38	3.38	2.30	3.21
38	4.10	7.35	3.25	5.21	2.85	4.34	2.62	3.86	2.46	3.54	2.35	3.32	2.26	3.15
42	4.07	7.27	3.22	5.15	2.83	4.29	2.59	3.80	2.44	3.49	2.32	3.26	2.24	3.10
46	4.05	7.21	3.20	5.10	2.81	4.24	2.57	3.76	2.42	3.44	2.30	3.22	2.22	3.05
50	4.03	7.17	3.18	5.06	2.79	4.20	2.56	3 72	2.40	3.41	2.29	3.18	2.20	3.02
60	4.00	7.08	3.15	4.98	2.76	4.13	2.52	3.65	2.37	3.34	2.25	3.12	2.17	2.95
80	3.96	6.96	3.11	4.88	2.72	4.04	2.48	3.56	2.33	3.25	2.21	3.04	2.12	2.87
100	3.94	6.90	3.09	4.82	2.70	3.98	2.46	3.51	2.30	3.20	2.19	2.99	2.10	2.82
200	3.89	6.76	3.04	4.71	2.65	3.88	2.41	3.41	2.26	3.11	2.14	2.90	2.05	2.73
1,000	3.85	6.66	3.00	4.62	2.61	3.80	2.38	3.34	2.22	3.04	2.10	2.82	2.02	2.66
∞	3.84	6.64	2.99	4.60	2.60	3.78	2.37	3.32	2.21	3.02	2.09	2.80	2.01	2.64

[1] Reproduced, by permission of the author and publishers, from George W. Snedecor, "Statistical Methods Iowa, 1940.

Table of F

of freedom of the greater variance

8		10		12		16		20		30		50		100		∞	
0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01	0.05	0.01
239	5,981	242	6,056	244	6,106	246	6,169	248	6,208	250	6,258	252	6,302	253	6,334	254	6,366
19.37	99.36	19.39	99.40	19.41	99.42	19.43	99.44	19.44	99.45	19.46	99.47	19.47	99.48	19.49	99.49	19.50	99.50
8.84	27.49	8.78	27.23	8.74	27.05	8.69	26.83	8.66	26.69	8.62	26.50	8.58	26.35	8.56	26.23	8.53	26.12
6.04	14.80	5.96	14.54	5.91	14.37	5.84	14.15	5.80	14.02	5.74	13.83	5.70	13.69	5.66	13.57	5.63	13.46
4.82	10.27	4.74	10.05	4.68	9.89	4.60	9.68	4.56	9.55	4.50	9.38	4.44	9.24	4.40	9.13	4.36	9.02
4.15	8.10	4.06	7.87	4.00	7.72	3.92	7.52	3.87	7.39	3.81	7.23	3.75	7.09	3.71	6.99	3.67	6.88
3.73	6.84	3.63	6.62	3.57	6.47	3.49	6.27	3.44	6.15	3.38	5.98	3.32	5.85	3.28	5.75	3.23	5.65
3.44	6.03	3.34	5.82	3.28	5.67	3.20	5.48	3.15	5.36	3.08	5.20	3.03	5.06	2.98	4.96	2.93	4.86
3.23	5.47	3.13	5.26	3.07	5.11	2.98	4.92	2.93	4.80	2.86	4.64	2.80	4.51	2.76	4.41	2.71	4.31
3.07	5.06	2.97	4.85	2.91	4.71	2.82	4.52	2.77	4.41	2.70	4.25	2.64	4.12	2.59	4.01	2.54	3.91
2.95	4.74	2.86	4.54	2.79	4.40	2.70	4.21	2.65	4.10	2.57	3.94	2.50	3.80	2.45	3.70	2.40	3.60
2.85	4.50	2.76	4.30	2.69	4.16	2.60	3.98	2.54	3.86	2.46	3.70	2.40	3.56	2.35	3.46	2.30	3.36
2.77	4.30	2.67	4.10	2.60	3.96	2.51	3.78	2.46	3.67	2.38	3.51	2.32	3.37	2.26	3.27	2.21	3.16
2.70	4.14	2.60	3.94	2.53	3.80	2.44	3.62	2.39	3.51	2.31	3.34	2.24	3.21	2.19	3.11	2.13	3.00
2.64	4.00	2.55	3.80	2.48	3.67	2.39	3.48	2.33	3.36	2.25	3.20	2.18	3.07	2.12	2.97	2.07	2.87
2.59	3.89	2.49	3.69	2.42	3.55	2.33	3.37	2.28	3.25	2.20	3.10	2.13	2.96	2.07	2.86	2.01	2.75
2.55	3.79	2.45	3.59	2.38	3.45	2.29	3.27	2.23	3.16	2.15	3.00	2.08	2.86	2.02	2.76	1.96	2.65
2.51	3.71	2.41	3.51	2.34	3.37	2.25	3.19	2.19	3.07	2.11	2.91	2.04	2.78	1.98	2.68	1.92	2.57
2.48	3.63	2.38	3.43	2.31	3.30	2.21	3.12	2.15	3.00	2.07	2.84	2.00	2.70	1.94	2.60	1.88	2.49
2.45	3.56	2.35	3.37	2.28	3.23	2.18	3.05	2.12	2.94	2.04	2.77	1.96	2.63	1.90	2.53	1.84	2.42
2.42	3.51	2.32	3.31	2.25	3.17	2.15	2.99	2.09	2.88	2.00	2.72	1.93	2.58	1.87	2.47	1.81	2.36
2.40	3.45	2.30	3.26	2.23	3.12	2.13	2.94	2.07	2.83	1.98	2.67	1.91	2.53	1.84	2.42	1.78	2.31
2.38	3.41	2.28	3.21	2.20	3.07	2.10	2.89	2.04	2.78	1.96	2.62	1.88	2.48	1.82	2.37	1.76	2.26
2.36	3.36	2.26	3.17	2.18	3.03	2.09	2.85	2.02	2.74	1.94	2.58	1.86	2.44	1.80	2.33	1.73	2.21
2.34	3.32	2.24	3.13	2.16	2.99	2.06	2.81	2.00	2.70	1.92	2.54	1.84	2.40	1.77	2.29	1.71	2.17
2.32	3.29	2.22	3.09	2.15	2.96	2.05	2.77	1.99	2.66	1.90	2.50	1.82	2.36	1.76	2.25	1.69	2.13
2.30	3.26	2.20	3.06	2.13	2.93	2.03	2.74	1.97	2.63	1.88	2.47	1.80	2.33	1.74	2.21	1.67	2.10
2.29	3.23	2.19	3.03	2.12	2.90	2.02	2.71	1.96	2.60	1.87	2.44	1.78	2.30	1.72	2.18	1.65	2.06
2.28	3.20	2.18	3.00	2.10	2.87	2.00	2.68	1.94	2.57	1.85	2.41	1.77	2.27	1.71	2.15	1.64	2.03
2.27	3.17	2.16	2.98	2.09	2.84	1.99	2.66	1.93	2.55	1.84	2.38	1.76	2.24	1.69	2.13	1.62	2.01
2.25	3.12	2.14	2.94	2.07	2.80	1.97	2.62	1.91	2.51	1.82	2.34	1.74	2.20	1.67	2.08	1.59	1.96
2.23	3.08	2.12	2.89	2.05	2.76	1.95	2.58	1.89	2.47	1.80	2.30	1.71	2.15	1.64	2.04	1.57	1.91
2.19	3.02	2.09	2.82	2.02	2.69	1.92	2.51	1.85	2.40	1.76	2.22	1.67	2.08	1.60	1.97	1.53	1.84
2.17	2.96	2.06	2.77	1.99	2.64	1.89	2.46	1.82	2.35	1.73	2.17	1.64	2.02	1.57	1.91	1.49	1.78
2.14	2.92	2.04	2.73	1.97	2.60	1.87	2.42	1.80	2.30	1.71	2.13	1.62	1.98	1.54	1.86	1.46	1.72
2.13	2.88	2.02	2.70	1.95	2.56	1.85	2.39	1.78	2.26	1.69	2.10	1.60	1.94	1.52	1.82	1.44	1.68
2.10	2.82	1.99	2.63	1.92	2.50	1.81	2.32	1.75	2.20	1.65	2.03	1.56	1.87	1.48	1.74	1.39	1.60
2.05	2.74	1.95	2.55	1.88	2.41	1.77	2.24	1.70	2.11	1.60	1.94	1.51	1.78	1.42	1.65	1.32	1.49
2.03	2.69	1.92	2.51	1.85	2.36	1.75	2.19	1.68	2.06	1.57	1.89	1.48	1.73	1.39	1.59	1.28	1.43
1.98	2.60	1.87	2.41	1.80	2.28	1.69	2.09	1.62	1.97	1.52	1.79	1.42	1.62	1.32	1.48	1.19	1.28
1.95	2.53	1.84	2.34	1.76	2.20	1.65	2.01	1.58	1.89	1.47	1.71	1.36	1.54	1.26	1.38	1.08	1.11
1.94	2.51	1.83	2.32	1.75	2.18	1.64	1.99	1.57	1.87	1.46	1.69	1.35	1.52	1.24	1.36	1.00	1.00

Applied to Experiments in Agriculture and Biology," 3d ed , Collegiate Press, Inc., of Iowa State College, Ames,

TABLE A26. FISHER'S TABLE OF THE DISTRIBUTION OF t FOR CERTAIN PROBABILITY LEVELS[1]

n	$P = 0.9$	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.05	0.02	0.01
1	0.158	0.325	0.510	0.727	1.000	1.376	1.963	3.078	6.314	12.706	31.821	63.657
2	0.142	0.289	0.445	0.617	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925
3	0.137	0.277	0.424	0.584	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841
4	0.134	0.271	0.414	0.569	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604
5	0.132	0.267	0.408	0.559	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032
6	0.131	0.265	0.404	0.553	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707
7	0.130	0.263	0.402	0.549	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499
8	0.130	0.262	0.399	0.546	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355
9	0.129	0.261	0.398	0.543	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250
10	0.129	0.260	0.397	0.542	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169
11	0.129	0.260	0.396	0.540	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106
12	0.128	0.259	0.395	0.539	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055
13	0.128	0.259	0.394	0.538	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012
14	0.128	0.258	0.393	0.537	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977
15	0.128	0.258	0.393	0.536	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947
16	0.128	0.258	0.392	0.535	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921
17	0.128	0.257	0.392	0.534	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898
18	0.127	0.257	0.392	0.534	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878
19	0.127	0.257	0.391	0.533	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861
20	0.127	0.257	0.391	0.533	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845
21	0.127	0.257	0.391	0.532	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831
22	0.127	0.256	0.390	0.532	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819
23	0.127	0.256	0.390	0.532	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807
24	0.127	0.256	0.390	0.531	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797
25	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787
26	0.127	0.256	0.390	0.531	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779
27	0.127	0.256	0.389	0.531	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771
28	0.127	0.256	0.389	0.530	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763
29	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756
30	0.127	0.256	0.389	0.530	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750
∞	0.12566	0.25335	0.38532	0.52440	0.67449	0.84162	1.03643	1.28155	1.64485	1.95996	2.32634	2.57582

[1] Table A26 is abridged from Table III of R. A. Fisher and F. Yates, "Statistical Tables for Biological, Agricultural and Medical Research," Oliver & Boyd, Ltd., Edinburgh, 1938, by permission of the authors and publishers.

TABLE A27. NATURAL TRIGONOMETRIC FUNCTIONS

Angle (degrees)	sin	tan	cot	cos	Angle (degrees)	Angle (degrees)	sin	tan	cot	cos	Angle (degrees)
0	.000	.000	.....	1.00	90	23	.391	.424	2.36	.920	67
½	.009	.009	115	1.00	89½	23½	.399	.434	2.30	.917	66½
1	.017	.017	57.3	1.00	89	24	.407	.445	2.25	.914	66
1½	.026	.026	38.2	1.00	88½	24½	.415	.456	2.19	.910	65½
2	.035	.035	28.6	.999	88	25	.423	.466	2.14	.906	65
2½	.044	.044	22.9	.999	87½	25½	.431	.477	2.10	.903	64½
3	.052	.052	19.1	.999	87	26	.438	.488	2.05	.899	64
3½	.061	.061	16.4	.998	86½	26½	.446	.499	2.01	.895	63½
4	.070	.070	14.3	.998	86	27	.454	.510	1.96	.891	63
4½	.078	.079	12.7	.997	85½	27½	.462	.521	1.92	.887	62½
5	.087	.087	11.4	.996	85	28	.469	.532	1.88	.883	62
5½	.096	.096	10.4	.995	84½	28½	.477	.543	1.84	.879	61½
6	.105	.105	9.51	.995	84	29	.485	.554	1.80	.875	61
6½	.113	.114	8.78	.994	83½	29½	.492	.566	1.77	.870	60½
7	.122	.123	8.14	.993	83	30	.500	.577	1.73	.866	60
7½	.131	.132	7.60	.991	82½	30½	.508	.589	1.70	.862	59½
8	.139	.141	7.12	.990	82	31	.515	.601	1.66	.857	59
8½	.148	.149	6.69	.989	81½	31½	.522	.613	1.63	.853	58½
9	.156	.158	6.31	.988	81	32	.530	.625	1.60	.848	58
9½	.165	.167	5.98	.986	80½	32½	.537	.637	1.57	.843	57½
10	.174	.176	5.67	.985	80	33	.545	.649	1.54	.839	57
10½	.182	.185	5.40	.983	79½	33½	.552	.662	1.51	.834	56½
11	.191	.194	5.14	.982	79	34	.559	.675	1.48	.829	56
11½	.199	.203	4.92	.980	78½	34½	.566	.687	1.46	.824	55½
12	.208	.213	4.70	.978	78	35	.574	.700	1.43	.819	55
12½	.216	.222	4.51	.976	77½	35½	.581	.713	1.40	.814	54½
13	.225	.231	4.33	.974	77	36	.588	.727	1.38	.809	54
13½	.233	.240	4.17	.972	76½	36½	.595	.740	1.35	.804	53½
14	.242	.249	4.01	.970	76	37	.602	.754	1.33	.799	53
14½	.250	.259	3.87	.968	75½	37½	.609	.767	1.30	.793	52½
15	.259	.268	3.73	.966	75	38	.616	.781	1.28	.788	52
15½	.267	.277	3.61	.964	74½	38½	.623	.795	1.26	.783	51½
16	.276	.287	3.49	.961	74	39	.629	.810	1.23	.777	51
16½	.284	.296	3.38	.959	73½	39½	.636	.824	1.21	.772	50½
17	.292	.306	3.27	.956	73	40	.643	.839	1.19	.766	50
17½	.301	.315	3.17	.954	72½	40½	.649	.854	1.17	.760	49½
18	.309	.325	3.08	.951	72	41	.656	.869	1.15	.755	49
18½	.317	.335	2.99	.948	71½	41½	.663	.885	1.13	.749	48½
19	.326	.344	2.90	.946	71	42	.669	.900	1.11	.743	48
19½	.334	.354	2.82	.943	70½	42½	.676	.916	1.09	.737	47½
20	.342	.364	2.75	.940	70	43	.682	.933	1.07	.731	47
20½	.350	.374	2.67	.937	69½	43½	.688	.949	1.05	.725	46½
21	.358	.384	2.61	.934	69	44	.695	.966	1.04	.719	46
21½	.366	.394	2.54	.930	68½	44½	.701	.983	1.02	.713	45½
22	.375	.404	2.48	.927	68	45	.707	1.00	1.00	.707	45
22½	.383	.414	2.41	.924	67½						
Angle (degrees)	cos	cot	tan	sin	Angle (degrees)	Angle (degrees)	cos	cot	tan	sin	Angle (degrees)

Table A28. Perpetual Calendar

Gregorian System. 1700–2099

1700	1800	1900	2000	J	F	M	A	M	J	J	A	S	O	N	D
21 49 77	17 45 73	13 41 69 97	25 53 81	4	7	7	3	5	1	3	6	2	4	7	2
22 50 78	18 46 74	14 42 70 98	26 54 82	5	1	1	4	6	2	4	7	3	5	1	3
00 23 51 79	19 47 75	15 43 71 99	27 55 83	6	2	2	5	7	3	5	1	4	6	2	4
24 52 80	20 48 76	16 44 72	00 28 56 84	7	3	4	7	2	5	7	3	6	1	4	6
25 53 81	21 49 77	17 45 73	01 29 57 85	2	5	5	1	3	6	1	4	7	2	5	7
26 54 82	22 50 78	18 46 74	02 30 58 86	3	6	6	2	4	7	2	5	1	3	6	1
27 55 83	00 23 51 79	19 47 75	03 31 59 87	4	7	7	3	5	1	3	6	2	4	7	2
28 56 84	24 52 80	20 48 76	04 32 60 88	5	1	2	5	7	3	5	1	4	6	2	4
01 29 57 85	25 53 81	21 49 77	05 33 61 89	7	3	3	6	1	4	6	2	5	7	3	5
02 30 58 86	26 54 82	22 50 78	06 34 62 90	1	4	4	7	2	5	7	3	6	1	4	6
03 31 59 87	27 55 83	00 23 51 79	07 35 63 91	2	5	5	1	3	6	1	4	7	2	5	7
04 32 60 88	28 56 84	24 52 80	08 36 64 92	3	6	7	3	5	1	3	6	2	4	7	2
05 33 61 89	01 29 57 85	25 53 81	09 37 65 93	5	1	1	4	6	2	4	7	3	5	1	3
06 34 62 90	02 30 58 86	26 54 82	10 38 66 94	6	2	2	5	7	3	5	1	4	6	2	4
07 35 63 91	03 31 59 87	27 55 83	11 39 67 95	7	3	3	6	1	4	6	2	5	7	3	5
08 36 64 92	04 32 60 88	28 56 84	12 40 68 96	1	4	5	1	3	6	1	4	7	2	5	7
09 37 65 93	05 33 61 89	01 29 57 85	13 41 69 97	3	6	6	2	4	7	2	5	1	3	6	1
10 38 66 94	06 34 62 90	02 30 58 86	14 42 70 98	4	7	7	3	5	1	3	6	2	4	7	2
11 39 67 95	07 35 63 91	03 31 59 87	15 43 71 99	5	1	1	4	6	2	4	7	3	5	1	3
12 40 68 96	08 36 64 92	04 32 60 88	16 44 72	6	2	3	6	1	4	6	2	5	7	3	5
13 41 69 97	09 37 65 93	05 33 61 89	17 45 73	1	4	4	7	2	5	7	3	6	1	4	6
14 42 70 98	10 38 66 94	06 34 62 90	18 46 74	2	5	5	1	3	6	1	4	7	2	5	7
15 43 71 99	11 39 67 95	07 35 63 91	19 47 75	3	6	6	2	4	7	2	5	1	3	6	1
16 44 72	12 40 68 96	08 36 64 92	20 48 76	4	7	1	4	6	2	4	7	3	5	1	3
17 45 73	13 41 69 97	09 37 65 93	21 49 77	6	2	2	5	7	3	5	1	4	6	2	4
18 46 74	14 42 70 98	10 38 66 94	22 50 78	7	3	3	6	1	4	6	2	5	7	3	5
19 47 75	15 43 71 99	11 39 67 95	23 51 79	1	4	4	7	2	5	7	3	6	1	4	6
20 48 76	16 44 72	12 40 68 96	24 52 80	2	5	6	2	4	7	2	5	1	3	6	1

1.

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

2.

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

3.

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

U. S. shift to Gregorian cal. Sept. 14, 1752

4.

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

5.

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

6.

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

7.

S	M	T	W	T	F	S
						1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29
30	31					

Find the year in the large table at upper left. Follow over to extreme right, and find numeral under desired month. This numeral refers to the proper calendar for the month at the bottom.

Example: July 4, 1776, was Thursday.

TABLE A29. THE JULIAN DAY

Add together the year number, the month number, and the day of the month

Year	No.	Year	No.	Year	No.	Year	No.	Year	No.
1800	2378496	1845	2394932	1890	2411368	1935	2427803	1980	2444239
1801	78861	1846	95297	1891	11733	1936	28168	1981	44605
1802	79226	1847	95662	1892	12098	1937	28534	1982	44970
1803	79591	1848	96027	1893	12464	1938	28899	1983	45335
1804	79956	1849	96393	1894	12829	1939	29264	1984	45700
1805	80322	1850	96758	1895	13194	1940	29629	1985	46066
1806	80687	1851	97123	1896	13559	1941	29995	1986	46431
1807	81052	1852	97488	1897	13925	1942	30360	1987	46796
1808	81417	1853	97854	1898	14290	1943	30725	1988	47161
1809	81783	1854	98219	1899	14655	1944	31090	1989	47527
1810	82148	1855	98584	1900	15020	1945	31456	1990	47892
1811	82513	1856	98949	1901	15385	1946	31821	1991	48257
1812	82878	1857	99315	1902	15750	1947	32186	1992	48622
1813	83244	1858	99680	1903	16115	1948	32551	1993	48988
1814	83609	1859	2400045	1904	16480	1949	32917	1994	49353
1815	83974	1860	00410	1905	16846	1950	33282	1995	49718
1816	84339	1861	00776	1906	17211	1951	33647	1996	50083
1817	84705	1862	01141	1907	17576	1952	34012	1997	50449
1818	85070	1863	01506	1908	17941	1953	34378	1998	50814
1819	85435	1864	01871	1909	18307	1954	34743	1999	51179
1820	85800	1865	02237	1910	18672	1955	35108	2000	51544
1821	86166	1866	02602	1911	19037	1956	35473		
1822	86531	1867	02967	1912	19402	1957	35839		
1823	86896	1868	03332	1913	19768	1958	36204		
1824	87261	1869	03698	1914	20133	1959	36569		
1825	87627	1870	04063	1915	20498	1960	36934		
1826	87992	1871	04428	1916	20863	1961	37300		
1827	88357	1872	04793	1917	21229	1962	37665		
1828	88722	1873	05159	1918	21594	1963	38030		
1829	89088	1874	05524	1919	21959	1964	38395		
1830	89453	1875	05889	1920	22324	1965	38761		
1831	89818	1876	06254	1921	22690	1966	39126		
1832	90183	1877	06620	1922	23055	1967	39491		
1833	90549	1878	06985	1923	23420	1968	39856		
1834	90914	1879	07350	1924	23785	1969	40222		
1835	91279	1880	07715	1925	24151	1970	40587		
1836	91644	1881	08081	1926	24516	1971	40952		
1837	92010	1882	08446	1927	24881	1972	41317		
1838	92375	1883	08811	1928	25246	1973	41683		
1839	92740	1884	09176	1929	25612	1974	42048		
1840	93105	1885	09542	1930	25977	1975	42413		
1841	93471	1886	09907	1931	26342	1976	42778		
1842	93836	1887	10272	1932	26707	1977	43144		
1843	94201	1888	10637	1933	27073	1978	43509		
1844	94566	1889	11003	1934	27438	1979	43874		

Mo.	Common Year	Leap Year
Jan.	0	0
Feb.	31	31
Mar.	59	60
Apr.	90	91
May	120	121
June	151	152
July	181	182
Aug.	212	213
Sept.	243	244
Oct.	273	274
Nov.	304	305
Dec.	334	335

Example: What Julian day was June 12, 1871?

Year:	2404428
Month:	151
Day:	12
Ans.:	2404591

TABLE A30. COMMONLY USED CONSTANTS

	N	$\log_{10} N$
π	3.14159 26536	0.49714 98727
2π	6.28318 53118	0.79817 98684
3π	9.42477 79608	0.97427 11274
$\pi/2$	1.57079 63268	0.19611 98770
$1/\pi$	0.31830 98862	9.50285 − 10
$1/2\pi$	0.15915 49431	9.20182 − 10
π^2	9.86960 44011	0.99429 97454
$\sqrt{\pi}$	1.77245 38509	0.24857 49363
$\sqrt{2/\pi}$	0.79788 45608	9 90194 − 10
e	2.71828 18285	0.43429 44819 = M
$\log_e 10 = 1/M$	2.30258 50930	0.36221 56887
$1/e$	0.36787 94412	9.56570 55181 − 10
$\sqrt{2\pi}$	2.50662 82746	0.39908 99342
$1/\sqrt{2\pi}$	0.39894 22804	9.60091 00658 − 10
$P.E./\sigma$	0.67448 97500	9.67846 03565 − 10
$\sigma/P.E.$	1.48260	0.17102
$1/\sqrt{2}$	0.70710 67812	9.84948 50022 − 10

Rules for change of base with logarithms:

$$\log_e x = \frac{\log_{10} x}{\log_{10} e}$$

$$\log_e x = \left(\frac{1}{M}\right) \log_{10} x$$

$$\log_{10} x = M(\log_e x)$$

TABLE A31. THE GREEK ALPHABET

A, α . . .	*Alpha*	N, ν . . .	*Nu*
B, β . . .	*Beta*	Ξ, ξ . . .	*Xi*
Γ, γ . . .	*Gamma*	O, o . . .	*Omicron*
Δ, δ . . .	*Delta*	Π, π . . .	*Pi*
E, ϵ . . .	*Epsilon*	P, ρ . . .	*Rho*
Z, ζ . . .	*Zeta*	Σ, σ . . .	*Sigma*
H, η . . .	*Eta*	T, τ . . .	*Tau*
Θ, ϑ, θ . .	*Theta*	Υ, υ . . .	*Upsilon*
I, ι . . .	*Iota*	Φ, φ, ϕ . .	*Phi*
K, κ . . .	*Kappa*	X, χ . . .	*Chi*
Λ, λ . . .	*Lambda*	Ψ, ψ . . .	*Psi*
M, μ . . .	*Mu*	Ω, ω . . .	*Omega*

TABLE A32. LOGARITHMIC FACTORIALS

n	log (n!)	n	log (n!)	n	log (n!)	n	log (n!)
1	0.00000	26	26.60562	51	66.19064	76	111.27542
2	0.30103	27	28.03698	52	67.90665	77	113.16192
3	0.77815	28	29.48414	53	69.63092	78	115.05401
4	1.38021	29	30.94654	54	71.36332	79	116.95164
5	2.07918	30	32.42366	55	73.10368	80	118.85473
6	2.85733	31	33.91502	56	74.85187	81	120.76321
7	3.70243	32	35.42017	57	76.60774	82	122.67703
8	4.60552	33	36.93869	58	78.37117	83	124.59610
9	5.55976	34	38.47016	59	80.14202	84	126.52038
10	6.55976	35	40.01423	60	81.92018	85	128.44980
11	7.60116	36	41.57054	61	83.70550	86	130.38430
12	8.68034	37	43.13874	62	85.49790	87	132.32382
13	9.79428	38	44.71852	63	87.29724	88	134.26830
14	10.94041	39	46.30958	64	89.10342	89	136.21769
15	12.11650	40	47.91164	65	90.91633	90	138.17194
16	13.32062	41	49.52443	66	92.73587	91	140.13098
17	14.55107	42	51.14768	67	94.56195	92	142.09476
18	15.80634	43	52.78115	68	96.39446	93	144.06325
19	17.08510	44	54.42460	69	98.23331	94	146.03638
20	18.38612	45	56.07781	70	100.07840	95	148.01410
21	19.70834	46	57.74057	71	101.92966	96	149.99637
22	21.05077	47	59.41267	72	103.78700	97	151.98314
23	22.41249	48	61.09391	73	105.65032	98	153.97437
24	23.79271	49	62.78410	74	107.51955	99	155.97000
25	25.19065	50	64.48308	75	109.39461	100	157.97000

TABLE A33. SHORT TABLE OF RANDOM SAMPLING NUMBERS

7766	7520	1607	6048	2771	4733	8558	8681	5204	3806
9627	5293	3569	0457	4426	2857	3666	9156	6931	6157
4594	2563	6826	8102	2543	4032	6897	2012	0945	9709
6668	4104	4018	4544	8117	7664	5270	3014	0420	4232
8874	0822	0949	8697	7550	4154	9697	9045	4916	1235
8009	5708	7072	8045	8451	5777	1613	0399	2069	7909
7271	5633	6025	0745	9804	3333	7160	5150	7743	5221
6450	6850	0602	9518	2275	9221	6441	8899	4640	7742
0598	0564	9655	3988	5620	3286	6319	6392	5743	1111
6546	4417	4453	5125	1356	6011	5965	9253	1486	7503
2806	6217	4278	3170	1626	1746	9731	9289	7667	5209
6901	9464	9302	6404	8049	3653	8101	4498	8558	6238
3625	0749	5025	7327	3984	1635	5963	0970	7357	2033
2222	9942	1706	2907	6304	8022	7972	7852	6242	6269
7224	3014	3943	5982	4052	4243	5306	1530	7537	3233
7160	6043	0767	0230	6082	3637	4556	6654	8972	9697
7965	7435	8397	9741	6207	2297	6491	7961	0243	6897
6708	0600	2765	1911	0813	2268	3554	7976	4102	0414
4159	6804	3838	4255	9664	7044	3067	6720	7416	4748
6592	1846	2269	9136	7107	0676	9782	8016	2715	3932
2805	7999	3743	1655	7812	7223	0954	4397	7427	9120
9501	0400	8056	4148	5585	7497	7421	0640	6695	6127
3346	6596	1997	9417	0164	9718	5671	9765	7091	1920
4447	3427	6134	9130	4763	2301	2892	4251	4491	5772
0610	4363	0705	0969	4684	4202	5274	6660	0468	1814
2131	4792	1418	0080	9763	7306	0167	9688	6959	2250
9569	9413	5681	9632	8505	8948	6475	2934	6046	9640
1412	7690	5615	1776	8568	7209	9907	3541	8847	8752
5064	7408	1951	1033	7817	2626	2441	3795	3275	1319
4193	2082	0412	5519	4108	3333	5546	0177	9345	5269
6414	5111	4003	3695	2976	4939	7555	7374	2913	2705
2672	8616	7005	5736	0172	7472	2033	6308	8779	1270
0758	3869	9288	2397	6264	8352	8617	7869	2459	8591
4502	2535	2434	5018	1202	9081	2674	2467	2532	9689
4823	3965	2801	6179	8592	6763	6567	1016	5801	9288
3011	0939	7162	4443	3849	9142	2922	9191	6029	7631
6611	9238	2160	9339	8177	2180	3905	2977	9234	3434
0378	8311	0623	4299	2335	7044	5855	0186	5895	5642
9905	4972	6907	5633	6548	3412	8469	0559	8878	8671
9424	4750	8325	3871	1831	7268	1863	9963	1905	7484
7004	3469	1159	4841	8681	8751	9214	1145	4394	1160
5658	2963	5798	4691	8653	7427	7826	9971	2622	9886
9327	2129	3459	1165	1011	4805	1821	7999	2136	9308
1161	2217	1797	3906	5304	4087	6766	3063	1747	3836
6002	3340	3648	3765	1565	8483	6353	8232	4942	5721
4311	3087	1756	6612	3277	1269	6573	3096	0898	1103
5237	1667	5941	2504	6213	5797	9326	3079	8796	4220
0163	7150	0894	9009	7858	4812	7678	0835	8447	1524
0437	7497	0187	4907	2202	2318	5339	3290	4342	9375
0974	9130	4974	9757	8802	8514	6564	5485	0793	5675

TABLE A33. SHORT TABLE OF RANDOM SAMPLING NUMBERS.—(*Continued*)

3754	7829	9473	8264	8502	0364	5146	0609	4708	5229
9278	1828	8171	8788	3821	0923	8249	8431	6516	0911
9152	6396	7516	2959	4988	0943	6070	8342	5643	7476
0306	8452	1326	8892	2571	4860	1907	4843	0248	5283
1775	3205	8496	0201	6864	3375	0599	7516	8592	9823
4448	1897	3406	1429	8153	3408	1136	9173	9582	2866
3406	4332	0083	1214	5107	0912	8257	4015	5933	5520
4869	7491	5786	3633	9450	4572	6046	7844	2536	9502
5042	6524	1138	4001	6957	7220	8715	5082	8909	2384
0371	1656	8756	3369	3347	3534	0519	7230	2516	2674
2969	0056	8199	9383	4840	4135	7713	6317	4188	8073
4680	0551	7807	9470	9460	2253	0146	6082	9037	1862
1979	1845	0247	4813	2052	2758	6032	8288	6840	2677
3463	7252	3753	1178	2766	3207	2332	8262	8499	4501
0698	8601	2945	6077	3785	4647	4226	8959	9006	0964
2709	2447	0580	3375	1775	2038	3797	5163	7845	9397
6014	1671	2362	2315	8297	3930	6686	5835	9464	0916
7219	3355	3933	9312	3808	7579	6254	7075	7818	0295
6900	7276	4131	5402	3263	4026	5185	2862	8450	7749
0652	9020	6533	5737	6390	8723	8240	6442	4775	6040
3559	8683	0358	0118	0825	3360	7913	1403	4016	0202
1133	5094	3564	9818	0188	6367	2887	5038	1039	1658
1066	2065	4018	9132	3343	6165	1351	1312	7876	8452
8099	2678	7288	1970	9523	4070	7258	7276	3138	6818
5599	5836	0212	7112	8857	5894	6647	1660	3518	5780
6204	6540	1791	3190	3727	4500	5370	5231	8629	6291
8288	1891	5014	8442	9712	3435	4570	9493	1563	9165
7590	9691	1601	6615	0848	2885	1863	5682	1666	3398
7162	9599	9286	2819	2867	6533	9931	9217	4987	7722
9948	6283	0839	4175	8654	2005	6128	1306	6979	3152
5187	9791	4301	8481	5699	2522	0394	1538	8492	1812
5330	8112	2323	3056	1282	0543	4135	5819	6172	1017
6454	8783	7254	5267	9809	9964	9835	1111	5988	8017
8771	0872	6538	9975	4349	4106	6047	9630	4211	3234
1804	3896	2518	5665	8766	7161	0755	0886	3256	3198
8109	0020	3347	9221	6511	7593	6133	6123	2128	2735
9371	0132	4794	3110	5357	7242	4790	8002	9268	9733
6062	6416	7311	1167	5131	9955	9738	6038	1119	4832
7072	3929	8992	8062	6898	5499	5278	3407	0544	8772
5867	5384	8700	8017	5235	4094	9441	2381	8478	0981
1390	8293	7525	7188	8218	0131	3543	1679	8610	5737
4974	9904	7964	6038	0910	9364	4842	3873	3495	5511
9086	9898	1529	8544	7800	8523	1353	3312	5255	3096
8786	4498	5476	6266	9636	1897	3924	7298	3764	0906
7215	2019	6780	1005	4981	0787	8463	3784	6072	0940
2701	2584	8904	7799	9877	9015	0310	9330	0037	8215
9830	7090	3878	7553	7460	2845	9183	6429	9249	0246
0008	1130	3811	1862	1670	6389	9179	8571	7621	2169
5338	0351	6437	6148	5015	6174	5761	4690	0799	3291
6508	4163	0794	5801	1272	2814	0989	1130	3918	8596

SECTION B

STATISTICAL FORMULAS

The following list of statistical formulas is necessarily incomplete, but the attempt has been made to include those formulas which are in most common use. For further examples the student is referred to specialized texts.[1] The page references given in connection with the formulas indicate the points in the author's "Elements of Statistical Method"[2] where the applications of the formulas are illustrated

Measures of Central Tendency

1. Arithmetic mean, ungrouped data:

$$\bar{X} = \frac{\Sigma X}{N} \qquad \text{(page 61)}$$

2. Arithmetic mean, grouped data:

$$\bar{X} = \frac{\Sigma fX}{\Sigma f} = \frac{\Sigma fX}{N} \qquad \text{(page 85)}$$

3. Arithmetic mean, grouped data, short method:

$$\bar{X} = \bar{X}' + Ci\,\frac{\Sigma(fd)}{N} \qquad \text{(page 89)}$$

4. Weighted arithmetic mean:

$$\bar{X} = \frac{\Sigma(XW)}{\Sigma W} \qquad \text{(page 62)}$$

5. Location of median item; ungrouped data:

$$\text{Location of Med.} = \frac{N+1}{2} \qquad \text{(page 67)}$$

[1] Especially good for this purpose is J. W. Dunlap and A. K. Kurtz, "Handbook of Statistical Nomographs, Tables, and Formulas," World Book Company, Yonkers-on-Hudson, New York, 1932.

[2] McGraw-Hill Book Company, Inc., New York, 1943.

6. Location of median item; grouped data:

$$\text{Location of Med.} = \frac{N}{2} \qquad \text{(page 92)}$$

7. Mode:

$$\text{Mo.} = m + \frac{2g(\overline{X} - m) - Ci(g - 1)}{2(g - 1)} \qquad \text{(page 98)}$$

8. Mode:

$$\text{Mo.} = 3 \text{ Med.} - 2\overline{X} \qquad \text{(page 98)}$$

9. Mode:

$$\text{Mo.} = \frac{2\overline{X} - \alpha_3\sigma}{2} \qquad \text{(page 205)}$$

10. Mode:

$$\text{Mo.} = \overline{X} - \frac{\sigma[\sqrt{\beta_1}(\beta_2 + 3)]}{2(5\beta_2 - 6\beta_1 - 9)} \qquad \text{(page 206)}$$

11. Geometric mean:

$$\log M_g = \frac{\Sigma(\log X)}{N} \qquad \text{(page 70)}$$

12. Geometric mean; grouped data:

$$\log M_g = \frac{\Sigma f(\log X)}{N} \qquad \text{(page 99)}$$

13. Harmonic mean:

$$M_h = \frac{N}{\Sigma(1/X)} \qquad \text{(page 71)}$$

14. Harmonic mean; grouped data:

$$M_h = \frac{N}{\Sigma(f/X)} \qquad \text{(page 100)}$$

15. First quartile (position); ungrouped data:

$$\text{Position of } Q_1 = \frac{(N + 1)}{4} \qquad \text{(page 75)}$$

16. Third quartile (position); ungrouped data:

$$\text{Position of } Q_3 = \frac{3(N + 1)}{4} \qquad \text{(page 75)}$$

17. Position of first quartile; grouped data:

$$\text{Position of } Q_1 = \frac{N}{4} \qquad \text{(page 103)}$$

18. Position of third quartile; grouped data:

$$\text{Position of } Q_3 = \frac{3N}{4} \qquad \text{(page 103)}$$

19. Quadratic mean:

$$M_q = \sqrt{\frac{\Sigma(X^2)}{N}} \qquad \text{(page 73)}$$

20. Quadratic mean; grouped data:

$$M_q = \sqrt{\frac{\Sigma f(X^2)}{N}} \qquad \text{(page 101)}$$

Measures of Dispersion

21. Semi-interquartile range:

$$Q = \frac{Q_3 - Q_1}{2} \qquad \text{(page 129)}$$

22. Average deviation:

$$AD = \frac{\Sigma(|x|)}{N} \qquad \text{(page 131)}$$

23. Average deviation:

$$AD = \frac{2(B\bar{\bar{X}} - b)}{N} \qquad \text{(page 132)}$$

24. Standard deviation; ungrouped data:

$$\sigma = \sqrt{\frac{\Sigma(x^2)}{N}} \qquad \text{(page 137)}$$

25. Standard deviation; grouped data:

$$\sigma = \sqrt{\frac{\Sigma(fx^2)}{N}} \qquad \text{(page 140)}$$

26. Standard deviation; ungrouped data:

$$\sigma = \sqrt{\frac{\Sigma(X^2)}{N} - \bar{\bar{X}}^2} \qquad \text{(page 138)}$$

27. Standard deviation; grouped data; short method:

$$\sigma = Ci\sqrt{\frac{\Sigma(fd^2)}{N} - \left(\frac{\Sigma(fd)}{N}\right)^2} \qquad \text{(page 143)}$$

28. Relative dispersion; general formula:

$$\text{Relative dispersion} = \frac{100\ (\text{absolute dispersion})}{\text{Average}} \qquad \text{(page 151)}$$

29. Coefficient of variation:

$$V = \frac{100\sigma}{\bar{X}} \qquad \text{(page 151)}$$

30. Standard deviation:

$$\sigma = \sqrt{v_2} \qquad \text{(page 189)}$$

31. Standard deviation corrected for grouping error:

$$\sigma = \sqrt{\mu_2} \qquad \text{(page 197)}$$

PROBABILITY, THE NORMAL CURVE, AND MOMENTS

32. The mean of a probability distribution:

$$\bar{X} = np \qquad \text{(page 158)}$$

33. Standard deviation of probability distribution:

$$\sigma = \sqrt{npq} = \sqrt{n}\,\sqrt{pq} \qquad \text{(page 159)}$$

34. The probability of getting exactly r successes in n trials:

$$p = \frac{n!}{r!(n-r)!}\,(p^r q^{n-r}) \qquad \text{(page 161)}$$

35. Equation of the normal curve:

$$y = \left(\frac{N}{\sigma\sqrt{2\pi}}\right) e^{-x^2/2\sigma^2} \qquad \text{(page 167)}$$

36. The "unit" normal curve:

$$y = .39894e^{-x^2/2} \qquad \text{(page 179}n.\text{)}$$

37. The maximum ordinate of the normal curve:

$$y_0 = Ci\left(\frac{N}{2.5066\sigma}\right) \qquad \text{(page 179)}$$

38. First moment about the mean; ungrouped data:

$$v_1 = \frac{\Sigma x}{N} \qquad \text{(page 189)}$$

39. Second moment about the mean; ungrouped data:

$$v_2 = \frac{\Sigma x^2}{N} \qquad \text{(page 189)}$$

40. Nth moment about the mean; ungrouped data:

$$v_n = \frac{\Sigma x^n}{N} \qquad \text{(page 189)}$$

41. First moment about an arbitrary origin; grouped data:

$$v'_1 = \frac{\Sigma(fd)}{N} \qquad \text{(page 191)}$$

42. Second moment about an arbitrary origin; grouped data:

$$v'_2 = \frac{\Sigma(fd^2)}{N} \qquad \text{(page 192)}$$

43. Nth moment about an arbitrary origin; grouped data:

$$v'_n = \frac{\Sigma(fd^n)}{N} \qquad \text{(page 192)}$$

44. First moment about the mean; grouped data:

$$v_1 = 0 \qquad \text{(page 192)}$$

45. Second moment about the mean; grouped data:

$$v_2 = Ci^2(v'_2 - v'^2_1) \qquad \text{(page 192)}$$

46. Third moment about the mean; grouped data:

$$v_3 = Ci^3(v'_3 - 3v'_2v'_1 + 2v'^3_1) \qquad \text{(page 192)}$$

47. Fourth moment about the mean; grouped data:

$$v_4 = Ci^4(v'_4 - 4v'_3v'_1 + 6v'_2v'^2_1 - 3v'^4_1) \qquad \text{(page 192)}$$

48. Second moment; corrected for grouping:

$$\mu_2 = v_2 - \frac{Ci^2}{12} \qquad \text{(page 196)}$$

49. Third moment; corrected for grouping:

$$\mu_3 = v_3 \qquad \text{(page 196)}$$

50. Fourth moment; corrected for grouping:

$$\mu_4 = v_4 - \frac{v_2 Ci^2}{2} + \frac{7Ci^4}{240} \qquad \text{(page 196)}$$

51. Moments of probability distributions:

$$\bar{X} = np \qquad \text{(page 197)}$$

$$\sigma = \sqrt{npq} \qquad \text{(page 197)}$$

$$\alpha_3 = \frac{q - p}{\sigma} \qquad \text{(page 197)}$$

$$\alpha_4 = \frac{1}{\sigma^2} - \frac{6}{n} + 3 \qquad \text{(page 197)}$$

52. Alpha one:

$$\alpha_1 = \frac{v_1}{\sigma} = 0 \qquad \text{(page 193)}$$

53. Alpha two:

$$\alpha_2 = \frac{v_2}{\sigma^2} = 1 \qquad \text{(page 193)}$$

54. Alpha three:

$$\alpha_3 = \frac{v_3}{\sigma^3} \qquad \text{(page 193)}$$

55. Alpha four:

$$\alpha_4 = \frac{v_4}{\sigma^4} \qquad \text{(page 193)}$$

56. Alpha n:

$$\alpha_n = \frac{v_n}{\sigma^n} \qquad \text{(page 193)}$$

57. Criterion of curve type:

$$\beta_1 = \frac{{v_3}^2}{{v_2}^3} = (\alpha_3)^2 \qquad \text{(page 212)}$$

58. Criterion of curve type:

$$\beta_2 = \frac{v_4}{{v_2}^2} = \alpha_4 \qquad \text{(page 212)}$$

59. Criterion of curve type:

$$\kappa_2 = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} \qquad \text{(page 212)}$$

60. A measure of skewness:

$$\text{Sk.} = \frac{\bar{X} - \text{Mo.}}{\sigma} \qquad \text{(page 201)}$$

61. A measure of skewness:

$$\text{Sk.} = \frac{3(\bar{X} - \text{Med.})}{\sigma} \qquad \text{(page 203)}$$

62. A measure of skewness:

$$\text{Sk.} = \frac{Q_3 + Q_1 - 2\ \text{Med.}}{Q_3 - Q_1} \qquad \text{(page 203)}$$

63. A measure of skewness:

$$\text{Sk.} = P_{50} - \frac{P_{90} + P_{10}}{2} \qquad \text{(page 204)}$$

64. A measure of skewness:

$$\text{Sk.} = \alpha_3 = \sqrt{\beta_1} \qquad \text{(page 204)}$$

65. Distance from mean to mode:

$$\bar{X} - \text{Mo.} = \sigma\left(\frac{\alpha_3}{2}\right) \qquad \text{(page 205)}$$

66. A measure of kurtosis:

$$\text{Ku.} = \beta_2 = \alpha_4 \qquad \text{(page 207)}$$

67. Poisson distribution:

$$y = N\frac{A^x e^{-A}}{x!} \qquad \text{(page 218)}$$

68. Poisson distribution:

$$\log y = \log N + x(\log A) - .434294A - \log x!$$

(page 218)

Measures of Reliability

69. Probable error of any measure in terms of the standard error of the same measure:

$$PE_X = .6745\sigma_X \qquad \text{(page 241)}$$

70. Standard error of any measure in terms of the probable error of the same measure:

$$\sigma_X = 1.4826PE_X \qquad \text{(page 241)}$$

71. Standard error of the mean:

$$\sigma_M = \sigma_x \sqrt{\frac{U - S}{S(U - 1)}} \qquad \text{(page 235)}$$

72. Standard error of the mean:

$$\sigma_M = \frac{\sigma_x}{\sqrt{N}} \qquad \text{(page 236)}$$

73. Standard error of the mean, small samples:

$$\sigma_M = \frac{\sigma_x}{\sqrt{N - 1}} \qquad \text{(page 254)}$$

74. Standard error of the standard deviation:

$$\sigma_\sigma = \frac{\sigma_x}{\sqrt{2N}} = .707107\sigma_M \qquad \text{(page 243)}$$

75. Standard error of the standard deviation; small samples:

$$\sigma_\sigma = \frac{\sigma_x}{\sqrt{2(N - 1)}} = .707107\sigma_M \qquad \text{(page 255)}$$

76. Standard error of the standard deviation of any distribution, whether normal or not:

$$\sigma_\sigma = \sqrt{\frac{v_4 - v_2^2}{4v_2(N)}} \qquad \text{(page 243}n.\text{)}$$

77. Standard error of the median:

$$\sigma_{\text{Med.}} = 1.25331\sigma_M \qquad \text{(page 244)}$$

78. Standard error of α_3:

$$\sigma_{\alpha_3} = \sqrt{\frac{6}{N}} = 2.4495\sqrt{\frac{1}{N}} \qquad \text{(page 244)}$$

79. Standard error of α_4:

$$\sigma_{\alpha_4} = \sqrt{\frac{24}{N}} = 2\sigma_{\alpha_3} \qquad \text{(page 244)}$$

80. Standard error of a percentage:

$$\sigma_{\%} = \sqrt{\frac{pq}{N}} = \frac{\sqrt{pq}}{\sqrt{N}} \qquad \text{(page 246)}$$

81. Standard error of the semi-interquartile range:

$$\sigma_Q = .7867\sigma_M \qquad \text{(page 247)}$$

82. Standard error of the average deviation:

$$\sigma_{AD} = .605\sigma_M \qquad \text{(page 248)}$$

83. Standard error of either quartile:

$$\sigma_{Q_1} = \sigma_{Q_3} = 1.36263\sigma_M \qquad \text{(page 248)}$$

84. Standard error of β_2:

$$\sigma_{\beta_2} = \sqrt{\frac{24}{N}} = 4.899\sqrt{\frac{1}{N}} \qquad \text{(page 249)}$$

85. Standard error of $\sqrt{\beta_1}$:

$$\sigma_{\sqrt{\beta_1}} = \sqrt{\frac{6}{N}} = 2.4495\sqrt{\frac{1}{N}}$$

86. Standard error of a measure of skewness based on the β's (see Text, page 206):

$$\sigma_{Sk.} = \frac{1.225}{\sqrt{N}} \qquad \text{(page 249)}$$

87. Standard error of a coefficient of variation:

$$\sigma_V = \frac{V}{\sqrt{2N}}\sqrt{1 + 2\left(\frac{V}{100}\right)^2} \qquad \text{(page 249)}$$

88. Approximate standard error of the coefficient of variation for small values of this coefficient:

$$\sigma_V = \frac{V}{\sqrt{2N}} \qquad \text{(page 250)}$$

89. Standard error of the difference between two measures, in terms of the standard errors of those two measures:

$$\sigma_{\text{Diff.}} = \sqrt{\sigma_1^2 + \sigma_2^2 - 2r_{12}\sigma_1\sigma_2} \qquad \text{(page 251}n.\text{)}$$

90. Standard error of the difference between two uncorrelated measures, in terms of the standard errors of those two measures:

$$\sigma_{\text{Diff.}} = \sqrt{\sigma_1^2 + \sigma_2^2} \qquad \text{(page 250)}$$

91. Standard error of the sum of two uncorrelated measures, in terms of the standard errors of those two measures:

$$\sigma_{\text{Sum}} = \sqrt{\sigma_1^2 + \sigma_2^2} \qquad \text{(page 253)}$$

92. Standard error of a coefficient of correlation:

$$\sigma_r = \frac{1 - r^2}{\sqrt{N}} \qquad \text{(page 400)}$$

93. Standard error of estimating Y from the linear regression equation:

$$S_y = \sigma_y \sqrt{1 - r^2} \qquad \text{(page 407)}$$

94. Standard error of the index of correlation:

$$\sigma_\rho = \frac{1 - \rho^2}{\sqrt{N - P}} \qquad \text{page 459)}$$

95. Square of the corrected standard error of estimating Y from the regression equation:

$$(S'_y)^2 = (S_y)^2 \left(\frac{N - 1}{N - P}\right) \qquad \text{(page 456)}$$

96. Standard error of a coefficient of multiple correlation:

$$\sigma_{R_{1.23\ldots n}} = \frac{1 - R^2_{1.23\ldots n}}{\sqrt{N - P}} \qquad \text{(page 478)}$$

97. Standard error of an index of multiple correlation:

$$\sigma_{\rho_{1.23\ldots n}} = \frac{1 - \rho^2_{1.23\ldots n}}{\sqrt{N - P}} \qquad \text{(page 478)}$$

98. Value of z:

$$z = \tfrac{1}{2}[\log_e (1 + r) - \log_e (1 - r)] \qquad \text{(page 400)}$$

99. Value of z:

$$z = 1.15129254 \log_{10} \left(\frac{1 + r}{1 - r}\right) \qquad \text{(page 401)}$$

100. Standard error of z:

$$\sigma_z = \frac{1}{\sqrt{n - 3}} \qquad \text{(page 401)}$$

101. Standard error of r in a universe in which there is no correlation:

$$\sigma_r = \frac{1}{\sqrt{n - 1}} \qquad \text{(page 421)}$$

Historical Data

102. Type equation of straight lines:

$$Y = a + bX \qquad \text{(page 278)}$$

103. Type equation of second-degree parabolas:

$$Y = a + bX + cX^2 \qquad \text{(page 281)}$$

104. Slope of a straight line:

$$b = \frac{\Sigma(xy)}{\Sigma(x^2)} \qquad \text{(page 299)}$$

105. Vertical intercept of a straight line:

$$a = M_y - b(M_x) \qquad \text{(page 299)}$$

106. Normal equations for straight line:

$$\begin{aligned} Na + b\Sigma X &= \Sigma Y \\ a\Sigma X + b\Sigma X^2 &= \Sigma(XY) \end{aligned} \qquad \text{(page 296)}$$

107. Normal equations for second-degree parabolas:

$$\begin{aligned} Na + b\Sigma X + c\Sigma X^2 &= \Sigma Y \\ a\Sigma X + b\Sigma X^2 + c\Sigma X^3 &= \Sigma(XY) \\ a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4 &= \Sigma(X^2Y) \end{aligned}$$

(pages 300, 441)

108. Normal equations for third-degree parabolas:

$$\begin{aligned} Na + b\Sigma X + c\Sigma X^2 + d\Sigma X^3 &= \Sigma Y \\ a\Sigma X + b\Sigma X^2 + c\Sigma X^3 + d\Sigma X^4 &= \Sigma(XY) \\ a\Sigma X^2 + b\Sigma X^3 + c\Sigma X^4 + d\Sigma X^5 &= \Sigma(X^2Y) \\ a\Sigma X^3 + b\Sigma X^4 + c\Sigma X^5 + d\Sigma X^6 &= \Sigma(X^3Y) \end{aligned}$$

(pages 309, 442)

109. Normal equations for reciprocal curve:

$$\begin{aligned} Na + b\Sigma X &= \Sigma\left(\frac{1}{Y}\right) \\ a\Sigma X + b\Sigma(X^2) &= \Sigma\left(\frac{X}{Y}\right) \end{aligned}$$

(page 303)

110. Normal equations for semilogarithmic curve:

$$\begin{aligned} Na + b\Sigma X &= \Sigma(\log Y) \\ a\Sigma X + b\Sigma(X^2) &= \Sigma(X \log Y) \end{aligned}$$

(page 306)

Correlation: Simple and Multiple; Linear and Curvilinear

111. Coefficient of correlation:

$$r = \sqrt{1 - \frac{S_y{}^2}{\sigma_y{}^2}}$$

(page 395)

112. Coefficient of correlation:

$$r = \sqrt{(b_{xy})(b_{yx})}$$

(page 405*n.*)

113. Coefficient of correlation:

$$r = \frac{\Sigma(xy)}{N\sigma_x\sigma_y}$$

(page 405)

114. Coefficient of correlation:

$$r = \frac{\Sigma(xy)}{\sqrt{(\Sigma x^2)(\Sigma y^2)}}$$

(page 405)

115. Coefficient of correlation:

$$r = \frac{\Sigma(XY) - N(M_x)(M_y)}{\sqrt{(\Sigma X^2 - NM_x^2)(\Sigma Y^2 - NM_y^2)}} \qquad \text{(page 405)}$$

116. Coefficient of correlation:

$$r = \frac{\dfrac{\Sigma(fd_xd_y)}{N} - \left(\dfrac{\Sigma(f_xd_x)}{N}\right)\left(\dfrac{\Sigma(f_yd_y)}{N}\right)}{(\sigma_x)(\sigma_y)} \qquad \text{(page 428)}$$

117. Square of the corrected coefficient of simple linear correlation:

$$(r')^2 = 1 - (1 - r^2)\left(\frac{n-1}{n-2}\right) \qquad \text{(page 398)}$$

118. Square of the corrected standard error of estimating from the linear regression equation:

$$(S_y')^2 = (S_y)^2\left(\frac{n-1}{n-2}\right) \qquad \text{(page 398)}$$

119. Simple linear regression equation:

$$y = r\frac{\sigma_y}{\sigma_x}x \qquad \text{(page 407)}$$

120. Simple linear regression equation:

$$y = \frac{\Sigma(xy)}{\Sigma(x^2)}x \qquad \text{(page 407)}$$

121. Simple linear regression equation:

$$Y - M_y = \frac{\Sigma(XY) - N(M_x)(M_y)}{\Sigma(X^2) - N(M_x)^2}(X - M_x) \qquad \text{(page 407)}$$

122. Index of correlation:

$$\rho = \sqrt{1 - \frac{S_y^2}{\sigma_y^2}} \qquad \text{(page 438)}$$

123. Square of the corrected index of correlation:

$$(\rho')^2 = 1 - (1 - \rho^2)\left(\frac{N-1}{N-P}\right) \qquad \text{(page 456)}$$

124. A beta coefficient:

$$\beta_{12.34} = b_{12.34}\left(\frac{\sigma_2}{\sigma_1}\right) \qquad \text{(page 475)}$$

125. Coefficient of multiple linear correlation:

$$R_{1.234} = \sqrt{1 - \frac{S^2_{1.234}}{\sigma_1^2}} \qquad \text{(page 477)}$$

126. Coefficient of multiple curvilinear correlation:

$$\rho_{1.234} \doteq \sqrt{1 - \frac{S^2_{1.f(234)}}{\sigma_1^2}} \qquad \text{(page 477)}$$

127. Normal equations, two independents, linear:

$$\begin{aligned} Na + b_2\Sigma X_2 + b_3\Sigma X_3 &= \Sigma X_1 \\ a\Sigma X_2 + b_2\Sigma(X_2)^2 + b_3\Sigma(X_2X_3) &= \Sigma(X_1X_2) \\ a\Sigma X_3 + b_2\Sigma(X_2X_3) + b_3\Sigma(X_3)^2 &= \Sigma(X_1X_3) \end{aligned} \qquad \text{(page 472)}$$

MISCELLANEOUS EQUATIONS

128. Sturges' rule for number of frequency classes:

$$m = 1 + 3.3. \log N \qquad \text{(page 45)}$$

129. Multiplier for logarithmic frequency classes:

$$r = \sqrt[n]{\frac{L}{F}} - 1 \qquad \text{(page 53)}$$

130. Charlier check for accuracy of computation:

$$\begin{aligned} \Sigma f(d+1) &= \Sigma(fd) + N \\ \Sigma f(d+1)^2 &= \Sigma fd^2 + 2\Sigma fd + N \\ \Sigma f(d+1)^3 &= \Sigma fd^3 + 3\Sigma fd^2 + 3\Sigma fd + N \\ \Sigma f(d+1)^4 &= \Sigma fd^4 + 4\Sigma fd^3 + 6\Sigma fd^2 + 4\Sigma fd + N \end{aligned}$$

(pages 91, 144, 195)

131. Chi square:

$$\chi^2 = \Sigma \frac{(f - f')^2}{f'} \qquad \text{(page 229)}$$

SECTION C

SUGGESTED EXERCISES

It is well-nigh impossible to understand statistical method without going through the process of statistical computation. Results cannot be correctly and understandably interpreted by those who have not seen them in the process of formation. For that reason it is important that every student of statistical method acquaint himself with the steps of actual statistical analysis. The number of problems which the student is to work out will depend considerably on the organization of the course. In some courses lectures are given three times a week for a semester, and the problems are all done on the outside. If one problem is assigned each time, paralleling the work of the class in the lecture room, this will make a total of some 40 problems. In some courses the class meets for lectures twice a week and for a laboratory exercise once a week. Under this plan it is usual to assign one problem each week, which would ordinarily amount to a dozen problems for the semester. In the latter case, of course, each assignment will be longer than in the former case.

There is, of course, no one "best" set of exercises for such classes. Hundreds of sets of problems can be arranged from the materials of this manual, and the teacher will always find additional problems here for use on examinations and as classroom illustrations. We suggest, however, two sets of exercises which may give a general idea of the type of work often covered in elementary courses. The first group consists of 12 weekly assignments, and the second group consists of 35 shorter assignments of daily work.

Group I

1. E2, E4, E5, E6, E10, E13, E18.
2. F6, F7.
3. G8.
4. G4 or G6.
5. H11, H13, H22.
6. I4, I7, I23.

7. I43 or I44.
8. J7, J8, J23.
9. K5, K6, K7, K8, K9, K10.
10. K18, K19, K20, K21.
11. K31, K32, K33, K34, K35, K36.
12. L27, L25.

Group II

1. D1, D2, D3.
2. D4, D5, D6.
3. D7, D9, D10 or D11.
4. D12, D13, D14.
5. E2, E5, E6, E8 or E9, E10 or E14 or E15.
6. F8*a*, F8*b*, F8*c*, F4, F5.
7. G1, G4*a* and G4*d*.
8. G4*b*, G4*c*, G6*c*, G6*d*.
9. G8.
10. H1, H3, H5.
11. H7, H10, H11, H13.
12. H22, H23.
13. I6, I7, I4, I8.
14. I9, I15, I16, I17.
15. I21, I23, I24, I27.
16. I44 or I43 (Two exercises).
17. Same as above.
18. J1, J2, J3, J6.
19. J7 or J9, J8 or J10.
20. J17 or J18, J23 or J24.
21. J34, J35.
22. K1, K2, K4.
23. K5, K6, K7.
24. K8, K9, K10, K11.
25. K26.
26. K22, K24, K25.
27. K31, K32, K33.
28. K34, K35, K36.
29. K38, K39.
30. L9, L10.
31. L11, L12.
32. L13, L14, L15.
33. L16, L17.
34. L19.
35. L25, L26, L28.

The teacher will, of course, feel free to substitute other problems from this manual at the proper points, whenever it is

necessary to give variety or to suit the interests of the particular class. Often local material can be substituted, the teacher supplying the figures and telling the student to follow the direcc-tions given in particular problems here. The order of exercises may occasionally be changed, or exercises omitted entirely. The outline given above is suggestive only.

SECTION D

GRAPHIC PRESENTATION

D1. On Jan. 1, 1930, there were in the United States the following numbers of families of the kinds mentioned:[1]

a. Native white families	21.0 million
b. Foreign-born white families	5.7 million
c. Negro families	2.8 million
Total families	29.5 million

Make a bar diagram of these data.

D2. Show the data of Prob. D1 by a pie diagram (also called a "sector diagram" or a "circle diagram"). Comment on the relative usefulness of the two methods of presenting the data.

D3. Show the data of Prob. D1 by squares. By cubes. Comment on these methods of presenting the facts.

D4. Convert the data of Prob. D1 to percentage form and make a percentage bar diagram.

D5. In Prob. K31 are figures representing the price of wheat from 1920 to 1933. Make a graph of these data.

D6. Make a graph of the data of Prob. K26 with the vertical axis broken at a point representing approximately 5 million spindles. What safeguards must be used in making such charts to protect readers from misinterpretation? Why would the graph otherwise be misinterpreted?

D7. In Prob. K28 are given figures showing the numbers of passenger automobiles registered in the United States from 1906 to 1918. Plot these data on ordinary graph paper. Plot them also on semilogarithmic paper. Note the difference in the appearance of the two charts. Explain. What is the special use of semilogarithmic charts in plotting historical data?

D8. Using the data of Prob. K28, plot on ordinary cross-section paper the logarithms of automobile registration for each year; that is, find the logarithms of the registration figures there given, and plot them. Compare the results with those of the two charts which were made in Prob. D7. Explain.

D9. Buy an outline map of the United States. Find figures by states showing potato production. (Try the most recent *Yearbook of the U. S. Department of Agriculture* or a recent *World Almanac* for figures.) Make a dot map showing the potato productions of the various states. Be careful to plan your work in such a way that the state which is filled in the most solidly is almost black, while the other states grade off gradually toward white.

D10. Procure an outline map of the United States. Show on it by the use of distinctive symbols (such as various kinds of crosshatching) the states which are in each of the following groups in 1864:

[1] Data from *World Almanac*, 1935, p. 248.

a. Northern States of the United States.
b. Confederate States of America.
c. Territory not then part of the union.

D11. In 1930 the percentage of illiteracy in the various states of the United States was as follows:[1]

Ala.	12.6	Maine	2.7	Ohio	2.3
Ariz.	10.1	Md.	3.8	Okla.	2.8
Ark.	6.8	Mass.	3.5	Ore.	1.0
Calif.	2.6	Mich.	2.0	Pa.	3.1
Colo.	2.8	Minn.	1.3	R. I.	4.9
Conn.	4.5	Miss.	13.1	S. C.	14.9
Del.	4.0	Mo.	2.3	S. D.	1.2
Fla.	7.1	Mont.	1.7	Tenn.	7.2
Ga.	9.4	Neb.	1.2	Tex.	6.8
Idaho	1.1	Nev.	4.4	Utah	1.2
Ill.	2.4	N. H.	2.7	Vt.	2.2
Ind.	1.7	N. J.	3.8	Va.	8.7
Iowa	0.8	N. M.	13.3	Wash.	1.0
Kan.	1.2	N. Y.	3.7	W. Va.	4.8
Ky.	6.6	N. C.	10.0	Wis.	1.9
La.	13.5	N. D.	1.5	Wyo.	1.6

Make a crosshatched map of the United States showing by distinctive symbols the states with a percentage of illiteracy under 2.5, those with 2.5 but under 5, those with 5 but under 10, and those with an illiteracy of 10 per cent and over. Be careful to get gradation of shade by proper use of symbols, so that there is continuous progression of shade in the legend.

D12. Make a frequency polygon of the data of Prob. G11.

D13. Make a frequency histogram of the data of Prob. G11.

D14. The following table shows (in billions of dollars) the amounts of various kinds of life insurance in force in various years.[2]

Year	Ordinary Insurance	Group Insurance	Industrial Insurance
1916	$19.4	$0.2	$ 4.7
1918	23.1	0.6	5.5
1920	32.8	1.6	6.9
1922	38.9	1.8	8.6
1924	48.5	3.1	10.9
1926	58.1	5.3	13.5

Make absolute and relative band charts of these data. Plot them also on semilogarithmic paper.

[1] Data from *World Almanac*, 1935, p. 390. An illiterate is a person 10 years of age or over who cannot write any language.

[2] Data from "Recent Economic Changes," p. 487.

D15. A questionnaire was sent to banks, asking each bank how it determined what interest rate to pay on its deposits. The replies indicated that the rates were fixed as follows:[1]

58 per cent by clearing house rule.
14 per cent by simple agreement among banks.
14 per cent by long-standing custom.
14 per cent by individual bank action.

a. Make a bar diagram to show these data.
b. Make a pie diagram to show these data.

D16. The following figures show the number of homes with radios in the United States in various years.

Year	Number of Homes (millions)	Year	Number of Homes (millions)	Year	Number of Homes (millions)
1923	1.0	1930	12.0	1937	26.7
1924	2.5	1931	14.0	1938	28.0
1925	3.5	1932	16.8	1939	28.7
1926	5.0	1933	20.4	1940	29.2
1927	6.5	1934	21.5	1941	29.7
1928	7.5	1935	22.9	1942	30.8
1929	9.0	1936	24.6		

a. Make a chart showing the data of this problem.
b. Show the data of this problem by a series of vertical bars.

D17. The largest college and university endowments in the United States are as follows:

	Endowment (millions)
Harvard	$148
Yale	103
Columbia	89
Chicago	73
Rochester	52
Texas	42
Massachusetts Institute of Technology	38
Duke	35
Princeton	33
Cornell	33

Make a bar diagram showing these data.

[1] Quoted by Lester V. Chandler, Monopolistic Elements in Commercial Banking, *Journal of Political Economy*, Vol. 46, February, 1938, p. 12.

SECTION E

SIGNIFICANT FIGURES

(Assume that all numbers in this section are approximations.)

E1. State exactly what is meant by the figure in each of the following statements:

a. The average distance from the sun to the earth is 93,000,000 miles.
b. Hartford, Connecticut, is in 41° 46′ north latitude.
c. George Washington was one of the wealthiest men in the country, owning 70,000 acres of land in Virginia.
d. Nicholas Longworth, then Speaker of the House of Representatives, died at the age of 61.
e. The Washington Monument is a tapering shaft 555 ft., 5⅛ in. in height.
f. The population of the United States at the time of the 1930 census was 123,000,000.
g. The data of the 1920 census show that the expectation of life for male babies at birth was 55.33 years as an average for the entire country.
h. A total of 3,707,500 income tax returns was recorded for individuals in 1930.
i. The bituminous coal production of the United States amounted to 3.1×10^8 net tons in 1932.
j. The world oat crop for 1932 has been estimated at 5 billion bushels.

E2. In 1929 there were 322 million bushels of potatoes produced in the United States on 2,900,000 acres of land. These potatoes had a value of $415,000,000.

a. Tell as accurately as you can the average yield per acre for the United States.
b. Tell as accurately as you can the average value per bushel for the United States.
c. Tell as accurately as you can the average value per acre in the United States.

E3. The distance from Albany, New York, to St. Louis, Missouri, by automobile road is 1092 miles. How long will it take a car to cover the distance if it travels at an average speed of 35 miles per hour? (How many significant figures in the latter figure?)

E4. In 1930 there were 5.3 people in the United States per passenger automobile registered. The total number of passenger automobiles registered was 23,059,262. From these figures compute as accurately as possible the 1930 population of the United States.

E5. What difference is there between the statement that the sun is 93,000,000 miles from the earth and the statement that the sun is 9.3×10^7 miles

from the earth? What are the advantages and disadvantages of each statement?

E6. Consult various sources to discover the measurement with the greatest number of significant figures of any which you can find. For example, investigate the results of attempts to measure the velocity of light, the diameter of the earth, the number of centimeters to the inch, etc. Find to how many significant figures one can make readings on the finest balances in your physics and chemistry laboratories. How many significant figures do students actually observe in courses in quantitative analysis, surveying, etc.?

E7. Look up the ages of the Presidents of the United States at the times of their inaugurations. (In cases where the same man served more than one term count his first inaugural only.) Compute as accurately as you can the average age of these men at the time of their first inauguration. Carry to as many places as the nature of the data warrants.

E8. The 1930 Census of Population in the United States was taken as of Apr. 1, 1930. The population as of that date is said to have been 122,775,046 persons. Comment.

E9. The American Jewish Committee estimated the 1931 Jewish population of Europe at 9,282,079 persons. Comment.

E10. A man wishes to determine the diameter of a steel water tower. He passes a tape line around it and finds that its circumference is 1084 inches. His tables tell him that the circumference is (approximately) 3.14159265359 times the diameter. What is the most accurate estimate of the diameter which he can make (assuming that the tower is actually circular in cross section)?

E11. Explain the difference between a continuous series and a discrete series from the standpoint of the meaning of the numbers.

E12. What is the difference between the number of decimal places and the number of significant figures?

E13. The 1930 census lists 12,048,762 radio sets in the United States on Apr. 1, 1930. The Columbia Broadcasting System estimated that there were 16,800,000 radio sets in the United States on Jan. 1, 1933. What was the increase in the number of sets?

E14. The 1935 *World Almanac* gives the following figures showing the numbers of people of each religious belief in North America:

Roman Catholics	40,000,000
Orthodox Catholics	1,000,000
Protestants	75,000,000
Jews	4,383,643
Mohammedans	20,000
Buddhists	180,000
Hindus	150,000
Confucianists, Taoists	600,000
Animists	50,000
Miscellaneous	25,000,000
Total	146,383,643

Comment. Find the total of these figures and compare your total with that given.

E15. The Hartford *Courant* of Jan. 1, 1935, states that the receipts of the local post office for the year just ended were "the best on record." Although the final figures for December were not yet available, the figures on monthly receipts for the year were given as follows:

January	$189,200.75
February	174,803.02
March	190,476.18
April	185,990.15
May	190,208.51
June	199,829.66
July	179,904.25
August	173,626.77
September	169,256.33
October	192,782.44
November	181,116.67
December (estimated)	250,000.00

The total for the year is given as $2,277,194.73. Comment.

E16. One yard is approximately equal to 91.44 cemtimeters. Compute as accurately as possible:

a. The number of centimeters in a foot.
b. The number of square centimeters per sq. ft.
c. The number of cubic centimeters per cu. ft.

E17. At the latitude of New York City the radius of the circle of latitude is 2.99×10^3 miles. Compute as accurately as you can the speed (in miles per hour), which results from the earth's daily rotation, of a point on the earth's surface at New York.

E18. In 1934 the United States government spent approximately $7,105,000,000. The United States population in 1934 has been estimated at 126,425,000 persons. Compute as accurately as possible the per capita government expenditures in 1934.

E19. The value of π is variously given as: 3.14, 3.142, 3.1416, 3.14159, 3.141593, 3.1415927, etc. If you are told that the earth's radius is 3959 miles, and you wish to compute the circumference, which of the above figures for π would you use? Why?

E20. In Prob. F9 are 180 figures taken from an actual research study. Add them and find the average. Then round off each of the original figures to two significant figures. Add the rounded numbers and find the average. In computing the average of a large number of figures, is it ever allowable to carry more significant figures in the answer than there were in the original figures? Why?

E21. In connection with the preceding problem read and report on J. R. Scarborough, "Numerical Mathematical Analysis," pp. 10–11; Johns Hopkins Press, 1930, and Raymond Pearl, "Medical Biometry and Statistics," 2d ed., pp. 362ff., W. B. Saunders Company, Philadelphia, 1930.

What rules would you feel safe in using in determining the number of significant figures to be retained in an arithmetic average?

E22. Comment on the following quotation.[1] "In 1808, Alexander Wilson, America's pioneer ornithologist, estimated that during a great flight of Pigeons which he saw in Kentucky, 2,230,272,000 birds passed in four hours."

E23. In explaining the meaning of the expression "an inch of rain," the *World Almanac* says in part,[2] "A rainfall of one inch over one acre of ground would mean a total of 3,630 cubic feet. As a cubic foot of pure water weighs about 62.4 pounds, the exact amount varying with the density, it follows that the weight of a uniform coating of one inch of rain over one acre of surface would be 226,512 pounds." State the final figure as it should have been given had the multiplication been carried out properly with approximate numbers.

E24. Recent measurements of the number of molecules in a mass of matter equal to its molecular weight, known as Avogadro's number, give the value 6.02331×10^{23}. The previously accepted value had been 6.0230×10^{23}, and the figure still appearing in many textbooks[3] is 6.064×10^{23}.

a. Write the newly obtained value in ordinary notation.
b. What is the percentage of error in the common textbook figure, assuming that the new determination is correct?
c. Find the common logarithm of the newly obtained value.

E25. A newspaper question-and-answer column received the query "How far on its journey around the sun does the earth travel in a day?" The answer[4] was, "Approximately 1,601,604 miles. The journey around the sun is 584,600,000 miles."

a. Assuming that the distance given for the length of the journey around the sun is correct, what would you give for the distance traversed in a day if you took 365 days for the length of the year?
b. What answer would you give if you took as the length of the year 365.25 days?
c. If you could find the length of the year correct to as many decimal places as you wanted, how many significant figures could you carry in your answer, basing your answer on the length of orbit given in the newspaper?

E26. If 1 yd. is 83.3 cm. to the nearest tenth centimeter, how many centimeters are there in 1 ft.?

[1] Frank M. Chapman, "The Travels of Birds," pp. 73–74, D. Appleton-Century Company, Inc., New York, 1916.
[2] *World Almanac*, 1942, p. 184.
[3] These figures are from *Science Supplement*, Vol. 96, No. 2481, July 17, 1942, p. 8.
[4] Hartford *Courant*, Sept. 16, 1935.

E27. According to *Science*,[1] the mass of the moon has recently been redetermined, indicating a mass that is 1/81.271 of the earth's mass. Convert this fraction to a number in standard notation.

E28. Write each of the following numbers in standard notation:

13,459
23,540,000
2
12
12.4
12.40
0.0000064

E29. Assuming that the two numbers 412 and 364 are both approximate numbers, multiply them together and give their product as accurately as possible.

E30. Assuming that both numbers are approximate, divide 261.9 by 131 and give the quotient as accurately as possible.

E.31. *Science*[2] states that the world production of oil for 1939 was 2,100,000,000 barrels and that this much oil has a heating value of about 2.5×10^{17} calories. The statement is also made that the earth receives energy from the sun a[illegible] rate of 2.5×10^{18} calories per minute.

a. What is the heating value of a barrel of oil?
b. How many times as much energy does the earth get from the sun in a minute as from the year's oil production?

E32. Read and report on the article The Accuracy of Measurement, which appeared in *Science*, Vol. 97, No. 2513, Feb. 26, 1943, pp. 10–12.

E33. Look up in Smithsonian Physical Tables, published by the Smithsonian Institution in Washington, the most recent figures on the most accurate physical measurements ever made, such as the measurement of the speed of light in a vacuum, etc. How many significant figures are there in these ultra-accurate physical measurements? On this basis, how many significant figures would a student be warranted in keeping in a measurement made in an ordinary college or university physics or chemistry laboratory?

E34. For an illustration of personal bias in measurements, read, and report on, Oscar E. Sette, Digit Bias in Measuring and a Device to Overcome It, *Copeia*, July 8, 1941, pp. 77*ff*.

[1] *Science*, Vol. 97, No. 2505, Jan. 1, 1943, p. 8.
[2] W. D. Coolidge, The Role of Science Institutions in Our Civilization, *Science*, Vol. 96, No. 2497, Nov. 6, 1942, p. 413.

SECTION F

THE FREQUENCY DISTRIBUTION

F1. In May, 1907, Ernest Thompson Seton traveled along the Canadian Pacific Railway and saw 26 bands of antelope along the southern side of the track. He records the numbers in these bands,[1] presumably in the order in which they were seen, as follows:

4, 14, 18, 8, 12, 8, 4, 1, 4, 5, 4, 6, 4, 18,
2, 6, 34, 6, 3, 1, 10, 25, 16, 3, 7, and 9.

a. Group these data in a frequency table using a class interval of 5. Group again using a class interval of 25. Group again using a class interval of 2.
b. Explain which of the three class intervals just used is the best, and give your reasons.

F2. A seventh-grade class in physiology and hygiene was given an examination on which the highest possible mark was 100. The 30 pupils in the class received the following marks:[2]

46, 75, 70, 65, 60, 87, 87, 83, 93, 70, 78, 50,
76, 76, 71, 59, 79, 98, 55, 85, 73, 94, 65, 76,
71, 97, 67, 70, 80, 78.

a. Group these data in a frequency table with a class interval of 2. Reclassify with a class interval of 5; and again with class intervals of 10 and 20.
b. What is the effect on the regularity of the change in frequencies from class to class as the size of the class interval is increased?
c. Group the data in a frequency table with a class interval of 5, so arranging the class limits that the class marks will be 45, 50, 55, 60, etc.

F3. In this book, or some other, select a section where the pages are made up mainly of solid text. Select 100 lines of printing, omitting those which are shorter than usual on account of indentations. Count the number of letters in each of these 100 full lines. Make a frequency table showing the number of times that each number of letters occurred. (Make your own rules as to the method of counting punctuation marks, etc., and stick to these rules. Skip any line about which there is serious question.)

[1] E. T. Seton, "The Arctic Prairies," p. 4, Charles Scribner's Sons, New York.

[2] Data from G. M. Ruch, "The Objective or New-Type Examination," p. 417, Scott, Foresman & Company, Chicago.

F4. Neifeld gives a table[1] showing the percentage of borrowers from personal finance companies whose borrowings were of various amounts. His figures follow:

Size of Loan	Percentage of Borrowers
$10– 50	17.0
60–100	32.3
105–150	15.7
160–200	17.1
220–240	3.6
250–300	14.3

a. What is the class interval in the table? Comment.
b. If a loan were outstanding for $210 in which class would it be listed?
c. If, as is probably true, the amounts of the loans were bunched around such points as $25, $50, $100, and other even numbers, with few loans of such odd amounts as $117.26, what precautions would you take in selecting the class limits of the table?

F5. Give five examples of discrete data and five of continuous data.

F6. In 1930 the Voluntary Defenders Committee of the Legal Aid Society of New York handled 1,235 cases. Of these, 289 defendants alleged that they had been beaten by the police. The records of 15 of these 289 are missing. The ages of the remaining 274 defendants[2] appear below:

28, 31, 24, 16, 25, 21, 22, 23, 21, 44, 19, 36,
25, 28, 17, 18, 24, 22, 27, 21, 23, 22, 41, 42,
16, 20, 22, 22, 39, 24, 26, 25, 22, 18, 24, 23,
22, 28, 23, 20, 28, 16, 16, 18, 37, 27, 31, 24,
35, 22, 38, 17, 32, 20, 17, 27, 16, 19, 37, 25,
32, 19, 24, 17, 19, 34, 29, 17, 20, 18, 29, 16,
44, 24, 22, 23, 19, 18, 17, 29, 18, 70, 19, 19,
20, 17, 19, 20, 25, 20, 20, 31, 16, 17, 19, 49,
16, 19, 27, 34, 21, 37, 18, 16, 31, 17, 17, 18,
24, 34, 32, 27, 21, 17, 16, 17, 19, 29, 26, 20,
20, 32, 20, 23, 18, 40, 27, 18, 24, 17, 33, 25,
18, 17, 32, 32, 23, 23, 24, 16, 17, 18, 38, 17,
17, 18, 19, 20, 40, 17, 18, 17, 18, 29, 19, 20,
29, 29, 19, 33, 38, 24, 43, 30, 56, 30, 27, 35,
38, 27, 17, 17, 16, 30, 16, 27, 30, 24, 28, 18,
20, 20, 17, 18, 21, 20, 27, 23, 22, 21, 28, 19,
16, 16, 17, 19, 25, 19, 32, 30, 16, 27, 17, 15,
17, 24, 26, 20, 30, 26, 32, 40, 22, 21, 17, 40,

(*Continued on page* 115)

[1] Neifeld, The Personal Finance Business, *Harper's*, New York, p. 200.

[2] From the *National Commission on Law Observance and Enforcement, Publication* 11, Report on Lawlessness in Law Enforcement, June 25, 1931 pp. 225–232.

26, 43, 22, 20, 29, 22, 37, 33, 20, 22, 38, 28,
29, 46, 37, 16, 32, 24, 15, 17, 18, 20, 23, 26,
34, 36, 30, 41, 23, 35, 18, 20, 35, 22, 23, 52,
18, 36, 22, 19, 17, 21, 24, 43, 22, 27, 19, 19,
21, 33, 21,29, 21, 16, 26, 35, 18, 26.

a. Classify these ages in a frequency table. Select the class interval in such a way that there will be from 10 to 15 classes. (It will usually be found desirable to select a class interval of 1, 2, 5, 10, 25, 50, 100, or some multiple of 100.)

b. State the class limits in two or three different but equivalent ways, keeping the class interval and the actual class limits the same.

F7. In Prob. G14 are tabulated the ages at marriage of 9907 urban women and of 6720 rural women. In Prob. G10 are tabulated the ages at marriage of 13,449 Philadelphia women.

a. Regroup the data of Prob. G14 so that the class intervals will coincide with those of Prob. G10

b. Regroup the data of Prob. G14 so that there are 5-year class intervals.

c. Try regrouping the data of Prob. G10 to make them comparable with the data of Prob. G14. Try regrouping the data of Prob. G10 into classes with a 5-year class interval. What are the advantages of short class intervals?

d. Since there is some advantage in having short class intervals, why should we ever group data into wide classes?

F8. In the 54th Annual National Archery Association Tournament, held in Storrs, Connecticut, in August, 1934, there were 107 men entered in the competition for the men's national title. The number of points scored by each contestant, arranged in array, follows:[1]

2902, 2767, 2722, 2634, 2508, 2283, 2227, 2220, 2210,
2197, 2186, 2186, 2104, 2054, 2018, 1982, 1972, 1955,
1949, 1935, 1928, 1928, 1926, 1902, 1893, 1872, 1867,
1861, 1849, 1847, 1835, 1805, 1802, 1769, 1767, 1756,
1740, 1739, 1720, 1702, 1694, 1692, 1691, 1689, 1674,
1672, 1669, 1667, 1663, 1653, 1638, 1575, 1559, 1557,
1550, 1545, 1534, 1510, 1487, 1487, 1458, 1447, 1435,
1422, 1417, 1401, 1395, 1391, 1386, 1379, 1350, 1322,
1297, 1290, 1277, 1258, 1245, 1226, 1198, 1169, 1156,
1155, 1133, 1125, 1095, 1071, 1047, 1032, 999, 995,
965, 937, 932, 897, 895, 858, 847, 700, 675,
668, 664, 607, 583, 430, 344, 337, 299.

a. Arrange these data in a frequency table with a class interval of 100, the class marks being multiples of 100.

[1] Data from Associated Press dispatch in the Hartford *Courant*, Aug., 18, 1934.

b. Arrange in a frequency table with a class interval of 250.
c. Arrange in a frequency table with a class interval of 10. Criticize.

F9. In the year 1927 the Bureau of Business Research of Harvard University collected operating expense statements from 180 department stores with sales over $1,000,000 a year.[1] The following figures are the gross margins of these 180 department stores:

30.39	34.74	35.26	30.74	31.00	34.80	33.81	36.15	34.04
31.97	34.08	39.38	35.88	35.04	30.16	34.33	31.87	24.34
33.75	33.33	34.24	34.22	32.69	31.69	33.39	34.02	34.76
32.24	41.84	33.80	34.05	29.73	38.59	35.83	34.82	30.14
32.63	35.03	33.36	34.63	37.12	34.29	24.68	32.56	33.16
34.23	34.46	33.76	29.57	33.59	35.14	29.70	33.70	29.75
32.90	35.37	32.02	35.28	33.41	27.02	35.33	35.20	29.21
31.67	28.55	30.58	34.57	32.85	35.56	32.14	35.31	32.73
28.45	28.40	32.90	35.15	34.39	31.58	27.42	37.32	35.61
37.76	34.20	35.45	30.62	36.42	35.13	32.67	36.89	33.73
32.31	33.22	35.06	30.85	13.56	37.55	35.60	35.33	37.41
35.62	33.52	34.62	33.44	34.64	31.04	28.64	34.40	38.45
36.85	44.64	34.29	33.31	27.76	23.08	33.39	32.01	28.33
33.46	36.23	32.33	33.44	31.07	28.49	22.92	34.13	33.44
30.14	33.16	34.38	32.55	34.64	33.75	32.82	32.93	29.57
32.39	34.30	28.63	34.34	34.91	34.32	37.47	30.80	20.70
33.38	29.93	30.18	33.62	30.96	32.80	31.36	27.81	36.07
29.90	31.07	29.25	30.35	31.89	29.34	29.39	32.90	33.29
31.19	32.91	31.25	34.97	31.14	33.87	31.61	31.28	32.16
32.44	34.75	36.50	27.20	41.89	29.18	33.95	28.42	34.08

a. Arrange these data in an array.
b. Arrange these data in a frequency table. First find the approximate range by inspection, and then select a class interval in such a way that the frequency table will contain about ten or twelve classes.
c. Explain the difficulties which would arise if we used too large a class interval. How would you know that the interval was too large. Explain also how you would know if the class interval was too small, and what difficulties would be encountered as a result.

F10. The following table shows the approximate numbers of telegraph poles remaining at the end of each year if one starts with 30,000 poles.[2] These were wooden poles treated with coal tar.

[1] From Brown, Bingham, and Temnomeroff, "Laboratory Handbook of Statistical Methods," pp. 101–103, McGraw-Hill Book Company, Inc., New York, 1931.

[2] Rough approximation based on Life Characteristics of Physical Property, *Iowa Engineering Experiment Station Bulletin* 103, p. 103.

Age of Poles (years)	Number of Poles Remaining	Age of Poles (years)	Number of Poles Remaining
0	30,000	11	12,600
1	29,800	12	9,750
2	29,650	13	7,200
3	29,200	14	4,800
4	28,300	15	2,850
5	27,400	16	1,800
6	25,800	17	900
7	23,800	18	300
8	21,600	19	150
9	18,750	20	0
10	15,600		

This table is in the form of an ogive. Recast the figures into the form of an ordinary frequency curve. Compute the average length of life, the median length of life, and the modal length of life. How could the median length of life be approximated from the data in their original form? Were the lengths of the lives of these poles approximately normally distributed?

F11. The numbers of students at City College, New York, who participated in various numbers of sports were found[1] to be as follows:

Number of sports.	0	1	2	3	4	5	6	7
Number of students.	164	63	24	40	26	7	1	1

Plot the frequency curve and comment on its shape. Is the distribution discrete or continuous?

Birth Order	Number of Cases	Birth Order	Number of Cases
1	635,820	14	3417
2	452,383	15	1755
3	303,510	16	907
4	207,452	17	469
5	144,793	18	271
6	104,447	19	139
7	76,218	20	78
8	55,667	21	26
9	38,706	22	18
10	27,144	23	18
11	16,947	24	11
12	10,635	25	6
13	6,042	26	3

[1] Daniel Harris, The Relation to College Grades of Some Factors Other Than Intelligence. *Archives of Psychology*, No. 131, p. 32.

F12. Of 2,086,882 children born in the United States in 1928, some were the first children of their mothers, some were the second children, etc. The number of these children who had each birth order is given in the table[1] at the bottom of page 117. For example, 635,820 of these 2 million children were first-born children.

Note that this is a J shaped distribution. In this sample there were as many twenty-third-born children as twenty-second-born children (18 of each). Would you expect this in other samples? Why or why not? Plot the first 12 classes on semilogarithmic paper. Describe the result. Plot classes 12 to 20 on semilogarithmic paper and describe the result. Compute the ratio of each number of cases in the table to the one preceding it.

F13. There follow a number of frequency tables, any one of which may be used as a basis for further computation. The tables are lettered as subheadings under Prob. F13 and are hereafter referred to as Prob. F13*a*, F13*b*, etc.

a. Heights, in inches, of 272 ten-year-old Mexican boys and 239 ten-year-old Mexican girls, as given by H. T. Manuel, Physical Measurements of Mexican Children in American Schools, *Child Development*, Vol. 28, September, 1934, p. 249.

Height (inches)	Number of Boys	Number of Girls
59	1	1
58	2	1
57	7	6
56	7	13
55	23	17
54	28	28
53	49	31
52	48	38
51	35	46
50	36	28
49	20	18
48	9	6
47	4	2
46	2	2
45	0	1
44	1	1
Total	272	239

[1] *Journal of the American Statistical Association*, December, 1933, p. 392.

b. This table shows the number of women who had various numbers of accidents when working for 5 weeks on 6-in. high-explosive shells. The data have been taken from Major Greenwood, Nerves and the Public Health, *Human Biology,* Vol. 4, May, 1932, p. 159.

Number of Accidents	Number of Women
0	447
1	132
2	43
3	21
4	3
5	2
Total.......	648

c. Pearl classified 1390 white women at a maternity hospital according to the total number of pregnancies that each had experienced. The results, taken from Raymond Pearl, Contraception and Fertility in 2000 Women, *Human Biology,* Vol. 4, September, 1932, p. 391, follow:

Number of Pregnancies	Number of Women
1	593
2	285
3	169
4	116
5	78
6	41
7	36
8	21
9	9
10	25
11	9
12	2
13	2
14	1
15	2
16	0
17	1
Total.......	1390

d. Boaz studied 483 girls to see at what age each one grew most rapidly. His figures, given in Franz Boaz, Studies in Growth, *Human Biology,* Vol. 4, September, 1932 p. 310, are:

Age of Maximum Rate of Growth (years)	Number of Girls
9–10	14
10–11	66
11–12	162
12–13	149
13–14	85
14–15	6
15–16	1
Total.......	483

e. Bowmar studied the influence on leaf growth of removing blossoms or fruits from prune trees. On some of the trees the blossoms were removed 16 days after full bloom; on others they were removed 30 days after full bloom. Data on the former appear in column *A*, and on the latter in column *B*. The data are from F. T. Bowmar, Influence of Early Times of Fruit Removal, *Journal of Pomology and Horticultural Science*, Vol. 19, August, 1941, p. 45. The table shows the number of shoots with various numbers of leaves.

Number of Leaves	Number of Shoots	
	A	*B*
2	11	15
3	84	144
4	120	304
5	303	557
6	337	453
7	129	125
8	39	48
9	16	25
10	10	19
11	14	3
12	12	6
13	5	6
14	2	2
15	6	4
Totals	1088	1712

f. The following table shows the ages at death from homicides in 37 New York counties from 1921–1930, by sex. Data are from

J. V. Deporte and Elizabeth Parkhurst, Homicide in New York State, *Human Biology,* vol. 7, February, 1935, p. 58.

Age (years)	Number of Homicides	
	Male	Female
1–14	28	24
15–19	37	23
20–24	94	28
25–34	268	72
35–44	246	64
45–54	125	37
55–64	55	18
65 and over	35	9
Age unknown	2	1
Totals	890	276

g. Twenty-three clutches of eggs of a species of salamander were collected at Woods Hole, Mass. W. G. Lynn and J. N. Dent, Notes on *Plethodon Cinerius* and *Hemidactylium Sculatum* on Cape Cod, *Copeia,* July 8, 1941, p. 113, give the number of clutches with various numbers of eggs as follows:

Number of eggs	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of clutches	1	0	1	1	0	2	10	2	0	0	4	1	0	1

h. Lester V. Chandler, in Monopolistic Elements in Commercial Banking, *Journal of Political Economy,* Vol. 46, February, 1938, p. 7, shows the number of commercial and savings banks in various cities of the United States in 1936. If a bank had branches in different cities, one branch, and only one, was counted in each city. The numbers of cities with various numbers of banks were

Number of banks	1	2	3	4	5	6	7	8	9	10	over 10
Number of cities	8,962	2,201	476	164	83	36	19	8	11	6	37

Total number of cities 12,003
Total number of banks 17,270

i. If we compute the percentage change in price for each of 834 common stocks listed on the New York Stock Exchange between Dec. 31, 1940, and Dec. 31, 1941, we find the number of issues

advancing or declining various amounts as follows. Data are from *The Exchange*, January, 1942, p. 15.

Per Cent Advance	Number of Stocks
60 and over	16
50–60	7
40–50	13
30–40	14
20–30	25
10–20	38
0–10	93
Per Cent Decline	
10–0	119
20–10	140
30–20	136
40–30	108
50–40	64
60–50	39
Over 60	22
Total........	834

j. The red-blood-cell counts (in millions) of 195 white full-term newborn babies born at Multnomah County Hospital, Portland, Ore., as given in *American Journal of Diseases of Children*, Vol. 62, December, 1941, p. 1194, were as follows:

Cell Count (millions)	Number of Babies
2.5	1
2.9	0
3.3	8
3.7	9
4.1	46
4.5	46
4.9	47
5.3	30
5.7	7
6.1	0
6.5	1
Total.......	195

k. The maximum daily flow of the Rhone River at Lyon, France, each year from 1816–1936 was measured in thousands of cubic meters per second. The number of years in which various maximum flows were recorded is given in the following table. The flows there given are class mid-points and are stated in units of a thousand cubic meters per second. Data are from E. J. Gum-

bel, The Return Period of Flood Flows, *Annals of Mathematical Statistics*, Vol. 12, June, 1941, p. 184.

Maximum Flow	Number of Years
807	1
1081	1
1355	7
1629	5
1903	13
2177	21
2451	19
2725	14
2999	9
3273	8
3547	6
3822	4
4096	2
4370	1
Total.......	111

l. The ages at their first walking of 217 children who "walked on all fours" are given by Wayne Dennis, The Age at Walking of Children Who Run on All Fours, *Child Development*, Vol. 5, March, 1934, p. 93, as follows:

Age (months)	Number of Children
8– 8.9	1
9– 9.9	9
10–10.9	20
11–11.9	29
12–12.9	60
13–13.9	32
14–14.9	30
15–15.9	14
16–16.9	8
17–17.9	1
18–18.9	8
19–19.9	2
20–20.9	0
21–21.9	0
22–22.9	2
23–23.9	1
Total.....	217

m. The number of times that the annual rainfall in Boston was at various levels is shown in the following table, which gives the rainfall in inches tabulated from original entries in *Annals of Mathematical Statistics*. Vol. 1. No. 2. pp. 126–127:

Rainfall (inches)	Number of Years
25–29	2
30–34	9
35–39	25
40–44	31
45–49	22
50–54	13
55–59	3
60–64	3
65–69	3
Total......	111

n. In 1942 there were in the United States 274 publishers who published five or more titles each, pamphlets not included. These publishers published 8107 books. The number of publishers publishing various numbers of books, derived from original detailed figures in *Publisher's Weekly*, Vol. 143, No. 3, Jan. 16, 1943, pp. 247–249, follows:

Number of Books	Number of Publishers
5–9	98
10–14	44
15–19	34
20–24	13
25–29	16
30–39	16
40–49	11
50–74	15
75–99	10
100–149	12
150–199	1
200–299	1
300–399	3
Total...............	274

o. In 400 throws of a coin there were 196 heads. Runs of one head, two heads, three heads, etc., in a row occurred the following number of times when the experiment was tried by A. C. Aitken, as recorded in his "Statistical Mathematics," p. 27, Interscience Publishers, Inc., New York, 1942.

Number of successive heads...........	1	2	3	4	5	6	7	8
Number of times......................	51	24	14	4	5	1	0	1

p. Treolar, in his "Outlines of Biometric Analysis," p. 19, Burgess Publishing Company, Minneapolis, 1935, gives the following distribution of birth weights of 402 female children of German parentage:

Weight (ounces)	Number of Children
77– 84.5	2
85– 92.5	20
93–100.5	45
101–108.5	74
109–116.5	85
117–124.5	62
125–132.5	61
133–140.5	26
141–148.5	13
149–156.5	9
157–164.5	4
165–172.5	1
Total.......	402

q. Harold Hotelling in his article on Frequency Distributions in the *Encyclopaedia of the Social Sciences*, Vol. 6, The Macmillan Company, New York, 1937, p. 484, gives the following numbers of retail hardware stores with various gross profit margins:

Profit Margin (per cent)	Number of Stores
Less than 4	3
4–8	10
8–12	16
12–16	30
16–20	20.5
20–24	15.5
24–28	5
Total......	100

r. R. S. and H. M. Lynd, "Middletown in Transition," p. 555, Harcourt, Brace and Company, New York, 1937, give the numbers of dwelling units in "Middletown" for which various amounts of rent were paid in April, 1930, as follows:

Monthly Rent	Number of Units
Under $15	603
$15–29	3449
30–49	1540
50–99	352
100 and over	10
Not reported	38
Total.......	5992

s. Gardiner C. Means, in studying price flexibility, investigated the price changes of 747 commodities included in the U.S. Bureau of Labor Statistics Wholesale Price Index for the 96 months of

1926–1933. Each month the price was listed, and the price could thus have changed 95 times. The numbers of commodities with various numbers of price changes (read roughly from Means' chart in his article, Price Inflexibility and the Requirements of a Stabilizing Monetary Policy, *Journal of the American Statistical Association*, Vol. 30, June, 1935, p. 402) were as follows:

Number of Times Price Changed	Number of Commodities
0–4	95
5–9	97
10–14	79
15–19	58
20–24	45
25–29	23
30–34	29
35–39	34
40–44	16
45–49	12
50–54	9
55–59	18
60–64	15
65–69	15
70–74	12
75–79	9
80–84	19
85–89	37
90–94	125
Total.......	747

t. The numbers of negro mothers of various ages in five southern states was as follows: The data are from S. J. Holmes and S. L. Parker, The Stabilized Natural Increase of the Negro, *Journal of the American Statistical Association*, Vol. 26, June, 1931, p. 161.

Age of Mother (years)	Number of Mothers
10–15	152
15–20	6,784
20–25	12,490
25–30	8,665
30–35	5,270
35–40	4,220
40–45	1,288
45–50	206
50–55	10
Total....	39,085

u. B. L. Shook, in his Synopsis of Elementary Mathematical Statistics, *Annals of Mathematical Statistics*, Vol. 1, August, 1930, p. 228, quotes figures of Pearson and Lee showing the number of days during 13 years when the barometer stood at various heights at Llandudno, England. The data follow:

Height of the Barometer	Number of Observations
28.35	1
28.55	2
28.75	8
28.95	30
29.15	74
29.35	166
29.55	368
29.75	509
29.95	656
30.15	580
30.35	353
30.55	140
30.75	30
30.95	5
Total......	2922

v. D. W. Lay, Ecology of the Opossum in Eastern Texas, *Journal of Mammalogy*, Vol. 23, May, 1942, p. 149, gives records on trapping opossums. Some of the animals were caught once, some twice, some three times, etc. The numbers of individuals caught various numbers of times follow:

Times Caught	Number of Opossums
1	58
2	22
3	13
4	8
5	5
6	1
7	3
Over 7	7
Total.....	117

The total catch was 376; so the average number of times caught was $376/117 = 3.21$.

w. C. W. Schwartz in Home Range of the Cottontail Rabbit in Central Missouri, *Journal of Mammalogy*, Vol. 22, November, 1941, p. 387, gives data on catching rabbits in live traps. Some

rabbits were caught once, some twice, etc. The following table shows the numbers of rabbits caught various numbers of times:

Times Caught	Number of Rabbits
1	127
2	52
3	31
4	10
5	11
6	8
7	5
8	2
9	1
Over 9	7
Total.....	254

Total catch was 637; average number of times caught, 2.5.

F14. B. L. Shook, Synopsis of Elementary Mathematical Statistics, *Annals of Mathematical Statistics*, Vol. 1, February, 1930, p. 28, records the numbers of even numbers in 170 samples of 100 numbers each, taken from the conscription drawings of 1918. The data follow:

51	42	49	53	49	46	47	51	57	48
49	51	55	50	46	53	46	47	46	54
60	59	42	42	58	43	53	49	54	53
53	46	47	50	55	50	48	47	44	51
48	57	49	52	57	56	45	64	37	58
51	53	51	49	39	54	51	56	44	41
42	46	50	56	42	54	50	45	47	58
50	52	53	55	52	48	50	53	45	48
51	55	47	45	55	51	47	54	48	46
52	60	52	53	49	52	46	62	43	48
54	50	51	50	50	53	44	54	51	45
53	47	44	48	55	45	55	45	55	50
52	55	54	56	42	49	45	55	45	55
44	37	44	53	52	50	51	47	56	44
54	56	50	53	49	52	60	48	50	51
56	45	50	51	53	44	47	54	46	54
42	44	49	43	57	46	48	48	49	48

a. Is this a continuous or a discrete series?
b. Select a class interval for these data in such a way that there will be 10–15 classes.
c. Make a frequency table from these data.

F15. Make a cumulative frequency table from the data of Prob. F13*u*.
F16. Make a frequency polygon from the data of Prob. F13*p*.
F17. Make a frequency histogram of the data of Prob. F13*p*.

F18. Glance through the various tables of Prob. F13 and pick out one good example of a mound-shaped curve that seems approximately symmetrical, one of a skewed mound-shaped curve, one of a J-shaped curve, and one of a U-shaped curve.

F19. Make an ogive of the data of Prob. F13*p*.

F20. Apply Sturges' rule to determine the number of classes into which the data of Prob. F13*u* should have been classified.

F21. The data of Prob. F13*f* are classified in unequal class intervals. Make a frequency histogram of these data, adjusting it for the inequality of class intervals.

F22. The data of Prob. F13*n* are badly skewed. Suppose that it is desired to classify these data into 12 logarithmic frequency classes (sometimes called geometric frequency classes). Set up the class limits.

F23. In Prob. F13*k* the class marks are given in place of the class limits. List the class limits for this table.

F24. The per capita net debt of 90 American cities of over 100,000 population is given in the U.S. Department of Commerce's *Financial Statistics of Cities*, 1938, Vol. II, *Summary Bulletin*, pp. 24*ff*. The figures for the individual cities are:

$167.59	$ 6.95	$ 78.67	$ 56.84	$135.61	$28.88
63.16	8.51	57.44	10.41	172 82	7.85
21.57	24.51	9.44	32.05	95.21	39.84
22.32	25.26	165.86	34.46	1.98	76.57
59.77	34.66	15.98	134.27	6.76	27.66
30.53	21.53	2.15	15.06	17.94	56.29
42.86	7.16	19.04	25.88	22.75	62.96
13.74	35.34	30.79	4.36	54.39	19.60
45 71	9.26	2.33	66.68	25.30	49.90
40.84	6.76	29.30	29.56	77.41	36.93
97.24	152.42	16.22	11.44	6.62	1.78
15.34	9.36	11.07	2.76	6.03	54.23
37.22	23.27	96.27	10.33	86.01	6.58
2.66	26.79	2.28	4.22	1.60	17.96
3.80	80.11	64.83	3.79	10.33	15.23

Each of these figures represents the per capita debt of one city in dollars and cents.

a. In making a frequency table of these data, how many classes would you use? How would you reach this decision?

b. Arrange these data in an array.

c. Make three different frequency tables from these data, all with the same class interval but with the class marks at different points.

SECTION G

MEASURES OF CENTRAL TENDENCY

G1. Find the arithmetic mean of the 30 marks listed in Prob. F2.

G2. What was the average (arithmetic mean) size of the bands of antelope listed in Prob. F1?

G3. Arrange the 30 items of Prob. F2 in an array. Find the median mark. Compare it with the arithmetic mean.

G4. During the period from 1921 to 1925 records were kept of the lengths of time that white pupils in the Hagerstown, Maryland, schools were absent for various kinds of sickness. During this period 657 children were absent on account of measles. The number of pupils who missed various numbers of school days on this account was as follows:[1]

Duration of Sickness (Number of school days missed)	Number of Pupils[2]
0– 4.9	29
5– 9.9	195
10–14.9	241
15–19.9	117
20–24.9	52
25–29.9	10
30–34.9	6
35–39.9	3
40–44.9	2

a. What was the average time lost (arithmetic mean) on account of measles?

b. What was the median time lost?

c. What was the most common time (mode) lost?

d. Calculate the arithmetic mean by both the short and the long methods. Compare the answers.

e. Find the first and third quartiles. Find the third decile and the 13th percentile. Explain your answers as you would to a high-school pupil who had never studied statistics.

[1] Data from The Health of the School Child, U. S. Treasury Department, *Public Health Bulletin* 200, pp. 131*ff*.

[2] For our purposes we omit two pupils who missed over 45 days of school, since the exact time missed by them was not given in the source from which our data were taken.

G5. Each year from 1915 to 1924, inclusive, the number of passenger automobiles registered in the United States was larger than in the year before. The percentage increase of each year over the preceding year was as follows:[1]

Year	Increase over Preceding Year (per cent)
1915	42
1916	43
1917	42
1918	21
1919	21
1920	22
1921	14
1922	16
1923	24
1924	15

In 1914 the number of passenger car registrations was 1,625,739. In 1924 the number was 15,460,649.

a. Using the data in the table, compute the arithmetic average percentage increase for the 10-year period. Compute the geometric average of the rates of increase. Check them against the actual figures on numbers of registrations to see which average, when applied ten times, will give the proper increase in the number of registrations.

b. Compare the size of the arithmetic mean of these numbers and the geometric mean.

c. Note the advantage of the geometric mean in averaging rates of change. Look up the figures on the percentage increase in the population of the United States by 10-year periods (census figures). Compute the average rate of increase per decade, using first the arithmetic mean and then the geometric mean. Test the results by applying your averages to the actual population figures one decade at a time to see if you end with the proper size of population.

G6. The number of bushels of wheat bought and sold on the future market of the Chicago Board of Trade varies considerably from day to day. From Jan. 3, 1921, to Dec. 31, 1932, the numbers of days when the volume of trading was at various levels is shown in the accompanying table:[2]

[1] Source: "Automobile Facts & Figures," 1934 ed., p. 10, National Automobile Chamber of Commerce.

[2] Data from Wheat Futures, *U. S. Department of Agriculture Statistical Bulletin* 41, September, 1933, p. 37. Omits 23 days over 105 million bushels.

Volume of Trading (millions of business)	Number of Days	Volume of Trading (millions of business)	Number of Days
0 - 5	4	55- 60	148
5 -10	105	60- 65	133
10-15	210	65- 70	81
15-20	369	70- 75	74
20-25	383	75- 80	42
25-30	394	80- 85	36
30-35	413	85- 90	28
35-40	377	90- 95	14
40-45	294	95-100	8
45-50	269	100-105	4
50-55	198		

a. Criticize the method of giving the class limits in this table.
b. Assume that the classes include the lower but not the upper limits as given; *i.e.*, assume that the first class is "zero but under 5," the second class is "5 but under 10," etc. Compute the arithmetic average volume of trading.
c. What was the median volume of trading? Explain the meaning of your answer in simple terms.
d. What was the modal volume of trading? Explain your answer in simple terms.

G7. The moths of 20 fall cankerworms were killed and their ovaries opened. The number of eggs in the ovaries of each moth was counted. The number of eggs found were as follows:[1]

158, 272, 127, 184, 213, 135, 140, 220, 200, 130,
111, 160, 193, 131, 281, 242, 116, 281, 192, 217.

a. What was the average number of eggs per moth (arithmetic mean)?
b. Arrange the items in an array. Find the median and the quartiles.
c. Try using the value $(N + 1)/2$ and the value $N/2$ in determining the location of the median. In which instance are there actually just as many cases below as above the median item?

G8. Find the average age of the 274 men whose ages appear in Prob. F6. Arrange the data in a frequency table and compute the arithmetic mean from the frequency table. Do the two arithmetic means coincide? Discuss. Try the same experiment with the median, finding it first from the ungrouped and then from the grouped data.

[1] Data from *Useful Birds and Their Protection*, p. 170, Massachusetts State Department of Agriculture, 1907.

G9. The distance by automobile road from Cleveland, Ohio to Charleston, W. Va., is 300 miles.[1] Suppose that the distance is divided into three stages of 100 miles each. One man drives the first stage of 100 miles at the rate of 25 miles per hour. When he reaches the first division point, a second man starts and drives the second 100 miles at 50 miles an hour. The third man thereupon drives the remaining 100 miles at 20 miles per hour. What was the average rate of speed for the entire distance from Cleveland to Charleston? Check your answer by computing the total elapsed time for the three stages, and dividing 300 by this time to find the average number of miles per hour. Try averaging the three rates (25 miles per hour, 50 miles per hour, and 20 miles per hour) in different ways to see what average or what averages give proper answers. Try the arithmetic mean of the speeds, the geometric mean, the median, and the harmonic mean.

G10. In 1931 there were 13,449 marriages in Philadelphia. The numbers of the brides and of the grooms who were of various ages are given in the following table:[2]

Age Period	Number of Men	Number of Women
14–16	3	163
17–19	194	2252
20–22	3015	4559
23–25	3353	2726
26–28	2355	1335
29–31	1444	741
32–34	870	392
35–37	605	353
38–40	449	246
41–43	265	193
44–46	231	152
47–49	163	99
50–52	146	84
53–55	109	51
56–58	68	37
59–61	62	26
62–64	38	17
65–67	41	11
68–70	38	12

a. Compute the arithmetic average, the median, and the mode of the ages for men and for women. Compare the sexes in respect to age of marriage.

b. This table seems to show that more women get married between the ages of 17 and 19 than between the ages of 14 and 16. It also

[1] *World Almanac*, 1934, p. 787.

[2] From Bossard, *American Journal of Sociology*, Vol. 38, p. 539.

seems to show that more women get married between the ages of 68 and 70 than between the ages of 65 and 67. Do you feel as confident of the comparison in the one case as in the other? Why or why not? If you were describing the distribution of the number of men who are married at various ages would you say that there is a general tendency for the number of men to get smaller and smaller as the age increases beyond about 24 years, except for a slight increase between the ages of 65 and 67? Comment.

G11. The following table shows the age distribution of 1143 horses on 414 New Hampshire farms in 1930.[1] On these farms there were 27 horses whose ages were unknown. They are not listed in the table.

Age (yrs)	Number of Horses	Average Age
1– 4	12	2.7
5– 9	223	7.6
10–14	435	12.0
15–19	272	16.3
20–24	161	20.8
25–29	34	25.8
30 and over	6	31.7

a. Note that in this table we are given not only the number of horses in each age group but also the average age of the horses in each age group. Do these averages coincide with the class marks? If not, are they constantly above or constantly below the class marks? Does the relationship of the class mark to the class average vary regularly throughout the table?
b. Compute the average age of these horses from the first two columns of the table by the usual short method.
c. Compute the average age of these horses from the last two columns by weighting the group averages by the numbers of horses in the groups. Compare with the average obtained by the usual method. Which one is more nearly the real average age?
d. Does it seem reasonable to find more horses between the ages of 10 and 14 than between the ages of 5 and 9? Explain.
e. If you were to pick a horse at random from this group what age would it be most likely to have? Is this the arithmetic mean, the median, the mode, or some other average?

G12. In a storage-battery plant records were kept of the number of cases of lead poisoning among the employees and of the length of time that

[1] Data from M. G. Eastman, An Economic Study of Dairy Farming in Grafton County, New Hampshire, 1930, *N. H. Agricultural Experiment Station Bulletin* 260, p. 21.

each poisoned employee received compensation. Between June, 1928, and November, 1930, the distribution of cases was as follows:[1]

Number of Days Compensated	Number of Cases	Number of Days Compensated	Number of Cases
0– 9	1	80– 89	15
10–19	12	90– 99	13
20–29	13	100–109	7
30–39	24	110–119	4
40–49	26	120–129	5
50–59	39	130–139	2
60–69	28	140–149	1
70–79	23		

a. Compute the total number of days for which compensation was paid by this plant.
b. Having the total number of days lost, and taking from the table the number of men affected, compute the average number of days lost per man. Note that you have found the mean from this table by the long method.
c. Find the mean from this table by the short method.
d. What was the modal length of compensation period. Explain the answer as you would to one who had studied no statistics.
e. Find the ninth decile from this table. If you were a workman affected by lead poisoning under the conditions of this example, what would this figure (the ninth decile) mean to you?
f. Find the geometric mean of the figures in the table. Compare it with the arithmetic mean.

G13. On June 22, 1907, Ernest Thompson Seton counted the eggs in 77 pelicans' nests on an island in Smith Rapids above Fort Smith, Canada.[2] He found that the number of eggs varied from 1 to 4, with the following numbers of nests in each group:

Number of Eggs	Number of Nests
4	3
3	5
2	65
1	4

[1] Lead Poisoning in a Storage Battery Plant, U. S. Treasury Department, *Public Health Bulletin* 205, p. 27. Two cases are omitted from the table in which employees received compensation for over 150 days, since the exact length of the compensation period is not given in the source noted.

[2] Data from E. T. Seton, The Arctic Prairies, *Scribner's Magazine*, p. 114. The data given here differ very slightly from those given by Seton, since there seems to be a discrepancy in his figures. His data are not in tabular form, and seem to be conflicting.

a. Note that these are discrete data. Compute the median and the mode.

b. In this case what is true of the "class limits"? What of the "assumption" that the items are located at the class marks?

G14. The following table gives the ages at marriage of the 9907 urban women and of the 6720 rural women of native white parentage who were under 40 years of age at the time of their marriage, who were married between Apr. 16, 1900, and Apr. 15, 1905, and who were living with their husbands at the time of the 1910 census.[1]

Age at Marriage	Number of Cases		Age at Marriage	Number of Cases	
	Urban	Rural		Urban	Rural
14	33	42	28	329	122
15	127	114	29	261	108
16	274	309	30	182	95
17	524	511	31	135	60
18	788	745	32	105	56
19	873	838	33	67	45
20	1098	833	34	62	26
21	1001	663	35	51	26
22	912	520	36	22	15
23	831	459	37	28	8
24	727	388	38	19	10
25	602	315	39	16	12
26	454	236	Totals	9907	6720
27	386	164			

a. Compute the arithmetic mean, the median, and the mode for rural and for urban women. Discuss the results.

b. Compute the quartiles for the rural and for the urban group. Interpret the answers.

c. Write a short paper giving the results of parts *a* and *b* above. Put the paper in the form of a brief report to be made to a group of club women who have not studied statistics. Keep technical terminology out of the report, but incorporate the ideas inherent in the measures found above.

d. Recast the table by grouping the cases with 5-year class intervals. (For example, 10–14, 15–19, etc.) Compute the average age of marriage from the new table. Compare the results with those obtained in part *a* above.

e. Repeat the operations of part *d* above, except use 10-year class intervals.

f. Repeat the operations of part *d* above, using 5-year class intervals, but with the class limits at other points. (For example, 12–16, 17–21, 22–26, etc.)

g. To what extent is the value of the mean influenced by the grouping of the data?

G15. In Prob. F8 are 107 scores arranged in array.

a. Compute the arithmetic mean of the scores without grouping. Also the median and the first and third quartiles.

b. Group the data in a frequency table with a class interval of 250. Compute the arithmetic mean, the median, and the quartiles from the grouped data. Compare the answers with those of part *a* above.

c. Compute the nine deciles for the ungrouped data. Count the actual number of cases in each of the 10 groups and see if the nine deciles have actually so divided the data that the number of cases is the same in each group.

d. Is the median just halfway between the two quartiles? That is, if we average the values of the two quartiles, do we get the median? Comment.

e. Is the mean larger or smaller than the median? What is the significance of the fact that they are not identical in size?

G16. There follow a number of frequency tables, any one of which may be used as a basis for the computation of measures of central tendency, etc. The tables are lettered as subheadings under Prob. G16 and are hereafter referred to as Prob. G16*a*, Prob. G16*b*, etc.

a. Leprosy Admissions to the Hospital at Specific Ages in the Whole Population of Hawaii, 1890–1930. From *Public Health Bulletin* 212, p. 30, U. S. Treasury Department.

Age at Admission	Number of Cases Admitted		
	Male	Female	Total
0– 4	1	1	1
5– 9	96	62	158
10–14	332	227	559
15–19	381	291	672
20–24	324	175	499
25–29	240	130	370
30–34	181	99	280
35–39	180	73	253
40–44	122	66	188
45–49	112	51	163
50–54	98	66	164
55–59	54	20	74
60–64	68	28	96
65–69	34	2	36
70 and over	41	13	54
Totals	2264	1304	3567

b. Number of Passages through Canals at Sault Ste. Marie by Vessels of Various Classes; Season of 1930. From *Statistical Report of Lake Commerce, U. S. Army Engineering Corps*, 1932, p. 6.

Tons Register	No. of Passages
0–1000	2785
1000–2000	1684
2000–3000	1255
3000–4000	1557
4000–5000	1164
5000–6000	1803
6000–7000	1300
7000 and over	135

c. Annual Sales of Grocery Stores Not Handling Meats, United States, 1929. Data adapted from 15*th Census of the United States*, 1930, *Distribution, Vol. I, Retail Distribution, Pt. I.* Table 4A, p. 59 and Table 4B, p. 63.

Volume of Sales (thousands of dollars)	Number of Stores
$500 and over	22
300–500	58
200–300	139
100–200	1583
50–100	15196
30–50	23865
20–30	18290
10–20	32227
Under $10	100496
Total	191876

d. When temperature readings were taken 12 times a day in various rooms of textile plants, the number of times that each temperature was found was as in the following table. The data are from The Health of Workers in a Textile Plant, *Public Health Bulletin* 207, U. S. Treasury Department, p. 9.

Temperature, (°F.)	Weaving Rooms, July–Aug.	Weaving Rooms, Feb.–Mar.	Carding Rooms, July–Aug.	Carding Rooms, Feb.–Mar.
92	0	0	12	0
91	0	0	33	0
90	0	0	49	0
89	1	0	60	0
88	5	2	62	2
87	32	6	41	16
86	99	12	61	43
85	116	28	31	31
84	115	67	10	59
83	66	94	6	68
82	28	128	6	62
81	3	70	0	26
80	7	42	0	33
79	1	18	0	21
78	0	7	0	5
77	0	6	0	3
76	0	1	0	0
75	0	0	0	2

e. In Prob. G4 are given data showing the number of children missing various numbers of days of school on account of measles. Here are given similar data from the same source on the number of children missing various numbers of school days for colds and for earache, and also the number for all causes of sickness combined.

Duration of Sickness (number of school days missed)	Number of Cases Involving		
	Colds[1]	Earache	All Causes[2]
0– 4.9	9561	612	29,453
5– 9.9	1117	27	4,376
10–14.9	131	6	1,204
15–19.9	29	0	469
20–24.9	8	1	206
25–29.9	3	0	98
30–34.9	2	0	56
35–39.9	0	0	48
40–44.9	0	0	28

[1] Omitting one case of cold from which the pupil missed over 45 days, the exact length of absence not being given in the source quoted.

[2] Omitting 55 cases of illness from which the pupil missed over 45 days, the exact length of absence not being given in the source quoted.

f. The distribution of family incomes in the United States in 1930, as estimated by Daniel Starch and quoted in Neifeld, The Personal Finance Business, *Harper's Magazine*, p. 110, is given in the following table. The average income of the families in each group is also given.

Annual Income	Number of Families (000)	Average Income
$10,000 and over	420	$22,500
$5,000–$9,999	2,220	6,750
3,000– 4,999	6,000	3,600
2,000– 2,999	11,100	2,500
1,000– 1,999	8,700	1,600
0– 999	1,560	750

g. The number of asthma sufferers whose first attacks came at various ages is given in the following table. The data are from M. Walzer, "Asthma and Hay Fever in Theory and Practice," Part II, p. 372, Chas. C. Thomas, Springfield, Illinois.

Age at Onset	Number of Cases
0– 5	298
5–10	113
10–15	64
15–20	61
20–25	70
25–30	81
30–35	77
35–40	64
40–45	53
45–50	40
50–55	35
55–60	24
Over 60	20

h. The numbers of independent stores charging various prices for Grape Nuts, in Chicago in 1930, are given below. Group *A* consists of stores giving delivery service, etc. Group *B* consists of cash-and-carry stores. Data from E. Bjorklund and J. L. Palmer, "A Study of the Prices of Chain and Independent Grocers in Chicago," University of Chicago Press.

Price (cents)	Number of Stores in	
	Group *A*	Group *B*
10	1	2
11	1	0
12	0	2
13	4	2
14	2	2
15	14	20
16	16	34
17	13	19
18	34	31
19	3	3
20	7	3
21	1	0
22	0	0
23	0	0
24	0	0
25	1	0

i. The number of independent stores giving delivery service, etc., which were charging various prices for Karo sirup in Chicago in 1930 are given in the following table. The data come from the source quoted for Prob. G16 *h*.

Price (cents)	Number of Stores
10	16
11	12
12	55
13	5
14	2
15	22
16	0
17	0
18	1

j. The average weekly wages of 435 women employed in the Chicago stockyards in 1906 and of girls and women employed in Chicago corset factories during a representative week and during a busy week about 1908–1910, are given in the following table. The data are taken from Edith Abbott's article in the *Journal of Political Economy*, Vol. 21, pp. 153, 155.

Weekly Wage (dollars)	Stockyards	Chicago Corset Factories	
		Normal Week	Busy Week
Under $3.00	3	0	26
$ 3– 3.99	31	1	16
4– 4.99	94	11	36
5– 5.99	122	123	99
6– 6.99	80	29	49
7– 7.99	37	17	39
8– 8.99	33	16	30
9– 9.99	19	2	21
10–10.99	9	1	7
11–11.99	4	0	2
$12 and over	3	3	11

k. The Connecticut State Department of Labor made a study of working conditions in the fabricated metal industries of the state in the summer of 1934. They report the numbers of children of various ages who were doing homework in these industries as follows. Data from "Homework in the Fabricated Metal Industry in Connecticut," p. 16, mimeograph bulletin of Minimum Wage Division, Connecticut State Department of Labor Hartford, September, 1934.

Age (years)	Number of Children
2– 3.9	4
4– 5.9	10
6– 7.9	20
8– 9.9	34
10–11.9	44
12–13.9	66
14–15.9	64

l. The following table shows the numbers of dairy cows in Wisconsin with various annual butterfat productions, 1926–1928. Data from Hodgson, *Journal Dairy Science*, Vol. 15, p. 214.

Annual Butterfat Production (pounds)	Number of Cows
125–174.9	655
175–224.9	1639
225–274.9	2486
275–324.9	2408
325–374.9	1616
375–424.9	762
425–474.9	291
475–524.9	103
525–574.9	26
575–624.9	9
625–674.9	5

m. The annual butterfat production of 3625 Guernsey cows is given in the following table. The data are from Gifford and Elting, *Journal of Dairy Science*, Vol. 11, p. 5.

Annual Butterfat Production (pounds)	Number of Cows
375	247
425	458
475	555
525	572
575	496
625	424
675	338
725	233
775	135
825	83
875	45
925	22
975	10
1025	7

n. The weights in milligrams of 2538 seeds of the long leaf pine ($\frac{1}{2}$ lb. of seed) were as follows. Data from Deen, *Journal of Forestry*, Vol. 31, p. 435.

Weight (milligrams)	Number of Seeds
10– 25	16
25– 40	68
40– 55	204
55– 70	233
70– 85	240
85–100	655
100–115	803
115–130	294
130–145	21
145–160	4

o. The numbers of calves of various breeds which had various birth weights are listed in the following table. The data are taken from Fitch *et al.*, *Journal of Dairy Science*, Vol. 7, p. 230.

Birth Weight (pounds)	Number of Calves			
	Jerseys	Holsteins	Guernseys	Ayrshires
40– 49	12	0	6	0
50– 59	42	0	22	8
60– 69	31	0	45	31
70– 79	9	8	23	53
80– 89	0	15	6	13
90– 99	0	16	0	8
100–109	0	23	0	0
110–119	0	11	0	0

p. The following table shows the numbers of inventors who took out various numbers of patents in a 10-year period in the United States. The data are from Carr, *American Journal of Sociology*, Vol. 37, p. 577.

Number of Patents	Number of Men
1	480
2	147
3	83
4	46
5	30
6	12
7	13
8	15
9	11
10	9

q. The following table shows the distribution of I.Q.'s of girl delinquents committed to the Wisconsin Industrial School. Data from Lumpkin, *American Journal of Sociology*, Vol. 38, p. 238.

I.Q.	Number of Cases
46– 55	4
56– 65	35
66– 75	89
76– 85	69
86– 95	30
96–105	21
106–115	3
116–125	1

r. The table below gives for various rural social classes the ages at marriage of women of native white parentage who were married between Apr. 16, 1900, and Apr. 15, 1905, and who were living with their husbands at the time of the 1910 census. The data are from Notestein, *American Journal of Sociology*, Vol. 37, p. 43. Only those women are included who were below 40 at marriage.

Wife's Age at Marriage	Women Whose Husbands Are		
	Farm Owners	Farm Renters	Farm Laborers
14	9	17	16
15	27	44	43
16	85	122	102
17	165	220	126
18	257	336	152
19	306	390	142
20	313	382	138
21	308	256	99
22	261	205	54
23	216	180	63
24	203	139	46
25	161	124	30
26	126	81	29
27	95	51	18
28	78	33	11
29	56	35	17
30	59	32	4
31	35	16	9
32	37	12	7
33	25	18	2
34	19	5	2
35	22	2	2
36	12	3	0
37	6	2	0
38	7	1	2
39	10	1	1
Totals	2898	2707	1115

s. The following table shows the number of men and of women marrying at various ages in Payne County, Oklahoma, during the period 1895–1932. The data are from Duncan *et al.*, *American Journal of Sociology*, Vol. 39, p. 472.

Age (years)	Number of Cases	
	Men	Women
Under 14	0	1
14–16	2	777
17–19	581	4,223
20–22	3,409	2,464
23–25	2,358	1,157
26–28	1,401	559
29–31	726	277
32–34	457	195
35–37	315	185
38–40	273	165
41–43	175	88
44–46	130	74
47–49	135	78
50–52	119	54
53–55	88	49
56–58	75	33
59–61	62	30
62–64	41	16
65–67	38	18
68–70	28	5
71–73	16	7
74–76	14	7
77–79	12	2
80 and over	10	1
Totals.	10,465	10,465

t. Ernest Thompson Seton states that 28 litters of black bear cubs were divided as follows as to size of litter. (From E. T. Seton, "Lives of Game Animals," Vol. II, Part I, p. 177.)

Number Cubs in Litter	Number of Litters
1	3
2	12
3	11
4	2

u. The following table shows the number of marriages in which the man was various numbers of years older (+) or younger (−) than the bride. The data cover all Philadelphia marriages in 1931 except for one case where the man was 38 years older, one where the bride was 25 years older, and one where the bride was 29 years older. The data are from Bossard, *American Journal of Sociology*, Vol. 38, pp. 540–541. The fact that the ages were given by the applicants introduces an unknown error into the data.

Years Man Older	Number of Cases	Years Man Older	Number of Cases
33	2	7	537
32	3	6	781
31	1	5	999
30	8	4	1250
29	2	3	1624
28	4	2	1624
27	6	1	1691
26	4	0	1421
25	10	− 1	528
24	9	− 2	284
23	13	− 3	189
22	22	− 4	104
21	12	− 5	90
20	25	− 6	40
19	36	− 7	44
18	43	− 8	21
17	47	− 9	13
16	62	−10	9
15	87	−11	14
14	115	−12	6
13	116	−13	6
12	176	−14	3
11	225	−15	1
10	330	−16	3
9	338	−17	1
8	467		

v. The following table shows the numbers of hours flown by various numbers of airplane pilots in the United States in 1 month in 1931. Data from *U. S. Bureau of Labor Statistics, Bulletin* 575, Table 5, p. 6.

Number of Hours Flown	Number of Pilots
15– 24.9	4
25– 34.9	8
35– 44.9	17
45– 54.9	23
55– 64.9	26
65– 74.9	71
75– 84.9	108
85– 94.9	109
95–104.9	59
105–114.9	26
115–124.9	6
125–134.9	3

w. The following table shows the distribution of hourly wage rates among laborers in the bread departments and among all wage earners in the cake departments of United States bakeries in 1931. The data are from *U. S. Bureau of Labor Statistics, Bulletin* 580 and are based on materials in Table 5, p. 11, and Table 6, p. 12.

Hourly Wage Rate (cents)	Number People Receiving this Rate of Wages	
	Bread Department	Cake Department
0– 9.9	1	0
10– 19.9	58	182
20– 29.9	206	904
30– 39.9	442	639
40– 49.9	478	310
50– 59.9	294	326
60– 69.9	79	199
70– 79.9	20	137
80– 89.9	2	55
90– 99.9	0	11
100–109.9	0	22
110–119.9	0	4
120–129.9	0	1
130–139.9	0	1
140–149.9	0	0
150–159.9	0	1

x. The following table shows the numbers of filling station employees who were receiving various hourly wages in 1931. The data are from *U. S. Bureau Labor Statistics Bulletin* 578, p. 9.

Hourly Wage (cents)	Number of Men
0– 9.9	18
10– 19.9	196
20– 29.9	438
30– 39.9	844
40– 49.9	774
50– 59.9	440
60– 69.9	177
70– 79.9	52
80– 89.9	17
90– 99.9	3
100–109.9	1
Total.....	2960

y. The following table shows the numbers of White Leghorn hens which produced various numbers of eggs during a year. The data are from *Storrs Agricultural Experiment Station Bulletin* 147, p. 246.

Number of Eggs	Number of Hens
0– 29	30
30– 59	36
60– 89	125
90–119	327
120–149	686
150–179	925
180–209	697
210–239	271
240–269	32
270–299	2

z. The following table shows the numbers of ground-personnel employees of aviation companies in the United States who were receiving various weekly wages in 1931. The data are from the *U. S. Bureau of Labor Statistics Bulletin* 575, p. 33.

Weekly Earnings	Number of Cases
$ 0–$ 9.99	36
10– 19.99	387
20– 29.99	881
30– 39.99	1018
40– 49.99	490
50– 59.99	140
60– 69.99	35
70– 79.99	5
80– 89.99	6

G17. In Prob. G14 we computed the arithmetic mean and the median of the ages of urban and of rural women at marriage. Compute the mode from the median and the mean, and compare it with the value which one gets when he computes the mode directly from the tabulated data.

G18. Using the data of Prob. F9, compute the arithmetic mean of the gross margins. Also compute the median gross margin. Explain your answers.

G19. In the introductory part of Prob. F13*n* we are told that the 274 publishers published a total of 8107 books. We can compute directly that the average number of books per publisher was 29.6. Compute the average from the frequency table of that problem to see how closely the result coincides with the actual average.

G20. Use the data of Prob. F13*p*.

a. Compute the arithmetic average weight.
b. Apply the Charlier check to your computations.

G21. In Problem F19 you made an ogive. Find the median and the quartiles graphically from this ogive.

G22. When finding the average of several rates we may use either the arithmetic or the harmonic mean. The answers will differ, but either answer may be the "correct" one for our purposes. Suppose we have 20 or 30 figures, each of which is a "population per square mile." Under what circumstances would we take the arithmetic mean, and under what circumstances the harmonic mean, of these 20–30 numbers?

G23. Professor Selden D. Bacon is quoted in the Hartford *Times* of Oct. 19, 1942, as criticizing attempts to appraise the morale of the nation as a whole: "Statements dealing with the psychology of 132,000,000 people which strike an average are almost certain to be misleading. If one were to average all the shipyards in the country, one would find the geographical center of the shipyard industry somewhere in Iowa. Average statements about family income or happiness or morale are likely to be just as ridiculous. Averages can be used and they can be abused."

What would have to be true of the distribution of family incomes if a statement of average income were to be "just as ridiculous" as the quoted statement concerning the geographical center of the shipbuilding industry? In what sorts of distributions is the arithmetic mean especially likely to mislead? In such cases what could be used as an alternative?

SECTION H

MEASURES OF DISPERSION

H1. Find the range in the ages of the men listed in Prob. F6.

H2. What was the range in the grades received on the examination mentioned in Prob. F2?

H3. Find the semi-interquartile range of the data of Prob. F6. Explain your answer.

H4. Find the semi-interquartile range of the data of Prob. F2.

H5. Convert your answers to Probs. H3 and H4 to relative dispersion. Compare the two measures of relative dispersion. Explain.

H6. Find the average deviation in the size of antelope herds listed in Prob. F1.

H7. Find the standard deviation in the size of antelope herds listed in Prob. F1.

H8. In Prob. G10 are two frequency distributions, one of the ages of brides and one of the ages of grooms. Compute the semi-interquartile range of each distribution. Find the relative dispersion based on the semi-interquartile range. Why is it useful to state dispersion in relative terms even in a case like this one, in which both series are stated in the same unit (years)?

H9. Find the average deviation of each of the two frequency distributions in Prob. G10.

H10. Find the standard deviation by the long method of the distribution of men's ages in Prob. G10.

H11. Find the standard deviation of the distribution of men's and of women's ages from Prob. G10, using the short method.

H12. Compare the values of the semi-interquartile range found in Prob. H8, of the average deviation found in Prob. H9, and of the standard deviation found in Prob. H11. What relative sizes would these measures have in a normal distribution?

H13. Compute the coefficient of variation for each of the series of Prob. G10, using the standard deviations computed for Prob. H11. Explain the meaning of your coefficient. Use nontechnical terms.

H14. In Prob. G16*l* are given figures showing the butterfat production of dairy cows in Wisconsin. In Prob. G16*y* are given figures showing the annual egg production of White Leghorn hens. Were the hens more variable in production than the cows or less so? (Explain why it is necessary to use relative rather than absolute measures of dispersion. Use the coefficient of variation.)

H15. In Prob. F8 are given the scores made by 107 contestants in an archery contest. Compute the standard deviation of the scores. What

proportion of the scores would lie within one standard deviation of the mean in a normal distribution? Within two standard deviations? Within three standard deviations? Check the original data to see what proportion of the scores actually does lie within one, two, and three standard deviations.

H16. Compute the standard deviation of the wages paid to workers in the cake departments of United States bakeries (Prob. G16*w*). Compute the standard deviation of the wages paid to filling station employees in the United States (Prob. G16*x*). In which case was there the greater (relative) variability in wages? Explain your answers.

H17. The coefficient of variation in the weights of newborn rats is said to be:[1]

Males	13.6
Females	9.9

Explain the meaning of these figures.

H18. In 1930 the standard deviation of the gross incomes on 32 specialized dairy farms in Eastern Connecticut was $2429. Explain.

H19. The average gross income on the farms mentioned in Prob. H18 was $3778. Compute the coefficient of variation.[2]

H20. The yield per acre of corn was ascertained for each of 680 dairy farms in Kane and McHenry Counties in Illinois in 1912. It was found[3] that there was a coefficient of variation of 38.3 per cent in the yields. Explain.

H21. Since the range includes 100 per cent of the cases and the interquartile range includes 50 per cent of the items (twice the semi-interquartile range) why is it that the range is not twice the interquartile range?

H22. The standard deviation in the heights of the women students who entered Hollins College in 1920 was 2.35 in. The arithmetic mean of the heights was 63.24 in.[4]

a. What was the coefficient of variation in the heights? Explain the meaning both of the standard deviation itself and of the coefficient of variation.

b. If the heights of these women were normally distributed, what was the average deviation of the heights? The semi-interquartile range? The approximate range?

H23. Mr. *A* and Mr. *B* take different examinations in algebra. Mr. *A* gets a grade of 82. The average mark on the examination which he took was 96 and the standard deviation was 8. Mr. *B* got a mark of 48 on an examination on which the average mark was 90 and the standard deviation was 40. Which man did relatively better work?

H24. Hugh S. Rice has counted the numbers of asteroids, among those well-known by the Astronomisches Rechen-Institut at Dahlem, and has

[1] H. H. Donaldson, The Rat, *Memoirs of Wistar Institute of Anatomy & Biology*, No. 6, p. 225, Philadelphia.

[2] Data from *Storrs Agricultural Experiment Station Bulletin* 191, p. 12.

[3] *Farm Economics*, Cornell University, May 15, 1926, p. 456.

[4] *Journal of the American Statistical Association*, Vol. 24, p. 42.

found how they are distributed in different magnitude groups at the time of opposition.[1] He found 1380 asteroids distributed as follows:

Magnitude at Opposition	Number of Cases
7	2
8	2
9	11
10	42
11	141
12	214
13	416
14	404
15	121
16	15
17	10
18	2
Total	1380

a. Compute the deciles, and explain your results.
b. Compute the standard deviation, and use it as a basis for describing the dispersion of the figures in words which could be understood by one who had not studied statistics.
c. Compute the semi-interquartile range. In a normal distribution what is the relationship between the sizes of the semi-interquartile range and the standard deviation? Is this relationship exhibited in the present example, or approximately so?

H25. Using the data of Prob. F9 (or of Prob. G18) compute the standard deviation of the gross margins of the 180 department stores. Explain your answer in nontechnical terms.

H26. What is the coefficient of variation of the gross margins in the preceding problem? Explain when you would use this coefficient of variation in preference to the standard deviation computed earlier.

H27. Compute the standard deviation of the weights of Prob. F13*p*. Apply the Charlier check to your computations.

H28. In Prob. G16*w* are given distributions of wage rates of bakery workers. Computation discloses that the standard deviation of the wage rates of the 1580 people in the bread department is 12.5 cents and that the standard deviation of the wage rates of the 2792 people in the cake department is 19.0 cents. The average wage in the bread department is 41.75 cents, and in the cake department is 40.00 cents. The average wage for all 4372 people in both departments is 40.60 cents. Without going back to the original frequencies, find the standard deviation of the wages of all 4372 people when the two groups are combined.

[1] Data from Hugh S. Rice, Miniature Worlds, *The Sky*, Vol. I, No. X, August, 1937, p. 5. Published by the American Museum of Natural History, New York.

H29. I. S. Wise and R. Davis, Relation of Birth to Behavior, *American Journal of Orthopsychiatry*, Vol. 11, April, 1941, p. 326, divided 380 children born at Mt. Sinai Hospital in New York City into three classes based on their I.Q.'s. Those with I.Q.'s below 75 were rated as inferior, and made up 15 per cent of the total. There were 60 per cent with I.Q.'s between 75 and 110, and they were classed as "dull or average." The 25 per cent with I.Q.'s above 110 were called "superior." Assuming that the I.Q.'s were normally distributed, what was the arithmetic average and what was the standard deviation of the distribution of I.Q.'s?

H30. Suppose we agree to call "tall" all people of a height over 72 in.; "medium," those with heights from 64 to 72 in.; and "short," those with heights below 64 in. On this basis we find the 200 students in a given college[1] divided as follows:

Height	Per Cent
Tall	8
Medium	87
Short	5

If we assume that the heights are normally distributed, find the average height and the standard deviation of the heights. Compare your answers based on these three percentages with the actual values derived from the original data, which are:

Arithmetic mean	67.53 in.
Standard deviation	2.66 in.

[1] Based on data for heights of 200 students classified in E. W. Sinnott and L. C. Dunn, "Principles of Genetics," 1st ed., p. 261, McGraw-Hill Book Company, Inc., New York, 1925.

SECTION I

PROBABILITY AND THE NORMAL CURVE

I1. What are the chances that if we draw a card from a pack it will be a 10-spot? The 10 of spades? A spade?

I2. What are the chances of getting heads each time in three successive throws of a coin?

I3. Give an example of statistical (empirical) probability and one of a priori probability.

I4. What is the distinction between statistical and a priori probability?

I5. What do we mean when we say that two or more events are "independent"?

I6. What are the a priori chances that you will die on a Sunday? If the statistical probability turned out to be higher than this a priori probability how would you explain it?

I7. In the United States in 1930 11.9 per cent of the marriageable men (single men 15 years old and over) were widowers. At this time 29.6 per cent of the marriageable women were widows. What proportion of the marriages would be between widowers and widows if there were no selection; that is, if the proportion were purely a chance affair? If 8 per cent of all marriages were between widowers and widows what would you conclude?[1] (There were 1,126,856 marriages in the United States in 1930.)

I8. In 1930 8.9 per cent of the people of the United States were between the ages of 20 and 24 years. At this time 0.4 per cent of the people of the United States were in the United States Army.[2] Would it be proper to conclude that the chance that a person chosen at random from the population of the United States in 1930 was between the ages of 20 and 24 *and* in the United States Army was:

$$0.089 \times 0.004 = 0.000356?$$

Comment.

I9. In a given smallpox epidemic 11.6 per cent of the people who contracted the disease died. In this epidemic 79 per cent of the people who contracted the disease had been vaccinated. If vaccination and recovery are independent, what proportion of the people would be both vaccinated and recovered? Actually 75 per cent of the people were both vaccinated and recovered. What do you conclude?[3]

I10. In 1922–1923 a study was made of farms in the town of Lebanon, Connecticut. Of the farms studied, 62.2 per cent were located on a particular type of soil. Of the farms studied, 48.4 per cent were operated by native-born farmers. If there were no relationship between soil type and

[1] The data of this problem are from the 1933 *Statistical Abstract*, pp. 44–45.

[2] Data from 1933 *Statistical Abstract*, pp. 38, 144.

[3] Data from *Biometrika*, Vol. 1, pp. 376*ff*.

nativity what proportion of the farms should have been operated by native-born farmers and located on this type of soil? Actually 37.2 per cent of the farms were both operated by native-born farmers and located on this type of soil. What do you conclude? (A total of 159 farms were studied.)[1]

I11. What is the difference between the point binomial and the normal curve?

I12. Expand the binomial ($\frac{1}{2} + \frac{1}{2}$) to the third power. Compare the coefficients with the distribution which one would expect from tossing three coins.

I13. Using Pascal's triangle find the numbers of times that various numbers of heads would be expected to appear in 262,144 throws of 18 coins. Make a graph of the data.

I14. Make a graph of the data tabulated in Prob. F8*b*. Compare this graph with the one made in Prob. I13.

I15. Find from the table of areas under the normal curve (Table A1 of this manual) the percentage of the cases which would be supposed to fall between a point 0.5σ above the mean and a point 1.3σ below the mean. Count the number of cases in Prob. F8 which actually do fall within this range. What percentage are they of the total cases? (The standard deviation of these data was computed in Prob. H15. Use your data from that problem if they are available. In case you do not have the results of that problem you may use the approximate figures: $\sigma = 550$ and arithmetic mean $= 1520$. Your own figures will be more accurate if you have them.)

I16. What proportion of the data would be found between the mean and a point 0.7 standard deviations below the mean in a normal curve? What proportion between points at -0.7σ and -1.4σ? Compare the actual numbers of cases within these ranges from Prob. F8, using the figures from Prob. I15 if necessary.

I17. In 1920 there were 85 women in the entering class at Hollins College. The average and standard deviation of their heights are given in Prob. H22. How many of these students had heights between 61 and 67 in. if the heights were normally distributed?

I18. Assume that the incomes of the farms mentioned in Prob. H18 and H19 were normally distributed. How many of the farms had incomes between \$3000 and \$4000?

I19. Assume that we have a large number of students whose average weight is 145 lb. We know that the weights are normally distributed. We find that 38.3 per cent of the students have weights between 132 and 158 lb. What is the standard deviation of the weights?

I20. A group of specialized tobacco farms in the Connecticut Valley were found to have an average net income of \$905 with a standard deviation of \$1409 in the year 1926.[2] If the incomes were normally distributed, what proportion of the farms had net losses (negative net incomes)?

I21. Suppose that we have a city in which there are 5374 single men who are 15 years of age or older. (This is the group which we called "marriageable" in Prob. I7.) How many of these men would you expect to find were widowers? (Base your answer on the data of Prob. I7.) If this city is

[1] Data from *Storrs Agricultural Experiment Station Bulletin* 139, p. 89

[2] *Storrs Agricultural Experiment Station Bulletin* 165, pp. 123, 142.

typical of conditions in the country as a whole, what are the chances that there will be as few as 610 widowers? That there will be as many as 700 widowers?

122. In a typical United States city of 12,000 people how many people would you expect to find between the ages of 20 and 24? (See Prob. 18.) What would you conclude about the city if you found 1250 such people? What are the chances of finding as few as 1040? As few as 1010? As many as 1100? As many as 1125?

123. Suppose that the figures of Prob. 19 are typical for smallpox fatality rates. If 240 people contract the disease in a given epidemic, how many deaths would you expect? Would it be reasonable to expect as few as 10 deaths? What are the chances of getting as many as 40 deaths by pure chance from data similar to those of Prob. 19?

124. Suppose that in the past the teacher of your statistics class has instructed 175 students in the course which you are now taking. Of these, 13 have failed the course. If you and your class mates are similar to your teacher's past material, how many of you can be expected to pass and how many to fail? If we set three standard deviations as the limit of reasonable variation, what is the smallest number that can be expected to fail? The largest number? (If you are not studying this book in a class, solve the problem as it would work out for a class of 30 students.)

125. Suppose that a manufacturer has been inspecting a given product, and that in the past he has had to discard 4 per cent of the units inspected on account of flaws. A new lot of the product consisting of 726 units comes up for inspection.

a. How many pieces should he expect to reject?
b. What is the standard deviation in the number of rejections?
c. Suppose he has to reject 45 units. What does he conclude?
d. What are the chances that he will reject as few as 22 units?

126. The Windham Community Memorial Hospital, Inc. of Willimantic, Connecticut, issued a prospectus in June, 1934, in which it gives figures describing the operations of its first 11 months. The statement is made, "125 babies were born in our Maternity Department among such modern sanitary surroundings that not a single death occurred from childbirth."

In 1933 the New York Academy of Medicine reported[1] that 342,000 cases of childbirth had resulted in the deaths of the mother in 2041 cases.

If the New York figures can be looked upon as typical experience under all sorts of conditions, how many maternal deaths would you have expected in the Willimantic hospital? How likely is it that a record such as theirs would arise from pure chance rather than from "modern sanitary surroundings"?

127. In an epidemic of infantile paralysis which took place in the eastern part of the United States in the fall of 1931, we have records on 927 children who contracted the disease. Of these, 408 received no serum and 104 of the 408 became paralyzed, while the other 304 recovered without paralysis. If the serum had no effect, how many cases of paralysis would you have expected among the 519 children who were given serum? What is the smallest number of paralysis cases that might have arisen from chance

[1] Editorial in Hartford *Courant*, Nov. 27, 1933.

rather than from serum? (Assume that 3σ marks the limit of reasonable chance occurrence.) Actually 166 of the children receiving serum were paralyzed.[1] What do you conclude as to the efficacy of the serum? What other factors might influence the result besides the effect of the serum?

I28. In Sec. G of this manual (and especially in Prob. G16) there are a number of frequency tables containing distributions of actual data. How many of these tables appear to exhibit the characteristics of normal distributions? In what way do they seem to vary most often from normality? How common do you think that real "normality" is? Why do we call it "normal"?

I29. The data of Prob. F8 may be tabulated as follows:

Score	Frequency
2700–2999	3
2400–2699	2
2100–2399	8
1800–2099	20
1500–1799	25
1200–1499	20
900–1199	15
600– 899	9
300– 599	4
0– 299	1
Totals	107

Fit a normal curve to these data. Plot a graph of the original data from the table, and add the computed normal curve to the graph for purposes of comparison.

I30. The following table[2] shows the distribution of the weights of 998 Harvard students between the ages of 18 and 25. Two extremely heavy cases are omitted.

Weight (kilograms) (mid-point)	Number of Cases	Weight (kilograms) (mid-point)	Number of Cases
93	2	66	151
90	2	63	154
87	7	60	142
84	12	57	85
81	25	54	58
78	34	51	22
75	60	48	5
72	100	45	1
69	138		

[1] Data from Associated Press dispatches in the Hartford *Courant*, May 11, 1932.

[2] From W. E. Castle, "Genetics and Eugenics," p. 61, Harvard University Press.

a Fit a normal curve to these data.
b. Plot the actual data on graph paper and superimpose the normal curve on the same diagram.
c. Compute the mean, the standard deviation, and α_3 and α_4. Interpret the two latter figures.
d. Compute the mode from the formula

$$\alpha_3 = \frac{2(\bar{X} - \text{Mo.})}{\sigma}$$

Obviously this formula becomes:

$$\text{Mo.} = \bar{X} - \left(\frac{\sigma}{2}\right)(\alpha_3)$$

e. Compute the quartiles and a measure of skewness based on the quartiles.
f. Compute a measure of skewness based on the arithmetic mean and the median.

I31. Inspect the data given in Prob. G16*x*. Decide whether they exhibit positive or negative skewness. Then compute the skewness by the following methods:

a. Based on the quartiles.
b. Based on the arithmetic mean and the median.
c. Based on the third moment about the mean.

Note whether or not each measure vindicates your a priori evaluation of the direction of the skewness. Compare the three measures to see how well they agree in size.

I32. In Sec. G (and especially in Prob. G16) are given a number of frequency tables. Look them over, and note how many seem to be symmetrical, how many positively skewed, and how many negatively skewed. Look up other frequency distributions in economic, biometric, educational, genetic, and other literature. Note these same facts relative to symmetry or direction of skewness. Is skewness rare or common? Is positive skewness more or less common than negative skewness, or are they equally common?

I33. Measure the skewness of the data of Prob. G16*y* by the method of moments. Explain the meaning of your answer.

I34. Measure the kurtosis (sometimes called the "excess") of the data of Prob. I30. Explain your answer.

I35. Measure the kurtosis of the data of Prob. I29. Explain your answer. Are the data mesokurtic, leptokurtic, or platykurtic?

I36. Compute the first to the fourth moments inclusive of the data of Prob. I30. Apply Sheppard's corrections. Compute the alphas, the betas, and κ_2. In what ways is this distribution similar to and in what ways different from a normal distribution?

137. Suppose that 500 students are given an objective test. The marks turn out to follow exactly a normal distribution. The arithmetic mean of the marks is 74 and $\sigma = 10$. What are the values of Q, Q_1, Q_3, AD, D_3, P_{97}, Mo., Med., Sk., α_3, α_4, β_1, β_2, κ_2, and the second moment about the mean?

138. Using the data of Table A2 of this manual, draw a normal curve on graph paper. Erect perpendiculars under the curve at distances one standard deviation each side of the mean. Count the squares between the perpendiculars and those outside the perpendiculars (in both cases staying between the curve and the base line). What proportion of the area is between the perpendiculars?

139. In the month of November, 1934, there were 17 births in the City of Willimantic, Connecticut. Of these, 13 were boys.[1] If male and female births are equally probable, how many boys would you have expected? If male and female births are equally probable, how often would we get as many as 13 male births out of 17 by chance?

140. In 1931 there were 2,112,760 births in the birth registration area of the United States. Of these 1,084,404 were males.[2] If this ratio of male births to total births is normal, what number of male births would you have anticipated in Prob. 139? How often would you get as many as 13 male births out of 17 by chance?

141. Is it reasonable to assume in the light of the figures of Prob. 140 that male and female births are equally probable? That is, how likely is it that we would get the birth ratio of Prob. 140 by chance if male and female births were equally probable?

142. In a given group of 260 children[3] it was discovered that 99 suffered from headaches. Out of the 260 children, 44 were found to have poor vision.

a. If headaches and poor vision were unrelated how many of the children would you expect to find with both headaches and poor vision.

b. How many cases combining headache and poor vision could you get from these 260 children before you would feel safe in saying that the combination was not a chance affair?

c. In this actual group of children it was discovered that 24 children had both headaches and poor vision. What do you conclude?

d. Is this a case of a priori probability or of statistical probability?

143. The following table shows the numbers of hens of various breeds which laid various numbers of eggs within a year:[4]

[1] Data from news item in the Hartford *Courant*, Dec. 30, 1934.

[2] Data from *World Almanac*, 1934, p. 293.

[3] Data from *Child Development*, Vol. 5, June, 1934, p. 194.

[4] Data from *Storrs Agricultural Experiment Station Bulletin* 147, p. 246; *Bulletin* 118, pp. 98, 36; and *Bulletin* 122, pp. 226, 265.

Number of Eggs per Year	Number of Birds with Given Production				
	White Leghorns	Rhode Island Reds	Wyandottes	Barred Rocks	White Rocks
0– 14	18	8	4	2	1
15– 29	12	4	2	2	1
30– 44	14	7	6	7	3
45– 59	22	11	11	4	3
60– 74	42	21	9	16	5
75– 89	83	30	28	10	20
90–104	118	63	43	19	35
105–119	209	84	64	37	31
120–134	311	124	82	65	32
135–149	375	151	94	78	33
150–164	445	160	125	80	57
165–179	480	137	130	76	42
180–194	406	143	111	91	41
195–209	291	87	85	76	17
210–224	185	48	54	48	21
225–239	86	31	32	25	6
240–254	26	16	12	13	3
255–269	6	6	7	2	0
270–284	2	1	3	4	1
285–299	0	0	0	0	0
300–314	0	0	1	0	0
Totals.........	3131	1132	903	655	352

Compute the higher moments of these curves. Are the distributions "normal"? In each case, when you test for normality, state what test you applied, and what it showed. Are the deviations from normality similar for the different breeds of fowls?

144. In the Willimantic Duckpin Bowling Sweepstakes, held in Willimantic, Connecticut, on Feb. 16, 1935, there were 45 entrants. Each entrant bowled 15 strings. The scores of these 675 strings were as shown in the table on page 163.[1]

a. Make a frequency table showing the numbers of strings at various scores. Use a class interval of 10 with the multiples of 10 at the lower limits of the classes; that is, 110–119, 120–129, etc.

b. Plot the tabulated data in a frequency curve. Is the curve approximately normal? Can you detect any nonnormal features by optical inspection?

c. Fit a normal curve to the data.

(*Problem continued on page* 164)

[1] Data of this problem from the Hartford *Courant*, Feb. 17, 1935.

Entrant Number	Score on String Number														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	123	93	126	116	139	125	155	134	152	121	119	145	138	118	131
2	116	128	119	103	147	138	120	151	136	144	147	126	142	112	107
3	108	99	121	132	103	106	104	119	144	152	122	104	122	118	127
4	99	115	122	132	133	96	148	125	117	124	106	126	129	118	108
5	154	147	113	128	104	119	102	125	135	124	127	121	98	117	153
6	91	130	126	136	127	108	124	108	125	139	107	116	129	140	130
7	119	109	125	132	138	122	138	142	125	158	118	118	112	117	112
8	126	106	111	107	135	116	120	136	117	102	131	131	122	105	133
9	100	125	144	113	118	123	124	135	132	117	132	116	121	112	147
10	114	149	142	135	119	103	117	116	127	123	149	143	108	105	137
11	113	92	112	112	127	110	118	91	136	119	112	133	115	101	96
12	117	127	133	131	121	118	110	130	108	114	101	120	131	129	121
13	114	135	114	109	152	111	134	102	136	125	145	136	101	150	132
14	146	155	133	145	119	120	129	98	117	136	114	129	105	113	111
15	100	111	140	148	127	114	124	122	139	97	105	94	131	113	110
16	157	117	151	135	117	120	95	121	135	123	140	131	151	122	134
17	115	139	104	113	122	105	111	146	116	149	154	142	146	150	138
18	120	134	145	114	171	151	130	129	142	123	130	100	113	112	116
19	104	122	140	99	140	110	112	119	118	124	162	121	131	180	142
20	147	128	105	105	112	132	100	140	127	142	118	124	111	117	120
21	125	118	134	156	129	105	154	137	113	120	125	116	114	121	121
22	119	126	125	126	137	108	113	129	108	147	116	141	96	125	122
23	120	124	127	152	137	123	132	133	136	145	114	113	120	118	135
24	123	108	118	116	139	117	143	104	126	109	117	165	115	134	127
25	107	114	118	101	130	105	109	149	123	131	128	121	106	131	119
26	136	128	134	147	102	121	102	137	149	122	114	127	132	139	120
27	129	104	112	139	135	130	132	140	104	144	92	130	126	129	124
28	119	119	108	117	111	119	137	118	133	123	147	120	126	123	120
29	129	138	132	123	145	134	140	116	147	119	120	131	125	103	138
30	98	148	127	124	97	149	112	118	127	125	125	129	134	94	110
31	124	162	109	137	133	144	115	135	114	104	116	146	96	110	133
32	151	121	120	140	100	117	113	130	128	143	122	113	118	113	128
33	137	144	129	140	142	113	102	123	118	101	135	124	107	134	114
34	131	104	119	127	112	110	110	141	136	114	133	124	138	128	102
35	145	84	139	137	156	100	128	119	134	144	150	119	119	162	122
36	135	110	108	122	144	133	131	148	109	163	120	115	119	122	131
37	107	143	107	147	139	114	129	130	147	140	113	129	126	109	136
38	120	145	112	134	120	126	128	111	108	121	145	100	114	119	127
39	110	182	102	121	147	136	117	137	114	107	114	91	102	114	130
40	111	123	137	129	127	119	130	156	99	104	132	121	91	116	121
41	132	127	149	139	115	128	129	100	123	130	138	132	123	120	127
42	118	158	140	104	132	114	111	129	139	124	154	113	134	115	162
43	127	105	137	122	130	142	120	129	145	172	95	151	120	115	121
44	112	118	122	137	143	112	114	113	111	123	154	142	104	142	132
45	156	126	120	104	114	137	122	125	112	126	107	131	122	119	129

d. Compute the mean and the median. Compute the higher moments. What evidences of abnormality do you now discover?
e. Compute the mode from the higher moments. Compare it with the mode found by grouping.
f. Plot your tabulated data on probability paper. In what ways does the diagram differ from that of a normal distribution?

145. Bortkiewicz, one of the early writers on the Poisson distribution,[1] gives the following figures which have almost become classic as an illustration of this type of distribution. Data were collected over a period of 10 years showing the numbers of deaths from horse kicks in each of 20 army corps. For these 200 corps-years the distribution of deaths was as follows:

Number of Deaths	Number of Times
0	109
1	65
2	22
3	3
4	1
Total	200

Fit a Poisson distribution to these data, and work out your estimates of the numbers of times each number of deaths would be expected to occur. Note that the probability of occurrence is very small, as always in cases where the Poisson distribution is used.

146. When 584 children were examined in Puerto Rico it was found that some of them had malformation of bones or skeletal signs of rickets while some showed no skeletal signs of rickets whatever. Of those who showed the symptoms, some had but one skeletal sign of the trouble, some showed two separate skeletal signs, and some showed several distinct indications. The following table[2] shows the numbers of children showing each number of signs of rickets:

Number of Skeletal Signs of Rickets	Number of Children
0	238
1	208
2	97
3	30
4	9
5	0
6	2

[1] Bortkiewicz, "Das Gesetz der kleinen Zahlen."

[2] Data from Effect of Tropical Sunlight on the Development of Bones of Children in Puerto Rico, *U. S. Department of Labor, Children's Bureau of Publication* 217, App. B, p. 110.

Fit a Poisson distribution to these data. Make a graph showing both the actual distribution and the calculated one.

147. On Nov. 28, 1937, at Newark, New Jersey, 58 runners finished in the 41st annual national A.A.U. senior cross-country championship. The times of those finishing, as reported in *The New York Times* the following day, were as follows:

32:57, 33:22, 33:53, 34:15, 34:26, 34:33, 34:38, 34:41, 34:57,
35:05, 35:21, 35:26, 35:31, 35:37, 35:39, 35:41, 35:46, 35:50,
35:51, 35:52, 35:53, 35:55, 35:58, 36:10, 36:16, 36:20, 36:35,
36:36, 36:41, 36:55, 37:03, 37:10, 37:17, 37:30, 37:33, 37:37,
37:44, 37:45, 37:47, 38:00, 38:06, 38:11, 38:14, 38:17, 38:18,
38:21, 38:41, 38:47, 38:55, 39:05, 39:06, 39:41, 40:07, 40:13,
40:17, 40:50, 42:36, 46:04.

In each case the first figure represents minutes and the second figure represents seconds; that is, the winner finished in 32 minutes and 57 seconds.

Group these times in various ways in an effort to discover whether or not the distribution is normal. Do different groupings all show the same thing? Try making larger and larger class intervals, and note the effect. Try varying the class limits with the class interval constant.

148. Fit a normal curve to the data of Prob. F9. Compare the actual frequencies with the graduated frequencies. Do you think that one can safely conclude that gross margins in general are not normally distributed, or is the agreement close enough so that these 180 stores may well have been selected at random from a larger group in which the gross margins were really normally distributed?

149. Apply the chi-square test to the data of the preceding problem. Interpret your results.

150. Compute the skewness of the data mentioned in Prob. I48. (Use α_3 as your measure of skewness.)

151. Using Table A4 in this manual (or the more complete tables in *Annals of Mathematical Statistics*, Vol. 1, No. 2, May, 1930), fit a skewed curve with the appropriate degree of skewness to the data of Prob. 148. Compare the original data and the graduated data. Apply the chi-square test, and explain your answers.

152. On Mar. 5, 1938, the national open and amateur downhill championship ski race was held at Stowe, Vermont. The times of the 50 contestants who finished were as follows:[1]

2:35.0, 2:40.6, 2:49.4, 2:55.6, 2:56.0, 2:56.2, 2:58.6,
3:04.2, 3:06.0, 3:08.0, 3:08.2, 3:10.0, 3:11.0, 3:11.2,
3:12.8, 3:15.2, 3:15.2, 3:17.4, 3:20.2, 3:21.6, 3:22.4,
3:27.0, 3:30.4, 3:35.0, 3:35.6, 3:42.6, 3:45.8, 3:47.6,

3:56.4, 3:59.2, 4:06.0, 4:09.4, 4:13.0, 4:13.2, 4:13.8,
4:14.8, 4:22.0, 4:31.2, 4:34.8, 4:38.4, 4:46.0, 4:57.2,
4:58.6, 5:13.0, 5:14.6, 5:18.2, 5:30.4, 6:04.2, 7:03.2,
9:50.6.

[1] Taken from *The New York Times*, Mar. 6, 1938, p. S 5.

The times for the 1¼-mile course are given in minutes, seconds, and tenths of seconds. For example, the last man to finish took 9 minutes, 50.6 seconds.

a. Make a frequency table and plot a frequency curve. Note by inspection the amount and direction of the skewness.
b. Compute α_3. Interpret your result.
c. Compute the arithmetic mean of the times. Using this mean and the value of α_3 computed in the preceding section of this problem, compute the value of the mode. Explain your answer.

153. The following systems of distributing letter grades in a five-letter grading system are said to be "based on the normal curve."[1] If we assume that the normal curve extends for a distance of three standard deviations on each side of the mean, what is your estimate of the number of each grade which should be given? Which of the illustrative distributions is most nearly normal?

PERCENTAGE OF MARKS IN EACH OF THE FIVE CLASSES UNDER VARIOUS SYSTEMS, ALL NORMAL CURVES

Mark	System 1	System 2	System 3	System 4
A	3	5	7	15
B	23	24	24	22
C	48	42	38	26
D	23	24	24	22
F	3	5	7	15

It will be noted that each of these four distributions is symmetrical. What in addition to symmetry is necessary in order that a distribution be normal?

154. The following data, originally collected by the Dutch botanist De Vries,[2] show the numbers of bulbous buttercups with various numbers of petals.

Number of Petals	Number of Plants
5	133
6	55
7	23
8	7
9	2
10	2

Fit a Poisson distribution to these data. Note that the classes must be stated as 0, 1, 2, 3, etc., instead of in their present form; that is, the value of x will, in this case, be the number of petals in excess of 5.

[1] Russell, "Classroom Tests," p. 308.

[2] This distribution, often quoted, is here taken from Arne Fisher, "Frequency Curves," p. 101, The Macmillan Company, New York, 1922.

155. Apply the chi-square test to determine how well the Poisson curve describes the data.

156. The accompanying table shows the distribution of lengths of the organ of Corti in 68 human ears.[1]

Length (millimeters)	Number of Cases
35–35.9	3
34–34.9	6
33–33.9	8
32–32.9	14
31–31.9	13
30–30.9	10
29–29.9	4
28–28.9	4
27–27.9	3
26–26.9	1
25–25.9	2
Total	68

Inspection shows that the data are negatively skewed.

a. Compute the skewness, using α_3 as the measure.
b. Regroup the data into groups with double the class interval. Recompute the value of α_3 with the regrouped data.
c. Fit a skewed curve to the data, using the table of skewed ordinates on pages 22–25.

157. Lay off a sheet of probability paper on an ordinary piece of unlined 8½- by 11-in. paper. Have the probability scale cover the range from 5 to 95 per cent

158. Put the data of Prob. F13*p* in proper form for plotting on probability paper.

159. Plot the data of the preceding problem on probability paper. Do the weights seem to be approximately normal in their distribution?

160. Compute the first to the fourth moments of the data of Prob. F13*p*. Apply the Charlier check to your computations.

161. Thirty dice are thrown many times, and each time we count the number of 6-spots.

a. What should be the arithmetic average number of sixes?
b. What should be the standard deviation in the number of sixes?
c. Should our distribution of numbers of sixes be symmetrical or skewed? What would be the value of α_3; the value of α_4?

[1] Data from Mary Hardy, Length of the Organ of Corti in Man, *American Journal of Anatomy*, Vol. 62, Jan. 15, 1938, p. 305.

162. On a priori grounds, what would you imagine is the value of α_4 for the distribution of Prob. F13*s*? Compute it.

163. Fit a Poisson curve to the data of Prob. F13*b*.

164. Fit a Poisson curve to the data of Prob. F13*o*.

165. Theoretically, what would have to be true of the data of Prob. F13*v* if a Poisson curve were to fit the data? Fit a Poisson curve to the data.

166. Fit a Poisson curve to the data of Prob. F13*w*.

SECTION J

MEASURES OF RELIABILITY

J1. Explain the relationship of a sample to the universe.

J2. What do we mean by a "random" sample?

J3. What is a standard error?

J4. Suppose that we have a given universe made up of 5000 ears of corn. Suppose we take every possible lot of 100 ears (the total possible number of combinations of 5000 things taken 100 at a time. How many such combinations are there?), and for each lot compute the average weight. This will give us many average weights, each from a lot of 100 ears. We next compute the standard deviation of these averages. In what way is the standard deviation of these averages different from the standard error of the mean of a single sample of 100 ears?

J5. Suppose that we have found the standard error of the mean in a sample of 100 cases. We wish to reduce the standard error to one-third its present size. If we continue to draw cases of a variability equal to that in our sample of 100, to what size must we increase our sample?

J6. What is the difference between a probable error and a standard error? How would you compute a probable error from a standard error? How would you compute a standard error from a probable error?

J7. Find the mean age at which men and at which women are admitted to the hospital in Hawaiian leprosy cases (Prob. G16*a*). What is the standard error of the mean in each case? Explain the meaning of these standard errors as you would to one who knew no statistics.

J8. Compute the standard deviations of the two frequency series of Prob. G16*a*. Compute the standard error of the standard deviations, and explain them.

J9. From Problem G16*d* compute the average temperature in carding rooms in July and August and the standard error of this mean. Do the same for February and March.

J10. From Prob. G16*d* compute the standard deviation of the temperatures in carding rooms in July and August and the standard error of the standard deviation. Do the same for February and March.

J11. Using the data of Prob. G16*x*, compute the standard error of the average wage, the standard error of the standard deviation of the wages, and the standard error of the coefficient of variation of the wages. Explain each answer.

J12. Compute the standard error of the median age of horses in Prob. G11. Explain.

J13. Find the median butterfat production from the data of Prob. G16*l*. Compute the standard error and the probable error of the median. Explain your answers.

J14. In Prob. I30 we computed the values of α_3 and α_4 from the weights of Harvard students. Compute the standard errors and the probable errors of these values. Explain them. Is it reasonable to assume that the original data were really symmetrical, or is there evidence which leads you to believe that the sample was drawn from a universe which was characterized by skewness?

J15. In Prob. I33 we found the skewness of a distribution by the method of moments. In order to do this, it was necessary to compute the value of α_3. Compute the standard error of this value. Is there evidence that the universe from which the sample was drawn was itself skewed?

J16. From Prob. I24 compute the percentage of students who have failed the course. Compute and explain the standard error of this relative frequency.

J17. In Prob. I26 we are given figures from which we can discover that 0.6 per cent of 342,000 cases of childbirth resulted in maternal deaths. Compute the standard error of this relative frequency. Explain your answer.

J18. Of the 408 children who received no serum in Prob. I27, 25.4 per cent became paralyzed. Compute and explain the standard error of this relative frequency. Also the probable error.

J19. Compute the semi-interquartile range of the data of Prob. G12. Find and explain the standard error and the probable error of the semi-interquartile range.

J20. In Prob. H9 we computed two average deviations. Find the standard errors of these figures, and explain them.

J21. In Prob. G4*e* we found the first and the third quartiles of a given distribution. Find the standard error of the first quartile, and explain it. Also the probable error.

J22. In Prob. I36 we computed the value of β_2 for a given distribution. Compute the standard error of this value, and explain it.

J23. Is there a significant difference between the sexes in the average age of admission to the hospital for leprosy? Use data of Prob. G16*a*, and compute the difference between the means and its standard error.

J24. Is there a significant difference between the temperature of weaving rooms and of carding rooms in February and March. Use data from Prob. G16*d*, computing the difference between the means and its standard error.

J25. Is the temperature of weaving rooms in February and March significantly more variable or significantly less so than it is in July and August. Use the data of Prob. G16*d*. Compute the difference between the standard deviations and the standard error of the difference.

J26*a*. Are the wives of farm owners significantly older at marriage on the average than are the wives of farm renters? (Use the data of Prob. G16*r*.) Is there a significant difference in the variability of ages at marriage?

J26*b*. Is there a significant difference in the skewness of the two distributions of Prob. J26*a*?

J27. Is there a significant difference in the average age of marriage for men and for women in Philadelphia? Use the data of Prob. G10. Do the Philadelphia women differ significantly from the urban women of Prob. G14, either as to average age of marriage or as to the variability of age at marriage?

J28. One hundred mica washers were measured and found to have the following thicknesses:

Thickness	Frequency
0.0080–0.00849	0
0.0085–0.00899	2
0.0090–0.00949	3
0.0095–0.00999	4
0.0100–0.01049	19
0.0105–0.01099	40
0.0110–0.01149	25
0.0115–0.01199	7

One hundred brass washers were found to have the following thicknesses:

Thickness	Frequency
0.0180–0.01849	1
0.0185–0.01899	7
0.0190–0.01949	11
0.0195–0.01999	21
0.0200–0.02049	27
0.0205–0.02099	16
0.0210–0.02149	10
0.0215–0.02199	5
0.0220–0.02249	2

In each case the thicknesses are given in inches.[1] If a brass washer and a mica washer are to be used in the same assembly what will be the average combined thickness? What will be the standard error of the sum?

J29. Suppose we study a sample of 100 manufacturers of a particular product. We find that the average advertising expense is $1.50 per unit sold and that the standard deviation is 22 cents. We find that the other selling expenses amount to 68 cents per unit sold and that the standard deviation is 7 cents. What are the average total selling expense and its standard error?

J30. Explain the modifications which must be made in the formulas for standard errors when we are dealing with small samples. Show by an illustration that the modification is minor when the number of cases is large.

J31. From Table A1 of this manual compute the chances against a random difference more than 1.7 times its standard error.

J32. Suppose that we count the number of young in 121 litters of mice. We find the average size of litter to be 4.59 and the standard deviation[2] to be 1.75. What is the standard error of the mean? The standard error of the standard deviation? The standard error of the median? The stand-

[1] Data from W. A. Shewhart, "Economic Control of Quality of Manufactured Product," p. 432, D. Van Nostrand Company, Inc., New York.

[2] These are actual values quoted from C. H. Forsyth, "Mathematical Analysis of Statistics," pp. 121, 124, John Wiley & Sons, Inc., New York.

ard errors of α_3 and α_4? The standard error of the semi-interquartile range? The standard error of the average deviation? The standard error of the third quartile? The standard error of the coefficient of variation? Note that all these can be computed from the three figures given. In this distribution 14.9 per cent of the litters contained two mice or less. What is the standard error of this relative frequency?

J33. From a knowledge of Prob. J32 what factors are of major importance in determining the reliability of statistical results? In general, how can we increase the reliability of our results?

J34. In a study of the weights of first-born and later-born children the following facts were discovered.[1]

First-born children:

Males:	
Average weight	3375.45 grams
Standard deviation of weights	440.40 grams
Number of cases studied	395
Females:	
Average weight	3222.49 grams
Standard deviation of weights	517.00 grams
Number of cases studied	417

Later-born children:

Males:	
Average weight	3539.81 grams
σ of weights	517.00 grams
Number of cases	423
Females:	
Average weight	3429.81 grams
σ of weights	470.40 grams
Number of cases	418

a. Is there a significant difference in the weight of male first-born and male later-born babies?
b. Are later-born female babies significantly different in weight from first-born females?
c. Are first-born females significantly more variable in weight than first-born males?
d. The standard deviations are identical for first-born females and later-born males. How about the coefficients of variation? Is there a significant difference in the latter?
e. Make other comparisons both between means and between standard deviations.

J35. The following figures[2] show the average lengths of middle fingers (in millimeters) of newborn babies at different seasons of the year. Each

[1] Data from *Human Biology*, Vol. 6, December, 1934, p. 631.
[2] Data from *Human Biology*, Vol. 6, December, 1934, p. 618.

mean is accompanied by its probable error and the number of cases measured is also stated in each case.

Months	Average Length	Probable Error of Average	Number of Cases
February–April	32.98	0.082	194
May–July	32.11	0.092	238
August–October	31.20	0.063	572
November–January	34.38	0.088	213

a. Compute the standard errors of the means.

b. Do the differences in the mean finger lengths arise from chance, or is it reasonably certain that there is actually seasonal variation in finger length?

J36. What was the standard deviation of finger lengths in each of the 4 periods for which figures are given in Prob. J35?

J37. In Prob. I43 are given several frequency distributions showing for various breeds of fowls the numbers laying various numbers of eggs per annum. Are there any significant differences in the means? Is the production of any one breed significantly more variable than that of another? Is there any significant skewness in any case? Is there any significant kurtosis in any case? Is the skewness in any one case significantly different from that in any other? Are there significant differences between medians?

J38. In an experiment designed to investigate factors affecting the hatchability of eggs, the experimenters started with 545 eggs from 47 White Leghorn pullets.[1] Of these but 520 turned out to be fertile. Of these 520 eggs, 300 hatched, and in the other 220 cases the embryo died in the shell. The mean weight of the 300 eggs which hatched was 55.05 ± .21 grams. The standard deviation was 5.54 ± .15 grams. For the 220 eggs in which the embryo died in the shell the average weight was 55.94 ± .24 grams, and the standard deviation was 5.30 ± .17 grams. Were the latter eggs significantly heavier? Significantly more uniform?

J39. The Bureau of Fisheries carried on tests at Beaufort, North Carolina, to determine whether or not barnacles preferred some colors to others.[2] The purpose, of course, was to find if paints of certain colors were to be preferred in painting the hulls of vessels. On 19 different days the experimenters submerged colored unglazed tiles, and at the end of the day a count was made of the number of barnacles on each tile. The results were as follows:

[1] Data from *Storrs Agricultural Experiment Station Bulletin* 109, p. 110.

[2] Data from *Bulletin of the Bureau of Fisheries*, Vol. 43, Part II, 1927, p. 243.

Date	Color of Tile				
	White	Buff	Dark Green	Red	Black
June 18	237	570	700	2840	720
June 19	520	864	572	1900	884
June 20	680	1120	1032	1800	2108
June 21	712	1587	2068	1344	2760
June 25	128	251	224	240	199
June 26	429	442	809	584	961
June 28	1164	1757	1836	1491	1800
June 29	166	760	1116	916	944
June 30	230	1040	1381	1168	1900
July 1	750	1100	1500	1800	2200
July 2	1400	2000	2700	3000	2500
July 3	1800	1600	2500	1800	500
July 6	1500	1900	1800	1500	1700
July 7	280	430	500	564	379
July 8	263	1130	1200	1500	458
July 9	400	713	470	591	520
July 16	122	212	262	252	300
July 17	40	224	300	448	316
July 18	53	173	180	176	196
Totals	9864	17873	21150	23914	21345
Average	519	941	1113	1259	1123

When the number of barnacles was small an exact count of their number was made. When the number was large a careful count was made on a random unit of area on the tile and from this count the number of barnacles on the tile was estimated to the nearest 100; that is, the entry for the red tile on June 19 signifies that the number of barnacles is estimated at between 1850 and 1950. Keep in mind this limitation on accuracy as you work this problem.

Compute the standard deviations for the various tiles. Are there significant differences in the mean number of barnacles? In the cases where the means are not significantly different, what could you do if you wanted to make certain whether the differences observed were really chance affairs or were due to barnaclean color preferences?

J40. In 5223 autopsies[1] there were 922 bodies which showed the presence of tuberculosis, 927 which showed the presence of syphilis, and 285 which showed the presence of cancer. Of these there were 73 bodies in which both tuberculosis and syphilis appeared, 9 in which were found both cancer and tuberculosis, and 8 in which were found both cancer and syphilis. In no case were all three found together. Comment on the difference

[1] Data from *Human Biology*, Vol. 4, No. 2, May 1932, p. 226.

between the actual and expected numbers of cases in which the diseases appear together.

J41. Chicago, with a male population of 987,759 people 15 years old and over and a female population in the same age group of 954,806, had an annual average of 292.1 male suicides and an annual average of 118.5 female suicides in the three years 1919–1921 inclusive.[1] Was the suicide rate for males significantly above that for females?

During the same period the average annual number of suicides by white people in California was 779, and the average for other people was 56. The total white population was 3,264,711; and the number of other people was 162,150. Was the difference in suicide rates significant?[2]

	N	$\bar{X}$	σ
Recreation 31 hours or more	48	41.2	15.0
Recreation 7 hours or less	68	34.4	12.7
No extracurricular activities	91	39.1	12.4
Two such activities	56	35.8	13.6
Four such activities	10	45.5	12.6
Best subject mathematics	72	43.3	11.8
Worst subject mathematics	113	31.4	10.9
Best subject social science	44	33.5	11.0
Worst subject social science	36	36.1	14.0
Best subject English	86	27.3	10.9
Worst subject English	55	45.9	14.6
Under 15.5 years old	24	43.6	15.7
19.5 years old and over	10	27.9	12.6
Parents unskilled	8	43.1	9.6
Parents professional	10	39.6	19.9
Only child	16	30.9	14.1
Five siblings	36	32.9	9.7
Height 5 ft. 9 in. and over	91	35.7	12.8
Height 5 ft. 3 in. and under	51	35.0	12.3
Believes in God	258	35.9	13.1
Unbeliever	70	34.9	12.5

[1] Cavan, "Suicide," p. 319, University of Chicago Press, Chicago.

[2] *Ibid.*, p. 37.

In these two cases are the results any more significant because they are based on a 3-year average than they would be if they had been based on 1 year alone? How is the difference shown in your method of handling the case?

J42. A study of the class which entered City College, New York, in February, 1929, showed that various things were associated with college grades. Some of the data of that study[1] are abstracted, in the table on page 175, showing the numbers of students studied, the average grade, and the standard deviation of the grades. The letter grades were evaluated on a system in which $A = 80$, $B = 50$, $C = 30$, $D = 20$, $E = 10$, and $F = 0$.

Within the various groups above, which differences in average grades are significant? Are the children of professional people significantly more variable in performance than are the children of unskilled people?

J43. Johansson found[2] that 51.52 per cent $\pm$ 0.142 per cent of cattle were born males. What is the meaning of the latter figure? How many cases must he have studied? Is it reasonable to assume that, with a large number of cases, we would get a 50–50 distribution of sexes?

J44. In a given breeding experiment, it was expected that ducks would be hatched in the ratio of 1 duck with a white bib to each 3 ducks without bibs. Of 86 ducks hatched there were actually 21 with white bibs and 65 bibless.[3] Are the actual proportions significantly different from those expected?

J45. Studies have been made of the relative proportions in which boys and girls appear in births which result from matings of people of different nationalities. We give here the figures as to sex ratios (number of male births per 100 female births) which have been discovered in the progeny of various crosses.[4] M means male, and F means female, so that Irish $M \times$ Swedish F would mean the cross of an Irish man with a Swedish wife. The probable errors of the ratios are also given.

Cross	Sex Ratio	Probable Errors
Italian $M \times$ Argentine F	105.72	0.95
Italian $M \times$ Italian F	100.77	0.41
Argentine $M \times$ Argentine F	103.26	0.69

What is the standard error of each percentage? Is the pure mating significantly less productive of males than either or both of the mixed matings? A cross of Spanish men with Argentinian wives gives the figures

[1] Harris, Relation to College Grades of Some Factors Other Than Intelligence, *Archives of Psychology*, No. 131 *passim*.

[2] *Zeitschrift fur Züchtung, Reihe B, Tierzüchtung und Züchtungsbiologie*, Vol. 24, No. 2, 1932, pp. 183–268.

[3] *Poultry Science*, Vol. 12, pp. 233–241.

[4] Raymond Pearl, "Studies in Human Biology," p. 108, Williams & Wilkins Company, Baltimore, 1924.

106.69 and 1.53. Is it significantly different from the pure Argentinian mating?

J46. In attempting to determine what forces sway men in their political decisions one student has investigated the relationship between rainfall in the East North Central States and the results of presidential elections.[1] If we classify years into those with rainfall above average and those with rainfall below average; and if we divide elections into those in which the party in power was continued and those in which it was changed; and if we determine the number of elections in each group, we find this:

	Party Changed	Party Continued
Rainfall above normal	1	11
Rainfall below normal	11	2

Compute the significance in the difference of the proportions. Explain your answer.

J47. A manufacturer of house dresses in Buffalo, New York, was sending out advertising matter by mail. He sent out exactly the same material to each of two groups of 1000 people, but in the one case he enclosed a white return envelope and in the other case the return envelope was blue. Nine per cent of the 1000 white envelopes were returned, and 12 per cent of the 1000 blue envelopes.[2] Was the difference significant?

J48. Three hundred and forty-eight high-school boys and 382 high-school girls were asked to indicate the things on which they disagreed with their parents. The following table gives the sources of disagreement with the percentage of boys and of girls who checked the item stated.[3]

Source of Disagreement	Percentage of	
	Boys	Girls
Use of automobile	35.6	29.6
Choice of friends	25.0	27.0
Spending money	37.4	28.8
Going out nights	45.1	47.6
Style of dress	15.8	24.6
Home duties	19.0	26.4
Church attendance	19.0	18.6

[1] Odegard, "American Public Mind," pp. 148–149.

[2] Starch, "Advertising Principles," p. 344, McGraw-Hill Book Company, Inc., New York.

[3] Lynd, "Middletown," selected from p. 522, Harcourt, Brace & Company, New York, 1929.

Which of the sources of disagreement show significant differences as between the sexes?

J49. Forty teachers graded an arithmetic paper. The average grade was 57.25 with a standard deviation of 13.7. A careful set of rules for grading the paper was then drawn up, and 48 teachers graded the paper with the aid of the rules. The average grade was now 65.21 and the standard deviation became 6.20. The same paper was scored in both cases.[1] Did the use of scoring rules in this case make a significant difference in the average grade? Was the variability in grading reduced significantly?

J50. The National Research Council published a bulletin some years ago in which they made the statement that, "Tidal observations at Fort Hamilton extending over a period of 35 years indicate no appreciable change in sea-level at that point during the period of observations." This statement was described as "erroneous" in a criticism which appeared immediately thereafter.[2] The critics argued that there had been appreciable change, and in support of their contention said, "As a matter of fact, the probable change in sea-level at Fort Hamilton between 1893 and 1927 is at the average rate of a rise of one foot in 214 years (by the least square method 0.0047 feet a year ± 0.06). Though the probable error of this result is great, it is more likely to be at the rate of 0.6 feet per century (.006 feet per year) as suggested by J. R. Freeman than to be with 'no appreciable change.'" In the light of your own knowledge of the meaning of probable errors, which side do you take in the controversy? Why?

J51. In Prob. I50 you computed a coefficient of skewness. Compute the standard error of this coefficient, and determine whether or not it is safe to conclude that the skewness was significant. Explain what we mean when we say that skewness is or is not "significant" in a particular case.

J52. The formula for the standard error of the coefficient of variation is complex, but, in cases where the size of the coefficient of variation is small, it may be simplified considerably (see Text, page 250). Justify the use of the simpler formula for small coefficients of variation.

J53. Studies of the metabolism rates of a group of boys and of a group of girls, both groups made up of children between the ages of 6 and 19 years, showed the following:[3]

	Number	σ	Coefficient of Variation (per cent)
Boys	307	3.14	6.5
Girls	281	2.98	6.8

[1] Ruch, "Objective or New Type Examinations," derived from data on p. 104, Scott, Foresman & Company, Chicago, 1929.

[2] Sea-level Change Near New York, *Science*, Vol. 71, No. 1838, Mar. 21, 1930, p. 319.

[3] Data from Joseph Berkson and Walter M. Boothby Studies of the Energy of Metabolism of Normal Individuals, *American Journal of Physiology*, Vol. 121, March, 1938, p. 682.

a. Was the average metabolism rate significantly higher for boys than for girls? (Note that the averages can be computed from the relationship of the coefficients of variation and the standard deviations. The latter are in calories per square meter of body surface per hour.)

b. Was the dispersion significantly greater for the boys than for the girls?

c. Was there a significant difference in the coefficients of variation?

J54. Twenty-two Moline elm trees were transplanted in full leaf in July, 1934, when the temperature was between 90 and 95°F. Half were sprayed with wax to impede transpiration, and the other half were untreated. Of the waxed trees, 10 lived and 1 died. Of the untreated trees, 1 lived and 10 died.[1] Have enough cases been studied here to convince you that the treatment was successful, or do you think it fair to suppose that when the experiment is tried again under similar conditions there will be no difference between treated and untreated trees? Justify your conclusions.

J55. W. W. Dalquist measured the hind feet of a number of snowshoe hares[2] and found the following:

Average length	117 mm.
Range of lengths	105–135 mm.
Average deviation	6.0 mm.
Coefficient of variation	6.0%

How many feet were measured to get these data? What was the standard deviation? Dalquist says that "the curves formed by the measurements were normal."

J56. Ellsworth Huntington[3] states that, of 2650 eminent persons, 382 were born in February and 149 in June. Are the differences between the actual percentages born in these months and the expected chance percentages significant?

J57. The birth months of 4960 children with I.Q.'s above 115 and of 3614 children with I.Q.'s below 84 follow:[4]

[1] Data from L. and J. Bush-Brown, "America's Garden Book," p. 227, Charles Scribner's Sons, New York, 1939.

[2] W. W. Dalquist, Geographic Variation in Northwestern Snowshoe Hares, *Journal of Mammalogy*, Vol. 23, May, 1942, p. 168.

[3] Ellsworth Huntington, "Season of Birth," p. 4, John Wiley & Sons, Inc., New York, 1938.

[4] R. Pintner and G. Forlano, Season of Birth and Intelligence, *Journal of Genetic Psychology*, Vol. 61, September, 1942, pp. 82–83.

Month of Birth	Number of Children with I.Q.'s of	
	115-199	45-84
January	401	349
February	359	332
March	416	332
April	438	280
May	413	297
June	418	274
July	458	309
August	422	315
September	458	301
October	404	303
November	396	256
December	377	266
Totals	4960	3614

Are the differences between the months significant?

J58. From 1858 to 1934 there were 320 births on Pitcairn Island. Of these, nine were twin births, or there was one twin birth for every 35.6 births. The usual ratio, according to H. L. Shapiro,[1] is one in 80. Is the difference between the actual and the expected proportion significant?

J59. When 10 Lincoln pennies were tossed 400 times, the following numbers of heads appeared the following numbers of times:[2]

Number of heads	10	9	8	7	6	5	4	3	2	1	0
Number of times	1	5	16	58	87	93	88	36	13	3	0

With 4000 opportunities, there were 2065 heads. Does this depart from 50-50 chances enough for us to assume that there was bias present?

J60. Using the expansion of $(\frac{1}{2} + \frac{1}{2})^{10}$, compute the numbers of times that various numbers of heads would have been expected to occur in the preceding problem. Compare the actual and theoretical frequencies. Apply the chi-square test. Interpret your results.

J61. Twenty-nine hills of unmulched Jersey short-stem sweet potatoes gave the following yields in grams: 493, 480, 179, 1277, 468, 298, 862, 782, 1580, 276, 832, 248, 994, 160, 247, 336, 446, 373, 657, 372, 460, 0, 1227, 153, 850, 260, 202, 368, and 85. Thirty-four hills of the same variety were

[1] H. L. Shapiro, "The Heritage of the Bounty," p. 247, Simon and Schuster, Inc., New York, 1936.

[2] Adapted from data of W. A. Bousfield and J. L. Barnes, Empirical Probability in a Free Two-choice Situation, *Journal of Psychology*, Vol. 12, 1941, p. 50.

mulched and gave the following yields in grams: 1093, 1361, 2158, 2615, 1945, 1453, 1567, 432, 2228, 433, 2063, 782, 1303, 2337, 2176, 2188, 1091, 236, 1552, 355, 547, 1561, 1721, 1496, 1867, 895, 321, 1673, 1434, 2037, 561, 1325, 1013, and 1044. The data are from *U. S. Department of Agriculture Technical Bulletin* 75, p. 6. Was there a significant difference between the yields of the mulched and the unmulched hills?

J62. The statement is made in *Time* of Sept. 3, 1934, that since 1900 there have been 348 cases of bubonic plague in California, with 239 deaths. For the rest of the United States there were 100 cases with 51 deaths. Was the proportion of deaths in California significantly higher than that in the rest of the country? How likely would it be that such a difference in proportions would arise by pure chance?

J63. Apply the chi-square test, and interpret your results, with the data of

a. Prob. I63.
b. Prob. I64.
c. Prob. I65.
d. Prob. I66.
e. Prob. I56*c*.
f. Prob. I48.
g. Prob. I46.
h. Prob. I45.
i. Prob. I44*c*.
j. Prob. I30*a*.

SECTION K

HISTORICAL DATA

K1. Correct the data of Prob. E15 for calendar variation. Rank the months in order of importance before and after the corrections are made. Is there any change in the order?

K2. Suppose that in the year in which you are working on this problem a corporation has the following output:

Month	Units of Output
January	27,416
February	26,291
March	27,800
April	30,117
May	32,149
June	32,006
July	31,119
August	29,974
September	26,792
October	25,777
November	25,998
December	26,876

a. Correct for calendar variation by the usual methods.

b. Correct for calendar variation allowing for the fact that this corporation is closed on all legal holidays. Use the legal holidays for your own state and for the current year.

K3. Look up and report on:

a. The history of the calendar.

b. Proposed changes in the calendar.

K4. What are the principal types of movements in historical data? Describe each carefully.

K5. The number of fire insurance companies which were members of the National Board of Fire Underwriters for the years 1918–1930, inclusive, is given in the table at the top of page 183.[1]

Plot these data on cross-section paper. Study the graph carefully, and decide where you could locate a straight line that would describe the data reasonably well. With a straightedge (preferably a transparent celluloid ruler) draw the straight line on the diagram.

K6. Select a point near each end of the straight line drawn in Prob. K5. By the method of selected points, calculate the equation of the line.

[1] Data from *Statistical Abstract*, 1933, p. 265.

Year	Number of Companies
1918	154
1919	159
1920	169
1921	175
1922	174
1923	184
1924	197
1925	210
1926	210
1927	213
1928	234
1929	246
1930	239

K7. Fit a line to the data of Prob. K6 by the method of least squares. Draw the new line on the diagram. Compare it with the one fitted by eye. Compare the equation of the least squares line with the equation of the line which was fitted by eye.

K8. Explain the meaning of each part of the equation which was found in Prob. K7.

K9. From the equation found in Prob. K7 estimate the number of companies in 1931. The actual number of companies in 1931 was 221. Explain the discrepancy. Estimate the number of fire insurance companies in 1890, using the equation computed in Prob. K7. Explain.

K10. Is the line found in Prob. K7 a better fit or a worse fit than that found in Prob. K5? Explain what you mean by "best fit." How do we measure "goodness of fit"?

K11. Compute a 5-year moving average of the values given in the table of Prob. K5. Plot the moving average on the diagram of the data. Note that the breaks in the curve tend to disappear. What are the values of the moving average for the years 1918, 1919, 1929, and 1930? Comment.

K12. The following table shows the expenses of the Western Union Telegraph Company each fifth year from 1870 to 1910. The data are in millions of dollars.[1]

Year	Expenses
1870	4.9
1875	6.3
1880	6.9
1885	12.0
1890	15.1
1895	16.1
1900	18.6
1905	21.8
1910	26.6

[1] From *Statistical Abstract*, 1933, p. 319.

Plot these data. Draw a freehand trend and compute its equation by the method of selected points. Fit a straight line by the method of least squares. Compare the two lines. In what units is the value of b given in the trend equation?

K13. Rewrite the trend equation of Prob. K12 so that it is stated in units of single years, and so that the origin is in 1900.

K14. The following table[1] shows the numbers of men on strike each year in coal mines. The figures are in hundreds of thousands.

Year	Number of Men	Year	Number of Men
1908	1.45	1920	2.82
1909	0.25	1921	1.51
1910	2.18	1922	6.03
1911	0.41	1923	1.97
1912	3.11	1924	1.29
1913	1.35	1925	1.86
1914	1.62	1926	1.75
1915	0.67	1927	1.93
1916	1.71	1928	0.87
1917	1.60	1929	0.58
1918	0.79	1930	0.41
1919	4.53	1931	1.11

Plot the data on graph paper. Compute a three-year moving average, and plot it on the same chart. Compute a 5-year and a 9-year moving average. Add them to the chart. What changes take place in the character of the moving average as the period is lengthened? What disadvantage becomes more and more apparent as the period is lengthened?

K15. The following table[2] shows the production (in thousands of short tons) of refined primary lead and secondary lead in the United States from 1890 to 1918.

Year	Production	Year	Production	Year	Production
1890	158	1900	368	1910	470
1891	198	1901	371	1911	487
1892	208	1902	368	1912	481
1893	224	1903	369	1913	462
1894	214	1904	393	1914	542
1895	236	1905	388	1915	550
1896	257	1906	405	1916	571
1897	282	1907	413	1917	611
1898	302	1908	397	1918	640
1899	298	1909	447		

[1] From *Statistical Abstract*, 1933, p. 677.

[2] From *Statistical Abstract*, 1933, p. 669.

Plot these data on graph paper. Compute the straight-line trend by least squares, and add the line to the chart. Explain the meaning of the various numbers in the trend equation.

K16. Shift the origin of the trend equation of Prob. K15 to the year 1900, and rewrite the equation.

K17. Plot the data of Prob. K15. Stretch a thread over the diagram and find what seems to be the best straight-line trend by eye. Draw this trend with a ruler. Select two points on the trend and compute the equation of the trend. Compare the results with those obtained in Prob. K15.

K18. The following table[1] shows the monthly mean temperature in New York City for the years 1908–1932, inclusive. All figures are in degrees Fahrenheit.

Year	Month											
	J	F	M	A	M	J	J	A	S	O	N	D
1908	32.0	28.1	41.4	50.6	61.3	71.6	76.8	72.5	67.8	59.6	44.7	35.2
1909	33.2	37.3	38.3	49.5	60.4	70.5	73.4	71.6	65.6	53.2	47.7	31.4
1910	32.4	31.4	44.7	54.0	60.2	68.0	77.8	72.2	68.4	58.2	41.6	28.0
1911	34.8	31.4	37.6	48.2	63.6	68.3	76.0	71.8	66.6	55.6	41.4	39.2
1912	23.5	28.4	36.8	49.0	60.7	68.4	74.0	70.7	65.9	58.5	46.6	38.5
1913	40.0	30.9	44.0	51.0	60.2	69.2	75.0	72.7	64.6	58.2	46.9	38.8
1914	31.4	25.3	35.8	46.6	63.6	67.6	71.1	73.7	66.2	59.0	44.0	31.5
1915	34.1	35.2	36.4	53.4	57.7	66.6	72.5	70.4	69.0	56.7	45.4	33.5
1916	35.4	27.7	32.2	47.1	59.8	64.2	73.8	73.6	66.0	57.2	44.8	33.8
1917	32.4	27.8	38.7	47.2	53.2	68.3	74.1	74.6	63.0	52.0	41.2	25.0
1918	21.6	29.6	41.2	49.8	64.0	66.4	72.7	74.8	62.8	58.6	45.7	39.0
1919	35.2	34.7	42.0	48.8	61.0	69.7	74.0	70.2	66.5	58.4	44.4	30.0
1920	24.1	29.1	40.6	47.7	57.8	67.6	72.5	72.8	67.4	60.4	44.2	37.8
1921	33.6	34.8	48.3	55.0	60.4	70.3	76.2	70.9	71.0	56.2	45.0	33.8
1922	29.0	34.5	41.3	51.0	63.6	70.6	72.7	71.4	67.4	56.6	45.4	34.3
1923	31.0	27.1	37.0	49.4	59.3	72.0	72.3	71.3	67.0	55.6	45.4	42.0
1924	32.5	29.2	38.7	48.1	56.2	66.8	72.6	72.9	63.0	56.8	44.1	34.0
1925	29.1	38.5	43.6	51.2	58.2	73.2	72.5	71.9	67.5	49.9	43.7	34.5
1926	31.8	29.4	35.1	46.0	58.5	64.8	73.2	72.8	64.8	54.2	44.0	29.2
1927	30.2	36.6	42.6	47.7	58.0	65.8	73.0	67.5	66.8	58.7	48.6	36.6
1928	33.8	32.9	38.3	47.5	58.4	66.6	75.0	74.4	63.8	57.8	46.6	39.4
1929	32.1	33.4	44.8	50.6	60.6	70.0	74.1	71.2	68.8	54.0	46.0	36.1
1930	33.6	37.0	39.8	47.6	62.6	71.8	75.0	72.8	71.2	55.0	45.8	35.0
1931	33.2	34.1	40.5	50.4	60.4	69.4	76.5	74.4	71.2	60.4	51.5	40.6
1932	42.8	36.0	37.0	48.4	60.9	69.0	73.9	74.8	67.7	56.6	43.7	39.0

Plot these data on graph paper. Find the average January temperature, the average February temperature, etc., for the period. Plot the average temperature cycle for New York City on a separate chart.

K19. From the data of Prob. K18 compute an index of seasonal variation of temperatures. "Correct" the data of the table in Prob. K18 by dividing the temperature of each month by the index of seasonal variation. Plot the "corrected" temperatures on a new chart. Note that, while there are still fluctuations in the data, they are not seasonal in character. These new data are temperatures "with seasonal eliminated."

K20. Compute the seasonal index of the temperatures tabulated in Prob. K18 by carrying out the following steps:

a. Compute a 12-month moving total of the data.
b. Compute a two-month moving total of the moving totals just computed. Center at the seventh month.
c. Divide each original temperature by the corresponding moving total. State the result in the form of a percentage.
d. Sort out these percentages by months and find the median January percentage, the median February percentage, etc.
e. Express each of these monthly medians as a percentage of the average of the 12 monthly medians. This is the index of seasonal variation.

Compare the results obtained here with those obtained in Prob. K19. Note that the more complicated but more exact method makes little difference unless there is a secular trend or some cycle other than the 12-month cycle.

K21. Using the index of seasonal variation computed in Prob. K20, eliminate the seasonal in the data of Prob. K18. This is done by dividing each monthly temperature by the appropriate monthly index. Plot the corrected data, and note that the seasonal has disappeared.

K22. The following table[1] shows the number of bunches of bananas (in thousands) imported into the United States each month during the years 1919–1928.

Year	Month											
	J	F	M	A	M	J	J	A	S	O	N	D
1919	2.3	2.2	2.8	2.8	4.5	4.6	4.2	2.8	3.2	2.9	2.5	2.2
1920	2.0	1.8	3.2	3.9	3.7	4.4	4.6	4.8	2.9	2.7	3.0	2.4
1921	1.9	2.2	3.2	4.2	3.8	5.2	4.7	4.6	4.0	3.6	3.4	2.7
1922	2.6	2.5	3.9	3.9	4.9	5.4	4.8	4.6	2.7	4.3	2.8	2.7
1923	2.6	2.4	3.4	4.4	4.9	4.9	4.7	3.9	3.4	3.4	3.1	2.9
1924	2.8	2.2	3.4	4.8	5.4	5.0	5.2	5.2	4.1	3.7	2.9	2.7
1925	3.1	2.0	4.0	4.0	6.2	7.4	6.2	5.8	5.5	4.4	3.6	3.2
1926	3.6	3.2	4.4	6.3	5.4	6.9	5.4	5.0	3.4	5.0	4.0	3.6
1927	3.1	3.5	5.3	5.6	6.2	6.9	6.0	6.3	4.9	4.7	4.4	3.9
1928	3.5	4.1	5.6	6.0	7.2	7.3	6.6	6.6	4.7	4.8	4.3	3.5

[1] From *U. S. Department of Agriculture Yearbook*, 1928, p. 1032, Table 501.

Plot these data on graph paper. Note that there is a secular movement as well as a seasonal movement. Compute the index of seasonal variation by the method of Prob. K20, and eliminate the seasonal by the method used in Prob. K21. Make a chart of the import data with seasonal eliminated.

K23. In Prob. K22 we computed the banana imports with the seasonal eliminated. The data show a secular trend. Eliminate the secular trend, and state the results as percentage deviations from the trend. Note that there are still fluctuations in the data, but that the fluctuations show no seasonal and no secular regularity. What do we call the fluctuations which remain?

K24. Compute a 12-month moving average of the data which are tabulated in Prob. K22. Center the moving average at the seventh month, and plot it on a chart of the original data.

K25. Compute the deviations from the moving average of Prob. K24 and the percentage deviations from the moving average.

K26. The following table[1] shows the numbers of active cotton spindles in New England for each year from 1915 to 1932. The figures are in millions.

Year	Spindles	Year	Spindles
1915	17.1	1924	17.1
1916	17.5	1925	16.0
1917	17.8	1926	15.5
1918	18.0	1927	15.0
1919	18.1	1928	13.8
1920	18.3	1929	12.5
1921	18.4	1930	11.4
1922	17.9	1931	9.7
1923	18.1	1932	8.6

Plot these data on graph paper. Draw freehand a curvilinear trend through the data. Select three points on the freehand trend and compute the equation of a second-degree parabola which passes through these three points. From the parabolic equation compute the estimated number of spindles for each year, and compare with the actual number.

K27. Fit a second-degree parabola to the data of Prob. K26 by the method of least squares. Add the parabolic curve to the diagram.

K28. The table[2] on page 188 shows the numbers of passenger automobiles registered each year from 1906 to 1918. The figures are in hundreds of thousands.

Plot these data on a chart. Draw a freehand curvilinear trend line. Select three points on this trend line, and compute the equation of the second-degree parabola which passes through these three points.

K29. Fit a second-degree parabola by the method of least squares to the data which are tabulated in Prob. K28. From the equation compute the

[1] From *Statistical Abstract*, 1933, p. 741.

[2] Data from "Automobile Facts and Figures," 1934 ed., p. 10, National Automobile Chamber of Commerce.

Year	Number of Automobiles
1906	1.1
1907	1.4
1908	1.9
1909	3.1
1910	4.6
1911	6.2
1912	9.0
1913	11.9
1914	16.3
1915	23.1
1916	33.0
1917	46.6
1918	56.2

estimated number of registrations for each year. Show the parabola on the chart of the original figures.

K30. Take the logarithms of the numbers of automobile registrations which are tabulated in Prob. K28. Make a chart showing the logarithm of registration for each year. Compare it with the chart of the original data. Fit a straight-line trend by methods of least squares to the logarithms. The trend equation will, as usual, be in the form:

$$Y = a + bX$$

But in this case the Y's are the logarithms of the registrations; that is, if the registration in any year is R, then $Y = \log R$. We could, then, state our trend equation in the form

$$\log R = a + bX$$

Work out this logarithmic trend equation, and add the trend line to the original chart.

K31. The table[1] at the top of page 189 shows the prices in cents per bushel of various grain crops. The prices quoted are average prices received by farmers on Jan. 15 of the year stated.

For each crop compute the price relatives for each year using the 1926 price as a base.

K32. Compute the simple aggregative index number of the grain prices listed in Prob. K31. Use a 1926 base.

K33. Compute the index number of grain prices from the price relatives of Prob. K31. Use the simple average of price relatives. Compute another index number using the median price relative.

K34. Compute the geometric mean of the price relatives for each year using the data of Prob. K31. Compare the results with those obtained by other methods in Probs. K32 and K33.

[1] Data from *World Almanac*, 1934, p. 358.

Year	Wheat	Corn	Oats	Barley	Rye
1920	234	144	80	134	153
1921	149	65	44	61	128
1922	95	45	32	44	70
1923	105	70	42	57	72
1924	97	74	43	56	64
1925	162	112	54	82	126
1926	158	70	40	60	88
1927	122	64	43	58	84
1928	115	75	49	74	88
1929	98	80	44	56	88
1930	108	77	43	54	86
1931	59	62	31	37	37
1932	44	34	23	36	37
1933	33	19	13	18	23

K35. The total United States production of the various grain crops listed in Prob. K31 was, in 1930, as follows:[1]

Crop	Production (millions of bushels)
Wheat	857
Corn	2060
Oats	1276
Barley	304
Rye	45

Use these figures for weights, and compute the weighted aggregative index number of grain prices with a 1926 base; the original prices appear in Prob. K31.

K36. Using the grain prices of Prob. K31 and the weights of Prob. K35, compute the weighted average of the price relatives. Compare this index number with the unweighted average of relatives. Compare it also with the weighted aggregative index number.

K37. Using the price relatives of Prob. K31 and the weights of Prob. K35, compute an index number of grain prices from the weighted geometric average of the relatives. Compare it with the other index numbers based on the same data.

K38. The weighted average farm price of apples in the United States in cents per bushel is given for the years 1913–1931 in the table[2] at the top of page 190. The table also gives for each year the index number of wholesale prices of all commodities, with 1926 as the base period.[3] Correct ("deflate") the apple prices with the index number, and explain the meaning of the corrected prices.

[1] Data from *World Almanac*, 1934, p. 343.

[2] Data from *Statistical Abstract*, 1933, p. 627.

[3] From *Statistical Abstract*, 1933, p. 280.

Year	Price	Index Number
1913	92	70
1914	62	68
1915	70	70
1916	89	86
1917	115	118
1918	138	131
1919	187	139
1920	135	154
1921	195	98
1922	109	97
1923	117	101
1924	122	98
1925	127	104
1926	88	100
1927	142	95
1928	110	97
1929	141	95
1930	103	86
1931	67	73

K39. Convert the index numbers of Prob. K38 to a 1913 base.

K40. What are the theoretical advantages and the practical disadvantages of the geometric mean as a basis for index numbers?

K41. What periods have been most commonly used as base periods for index numbers in the United States? Why was each period originally chosen, and why was it abandoned?

K42. What considerations enter into the decision as to the length of the base period to be chosen?

K43. Suppose that we have a factory which produces the following numbers of units of goods in each year (the years being numbered from the beginning of the firm's business).

Year	Output
1	1
2	3
3	2
4	4
5	3
6	5
7	4
8	6
9	5

Plot these data on cross-section paper. Draw a freehand trend. Measure the deviations from the trend. Square the deviations and compute the sum of the squared deviations. Try this with the line fitted by least

squares. See if you can fit any other straight line which will give you as small a sum of the squared deviations as you get around the line fitted by least squares. Explain what is meant by "least squares."

K44. Find out what you can about Fisher's "ideal" formula for index numbers. (See Irving Fisher, "The Making of Index Numbers," or extracts in other texts.) Can you apply the ideal formula to the data of Prob. K31 with the weights of Prob. K35?

K45. What difficulties are presented by the moving average when the original data are concave upward; that is, when they rise at a more and more rapid rate? When the original data are concave downward; that is, when there is a decrease at a more and more rapid rate or an increase at a less and less rapid rate?

K46. In cases similar to those mentioned in Prob. K45, what would be the advantages of a moving geometric average or a moving harmonic average? How would you go about computing them?

K47. The following table gives the index numbers of the wholesale prices of "all commodities" with 1910–1914 = 100 for the United States starting in 1798. Data are from *Farm Economics*, published by Cornell University.[1]

Year	0	1	2	3	4	5	6	7	8	9
1790									122	126
1800	129	142	117	118	126	141	134	130	115	130
1810	131	126	131	162	182	170	151	151	147	125
1820	106	102	106	103	98	103	99	98	97	96
1830	91	94	95	95	90	100	114	115	110	112
1840	95	92	82	75	77	83	83	90	82	82
1850	84	83	88	97	108	110	105	111	93	95
1860	93	89	104	133	193	185	175	162	158	151
1870	135	130	136	133	126	118	110	106	91	90
1880	100	103	108	101	93	85	82	85	86	81
1890	82	82	76	78	70	71	68	68	71	77
1900	82	81	86	87	87	88	90	95	92	99
1910	103	95	101	102	99	101	125	172	191	202
1920	226	143	141	147	143	151	146	139	141	139
1930	126	107	95	96	109	117	118	126	115	113
1940	115	127	144							

Test the data to see if you can discover regular cycles. Compare the index numbers with those given in Prob. K38, and explain the differences. In Prob. K12 are given the expenditures of the Western Union Telegraph Company for a number of years. Correct (deflate) these expenses with the above index numbers, and explain the meaning of the corrected figures.

[1] Pages 1586–1587, 1634, and scattering thereafter. To convert the U.S. Bureau of Labor Statistics 1926 index to the 1910–1914 base, multiply it by 1.460707.

K48. The following table gives the annual mean of Wolf's sun-spot numbers for the years 1750–1932. These numbers are roughly proportional to the spotted area of the sun, and a number of 100 corresponds to about $1/500$ of the sun's disk covered with spots.

Year	0	1	2	3	4	5	6	7	8	9
1750	83	48	48	31	12	10	10	32	48	54
1760	63	86	61	45	36	21	11	38	70	106
1770	101	82	66	35	31	7	20	92	154	126
1780	85	68	38	23	10	24	83	132	131	118
1790	90	67	60	47	41	21	16	6	4	7
1800	14	34	45	43	48	42	28	10	8	2
1810	0	1	5	12	14	35	46	41	30	24
1820	16	7	4	2	8	17	36	50	62	67
1830	71	48	28	8	13	57	122	138	103	86
1840	63	37	24	11	15	40	62	98	124	96
1850	66	64	54	39	21	7	4	23	55	94
1860	96	77	59	44	47	30	16	7	37	74
1870	139	111	102	66	45	17	11	12	3	6
1880	32	54	60	64	64	52	25	13	7	6
1890	7	36	73	85	78	64	42	26	27	12
1900	10	3	5	24	42	63	54	62	48	44
1910	19	6	4	1	10	47	57	104	81	64
1920	38	26	14	6	17	44	64	69	78	65
1930	36	21	11							

a. Find the length of the sun-spot period.
b. Plot these data on a diagram.
c. Smooth the data with a moving average.
d. Smooth with a progressive mean, using a 5-year period.

K49. Fit a semilogarithmic trend to the following data:

Year	1932	1933	1934	1935	1936	1937	1938	1939	1940	1941	1942
Output	124	140	158	179	202	228	258	291	330	372	421

K50. Fit a reciprocal trend to the following data:

Month	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.
Deaths	1070	684	501	396	327	279	243	215	193	175	160

K51. Correct the data of Prob. J57 for calendar variation.

K52. The following table[1] shows the number of lynx furs brought to the northern department of the Hudson Bay Company each year from 1821–1891. Figures are in thousands.

Year	0	1	2	3	4	5	6	7	8	9
1820	....	4.8	3.1	3.1	2.7	2.6	5.1	7.5	16.6	24.1
1830	22.8	9.4	4.6	3.1	3.9	6.3	18.9	34.7	53.7	42.3
1840	15.5	3.9	2.3	2.9	5.9	14.1	21.8	35.9	28.7	7.3
1850	2.2	1.4	1.2	1.5	5.1	13.1	16.5	25.4	14.0	8.5
1860	3.4	1.7	1.5	2.9	9.5	27.1	57.6	49.5	25.6	8.6
1870	4.3	1.7	2.5	5.9	10.1	25.7	28.8	20.7	10.6	9.9
1880	4.5	3.2	4.9	15.7	42.1	63.7	62.7	32.0	15.1	6.7
1890	3.2	3.9								

a. Plot the data on a graph.
b. Determine the period of the cycle.
c. Smooth the data with a moving average of appropriate length.

K53. The following table shows the weight, in pounds, of children of various ages. The ages are given in months.

Age (months)	Weight (pounds)	Age (months)	Weight (pounds)
27	27.8	99	52.2
33	29.0	105	54.3
39	30.6	111	56.1
45	32.6	117	58.1
51	34.8	123	62.4
57	36.5	129	65.2
63	37.8	135	66.2
69	39.4	141	69.2
75	41.5	147	74.2
81	43.5	153	79.2
87	47.7	159	82.4
93	49.5	165	87.3

These data[2] indicate that the weight increases with age, and that on the average the increases become somewhat greater toward the end of the period. Find a trend curve from which you can estimate the weight at any age.

[1] From Charles Elton and Mary Nicholson, The Ten-year Cycle in Numbers of Lynx in Canada, *Journal of Animal Ecology*, Vol. 11, November, 1942, pp. 228–229.

[2] A. P. Weinbach, The Human Growth Curve, *Growth*, Vol. 5, September, 1941, p. 239.

SECTION L

SIMPLE LINEAR CORRELATION

L1. What do we mean when we say that two things are "related"? To what extent do we imply cause and effect? What do we mean when we say that one thing is "caused by" another?

L2. The table[1] on pages 195 and 196 gives latex yields for each of 95 Hevea rubber trees during 9 of the 10 months of one growing season. The trees were located in Haiti. Yields are given to the nearest cubic centimeter. Trees were not tapped during July.

Classify the trees according to their November yields into the following groups:

100–149.9 cc.
150–199.9 cc.
200–249.9 cc.
etc.

Find the average December yield for the trees in each of these classes. Comment on the results obtained. Do you take it that the November yield and the December yield are related?

L3. On a separate work sheet make up a table showing for each tree of Prob. L2 the November yield and the yield for the rest of the season; that is, the season's total minus the November yield. Make a scatter diagram of the data. Comment on the distribution of the points on the scatter diagram. (Make November yield the independent variable.)

L4. Draw a freehand regression line through the scatter obtained in Prob. L3. Find the equation of the line by the method of selected points.

L5. Fit a straight line (the regression line) to the data by the method of least squares. Add the least squares line to the scattergram.

L6. Explain the meaning of each figure in the regression equation found in Prob. L5. What part of the equation do we call the "regression coefficient"?

L7. When we say (as in Prob. L3) that the November yield is to be thought of as the independent variable, what do we mean? Do we imply anything about cause and effect?

L8. If you could have data for but one month from which to estimate the season's yield of latex, which month would you choose?

L9. The table[2] on page 197 shows average temperature from Feb. 7 to Mar. 21 of each year from 1906 to 1925, and for the same years shows the total apple production in Virginia (in thousands of bushels). The temper-

[1] From *U. S. Department of Agriculture Technical Bulletin* 65, pp. 10–11.
[2] From *U. S. Department of Agriculture Technical Bulletin* 54, p. 12.

Tree Number	1924		1925							Total
	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	Aug.	
1	166	192	188	71	7	9	52	104	250	1,039
2	274	395	630	226	196	190	118	204	276	2,509
3	359	1,301	1,550	376	346	272	218	262	416	5,100
4	140	560	712	242	148	69	91	90	224	2,276
5	154	289	482	80	63	34	95	44	299	1,540
6	300	547	573	303	268	232	214	248	478	3,163
7	436	756	940	337	267	135	74	228	412	3,585
8	118	283	962	289	120	38	25	101	184	2,120
9	120	400	632	100	112	88	79	156	276	1,963
10	218	412	534	105	143	66	150	117	352	2,097
11	66	151	190	73	99	74	69	78	100	900
12	308	526	753	326	255	248	285	352	462	3,515
13	332	769	1,093	343	238	264	366	364	630	4,399
14	576	830	1,375	538	339	342	392	373	664	5,429
15	392	604	1,099	569	363	350	412	495	724	5,008
16	188	499	708	137	89	68	122	258	314	2,383
17	292	754	1,402	480	273	170	140	203	398	4,112
18	204	674	1,262	341	211	140	119	180	288	3,419
19	145	221	265	37	40	76	88	186	272	1,330
20	550	1,079	1,796	628	465	396	526	619	1,100	7,159
21	310	798	1,347	304	136	120	350	539	1,122	5,026
22	308	430	755	339	230	234	242	350	764	3,652
23	544	826	1,208	468	455	370	350	340	512	5,073
24	256	584	1,204	430	310	225	247	348	554	4,158
25	150	207	331	63	90	62	100	141	244	1,388
26	250	287	352	48	85	37	189	232	442	1,922
27	196	274	421	112	183	240	273	260	338	2,297
28	619	569	794	211	197	232	374	508	636	4,140
29	274	448	526	145	117	186	392	482	608	3,178
30	162	283	498	189	154	196	178	216	244	2,120
31	278	352	450	173	80	32	80	199	324	1,968
32	240	452	474	170	100	178	220	304	332	2,470
33	278	345	380	148	126	134	142	150	280	1,983
34	391	596	1,052	242	336	338	418	420	646	4,439
35	322	599	896	266	334	322	315	318	454	3,826
36	338	453	597	172	190	228	274	332	496	3,080
37	74	165	221	21	99	35	65	68	106	854
38	12	4	9	...	...	...	...	...	629*	654
39	535	839	1,462	525	376	290	382	496	706	5,611
40	490	624	960	316	312	198	350	382	494	4,126
41	453	711	946	368	190	200	340	360	464	4,032
42	134	220	590	132	54	52	71	112	180	1,545
43	154	242	326	31	91	108	217	236	270	1,675
44	356	523	758	314	268	196	254	314	436	3,419
45	296	458	708	252	254	254	296	312	508	3,338

(Continued)

* In August a new tapping cut was used on tree No. 38.

Tree Number	1924 Nov.	1924 Dec.	1925 Jan.	Feb.	Mar.	Apr.	May	June	Aug.	Total
46	385	560	949	356	334	378	386	336	441	4,125
47	306	486	286	7	0	0	13	0	79	1,177
48	360	599	708	204	216	104	144	258	460	3,053
49	335	611	755	286	224	278	268	276	438	3,471
50	324	564	942	290	334	308	328	327	446	3,863
51	288	623	993	276	153	106	104	124	188	2,855
52	214	370	645	238	320	186	224	220	266	2,683
53	405	648	836	380	312	354	402	368	536	4,241
54	625	823	1,286	427	530	388	564	588	864	6,095
55	280	566	1,068	147	87	124	318	228	283	3,101
56	164	326	418	86	128	132	196	160	280	1,890
57	226	596	722	352	98	166	300	310	410	3,180
58	821	1,285	1,908	614	452	410	528	654	946	7,618
59	467	668	886	105	138	152	206	264	268	3,154
60	294	458	532	161	140	84	160	228	236	2,293
61	326	477	870	330	396	324	362	412	415	3,912
62	533	614	972	307	290	250	356	324	390	4,036
63	224	561	714	57	95	84	160	242	210	2,347
64	248	393	484	21	142	188	258	258	228	2,220
65	396	565	796	158	90	146	330	466	512	3,459
66	262	386	602	172	184	220	236	268	514	2,844
67	482	686	832	238	250	252	322	302	358	3,722
68	598	979	1,582	396	484	316	288	366	404	5,413
69	204	157	172	3	0	0	24	99	189	848
70	288	577	825	72	78	52	86	182	234	2,394
71	326	775	774	74	42	36	110	228	224	2,589
72	447	617	608	49	88	144	236	264	322	2,775
73	443	516	690	196	228	122	186	288	244	2,913
74	640	807	1,272	507	514	412	464	548	918	6,082
75	324	476	640	178	132	96	144	174	210	2,374
76	554	1,109	1,528	433	432	564	614	656	556	6,446
77	266	370	558	183	184	212	224	272	378	2,647
78	272	448	600	208	100	114	112	200	244	2,298
79	310	294	658	98	70	108	162	300	358	2,358
80	350	626	875	322	298	280	268	360	548	3,927
81	266	536	699	211	201	210	240	274	270	2,907
82	244	401	470	139	53	49	84	172	178	1,790
83	260	552	823	262	80	128	202	332	186	2,825
84	292	392	583	207	124	114	140	190	252	2,294
85	463	765	925	250	280	374	384	444	604	4,489
86	344	507	778	197	142	152	218	272	364	2,974
87	856	1,504	1,994	812	676	588	736	1,040	1,916	10,122
88	252	364	502	153	151	162	182	200	208	2,173
89	296	523	744	383	271	288	322	364	482	3,674
90	278	289	362	170	53	128	160	238	264	1,942
91	136	268	270	101	58	74	72	110	88	1,173
92	597	701	1,102	245	278	434	398	532	826	5,113
93	280	608	898	270	184	140	194	332	206	3,112
94	222	428	602	174	113	164	178	222	320	2,423
95	176	390	463	107	160	55	170	262	232	2,015
Total........	30,707	51,375	74,612	22,652	19,196	17,878	22,517	27,615	39,897	306,449

ature figures are averages from several weather stations nearest to the Virginia apple sections.

Year	Temperature	Production
1906	36	5,500
1907	38	5,200
1908	37	8,900
1909	41	6,107
1910	39	12,100
1911	39	7,200
1912	34	15,000
1913	41	5,200
1914	32	15,300
1915	39	13,176
1916	37	13,299
1917	37	11,778
1918	43	10,068
1919	41	8,943
1920	35	13,744
1921	46	570
1922	41	8,960
1923	39	10,000
1924	36	14,500
1925	45	7,844

Arrange the temperatures in array. Put the 5 years of lowest temperature in one group, the next 5 years in a second group, and so on until the 20 years have been divided into 4 groups of 5 years each. Compute the average apple production for the 5 years of each group. Make a table showing these group averages.

L10. Make a scatter diagram of the data which are tabulated in Prob. L9. Draw a freehand straight line through the scatter, and compute the equation of the line by the method of selected points.

L11. Fit a straight line by the method of least squares to the data of Prob. L9. Show this regression line on a scatter diagram of the original data.

L12. Explain each term of the regression equation found in Prob. L11.

L13. Compute the coefficient of simple linear correlation (r) from the data of Prob. L9. Explain your result.

L14. Compute the standard error of estimate for the regression line found in Prob. L11. Explain your result.

L15. Compute and explain the coefficient of determination from Prob. L13.

L16. Carry through the z-transformation on the data of Problem L13. Explain your answers. Apply in addition the usual method of finding the probable error of a correlation coefficient; that is,

$$PE_r = .6745\sigma_r = .6745\frac{1 - r^2}{\sqrt{n}}$$

Find the probable error and the standard error of the coefficient of correlation by this method, and determine the limits within which you would expect to find the coefficients of other samples. Compare your results with those obtained by means of the z-transformation. Explain.

L17. Imagine that you have been assigned the problem of determining the relationship between early spring temperature and the size of the apple crop in Virginia. You are to make a report to a cooperative association of farmers who are shipping apples. They are not technically trained in statistical method or in terminology. Take the results which you have obtained in Probs. L9 to L16 inclusive, and prepare a short report for this group, giving your conclusions.

L18. In the following table we are given two series of variables. The independent variable is called X and the dependent variable is called Y. Values are given each year from 1910–1932.

Year	X	Y	Year	X	Y
1910	587	176	1922	814	434
1911	603	151	1923	853	420
1912	627	176	1924	870	455
1913	635	197	1925	891	440
1914	666	220	1926	910	437
1915	703	237	1927	926	527
1916	727	253	1928	940	599
1917	721	241	1929	948	466
1918	700	202	1930	974	677
1919	712	282	1931	982	752
1920	736	355	1932	1001	558
1921	779	367			

a. Compute the coefficient of correlation from these data. Compute the standard error of the coefficient by the usual method, namely,

$$\sigma_r = \frac{1 - r^2}{\sqrt{n}}$$

Test the reliability of the correlation coefficient also by means of the more accurate z-transformation. Test the coefficient to see if it differs significantly from zero.

b. Is it reasonable to assume that one would find a relationship such as the one you have discovered in a chance sample from a universe in which there was actually no correlation?

c. The values of X in the problem just studied are figures on the average attendance in New York City schools for the scholastic years ending in June of the years stated. The values of Y are figures showing the total numbers of people arrested and presented before the Magistrates Courts in New York City in the same years.[1] Does this knowledge alter your explanation of the correlation? How do you explain the fact that such a relationship existed in the sample if there is not a similar relationship in the universe?

d. Can you derive from this problem any conclusions as to sources of error in the correlation of historical data?

L19. The following correlation table shows the numbers of female students at Pratt Institute who had various combinations of chest measure and waist measure.[2] All measurements are in inches, and chest measures are given with the chest expanded.

Chest Measure (inches)	Waist Measure (inches)									
	20	22	24	26	28	30	32	34	36	38
28	3	13	3	1						
30	13	130	65	5	2					
32	2	205	418	85	2					
34	..	44	405	305	35	1				
36	..	...	40	152	88	16	1	2	1	
38	..	...	2	16	31	22	11	4	..	2
40	..	...	...	...	1	2	6	3		
42	..	. .	...	...	..	..	1	1		

From these data compute the coefficient of correlation between waist measure (independent) and chest measure (dependent). Compute the regression equation and the standard error of estimate. Explain your results.

L20. The table at the top of page 200 shows the number of oat plants with various combinations of straw diameter and breaking strength of straw.[3] Three hundred plants were tested. Diameters are given in decimillimeters and breaking strengths in grams.

a. Compute the coefficient of correlation and the regression equation by which one would estimate the breaking strength of the straw from the diameter.

[1] Data from the 1934 *World Almanac*, pp. 505, 475.

[2] Data from P. H. Nystrom, "Economics of Fashion," p. 455, Ronald Press, New York, 1928.

[3] Data from C. E. Leighty, *Cornell University Agricultural Experiment Station Memoir* 4, p. 196.

Diameter of Straw (class mid-points)	Breaking Strength (grams) (class mid-points)										
	10	30	50	70	90	110	130	150	170	190	210
3	1										
5	3										
7	7	9									
9	4	30	4	1							
11	..	10	20	4							
13	..	3	30	19	2						
15	..	..	7	14	16	4					
17	..	1	..	5	22	14	6				
19	..	..	..	..	7	19	9	4	1		
21	..	..	..	..	..	1	9	5		1	
23	..	..	..	..	..	..	..	3	2	1	
25	..	..	..	..	..	..	..	..	..	1	1

b. If we wish to estimate the breaking strength, is a knowledge of the diameter helpful to us? By how much can we reduce the error of our estimate by using the linear regression equation?

L21. The following table shows the numbers of male students at the Connecticut Agricultural College who had various combinations of height (in inches) and weight (in pounds).[1]

Weight (pounds) (class mid-points)	Height (inches) (class mid-points)								
	58.5	60.5	62.5	64.5	66.5	68.5	70.5	72.5	74.5
95	..	..	1						
105	..	..	1	..	..	1			
115	..	..	1	7	2				
125	..	1	2	12	9	6	1		
135	..	1	2	6	24	16	4	1	
145	1	..	..	6	14	17	8	2	
155	..	..	..	..	7	11	4	5	
165	..	..	..	..	2	5	3	4	
175	..	..	..	..	1	4	3	2	1
185	..	..	..	..	..	..	..	..	1
195	..	..	..	..	..	..	1		

[1] Data from E. W. Sinnott and L. C. Dunn, "Principles of Genetics," McGraw-Hill Book Company, Inc., New York, 1st ed., p. 261.

a. Compute the coefficient of correlation and its probable error.

b. Castle finds that the relation between height and weight in Harvard students[1] can be expressed as follows: $r = +.54 \pm .015$. Is the relationship significantly higher in one case than in the other? (Measure the reliability of the difference between the coefficients of correlation.)

L22. In the following table[2] there are shown 170 American cities distributed according to the sex ratio (number of males per 100 females of 25 years of age and over) and the percentage of the women who are married.

Number of Males per 100 Females (class marks)	Percentage of Married Women 25 Years Old and Over (class marks)						
	47.5	55.5	63.5	71.5	79.5	87.5	Totals
68.5	1	2	. .	. .	. .	. .	3
86.5	1	8	38	2	. .	. .	49
104.5	. .	1	38	39	1	. .	79
122.5	. .	. .	2	26	1	. .	29
140.5	. .	. .	. .	2	4	. .	6
158.5	. .	. .	. .	. .	1	1	2
176.5	. .	. .	. .	. .	. .	1	1
194.5	. .	. .	. .	. .	. .	1	1
Totals	2	11	78	69	7	3	170

a. Inspect the diagram and describe the general nature of the relationship.

b. Compute the value of r and its standard error. Compute the regression equation by which you would estimate for a city with a known sex ratio the percentage of women who are married. Compute the standard error of estimate.

c. Is it likely that this relationship is a chance affair?

L23. The data[3] in the table on page 202 show mean temperature on 41 successive days. There is also a record of the average per cent of butterfat in the milk from a herd of 10 cows on these same days.

a. Is the butterfat content of the milk related to the temperature? If so, how closely?

b. Subtract 4 from each of the figures on butterfat percentage. Correlate the temperature with the new figures; that is, butter-

[1] W. E. Castle, "Genetics and Eugenics," p. 61, Harvard University Press.

[2] Data adapted from E. R. Groves and W. F. Ogburn, "American Marriage and Family Relationships," p. 479, Henry Holt & Company, New York.

[3] From Ragsdale and Brody, *Journal of Dairy Science*, Vol. 5, p. 214.

Date	Temperature	Per cent Butterfat	Date	Temperature	Per cent Butterfat
Mar. 13	57	4.51	Apr. 3	64	4.65
Mar. 14	60	4.57	Apr. 4	65	4.58
Mar. 15	60	4.37	Apr. 5	65	4.67
Mar. 16	47	4.64	Apr. 6	64	4.60
Mar. 17	58	4.22	Apr. 7	61	4.83
Mar. 18	64	4.17	Apr. 8	55	4.55
Mar. 19	70	4.40	Apr. 9	39	5.14
Mar. 20	69	4.45	Apr. 10	41	4.71
Mar. 21	41	4.94	Apr. 11	46	4.69
Mar. 22	43	4.58	Apr. 12	59	4.65
Mar. 23	48	4.78	Apr. 13	56	4.36
Mar. 24	55	4.72	Apr. 14	56	4.82
Mar. 25	58	4.41	Apr. 15	62	4.65
Mar. 26	64	4.48	Apr. 16	37	4.66
Mar. 27	45	4.65	Apr. 17	37	4.95
Mar. 28	27	4.84	Apr. 18	45	4.60
Mar. 29	39	4.81	Apr. 19	57	4.68
Mar. 30	44	4.74	Apr. 20	58	4.65
Mar. 31	41	4.65	Apr. 21	60	4.60
Apr. 1	46	4.51	Apr. 22	55	4.46
Apr. 2	49	4.58			

fat minus 4. Compare the result with that obtained by correlating the original figures. Compute the regression equation and the standard error of estimate from the new figures, and compare with those for the original data.

L24. We have seen that misleading correlations may arise in time series merely from the fact that both series may be growing or shrinking through time (see Prob. L18). Is the relationship of Prob. L23 of this kind? How would you test the data to find out?

L25. The weights and the sitting heights of 608 newborn male babies were measured and correlated. It was found that $r = +.704$. When the same variables were correlated in the cases of 609 newborn female babies, it was found that $r = +.685$.[1]

a. Explain each figure.
b. Compute the probable error of each figure. Explain your answers.
c. Is there any reason for believing that this relationship is really more marked among male babies than among females? That is, is it reasonable to assume that this difference arose from chance?

L26. The following correlation table gives an hypothetical distribution.

[1] Data from *Human Biology*, Vol. 6, December, 1934, p. 621.

	5	5	5	5			
		10	10	10			
		5	15	15	5		
Y			10	20	10		
			5	15	15	5	
				10	10	10	
				5	5	5	5

X

a. Find the mean of each column; that is, the average value of Y for each of the values of X. Since the columns are symmetrical this will be easy. Locate these means on the columns themselves. Draw a line through these means of columns.
b. Find the means of the rows; that is, the average value of X for each of the values of Y. Locate these means of the rows on the diagram and draw a straight line through the points.
c. One of these lines is the regression of Y on X; the other is the regression of X on Y. Which is which? Explain the difference in the meanings of the two lines.
d. Explain why values of X cannot be estimated from the first line.
e. Explain what happens to the two regression lines as we approach perfect correlation.

L27. The following data[1] show the weights (in kilograms) and the volumes (in cubic decimeters) of the bodies of 18 children.

Weight	Volume	Weight	Volume
17.1	16.7	15.8	15.2
10.5	10.4	15.1	14.8
13.8	13.5	12.1	11.9
15.7	15.7	18.4	18.3
11.9	11.6	17.1	16.7
10.4	10.2	16.7	16.6
15.0	14.5	16.5	15.9
16.0	15.8	15.1	15.1
17.8	17.6	15.1	14.5

a. Plot these data on a scattergram.
b. Compute the regression of Y on X.
c. Compute the regression of X on Y.
d. Explain the difference between the two regression equations.
e. Interpret each regression coefficient.
f. Why is it that if you have found the regression equation,

$$Y = 10 + 2X,$$

you can not turn it around and say, $X = -5 + 0.5Y$.

[1] Data from *Human Biology*, Vol. 5, pp. 651–652. The figures given here include all cases from 5.0 to 7.9 years of age.

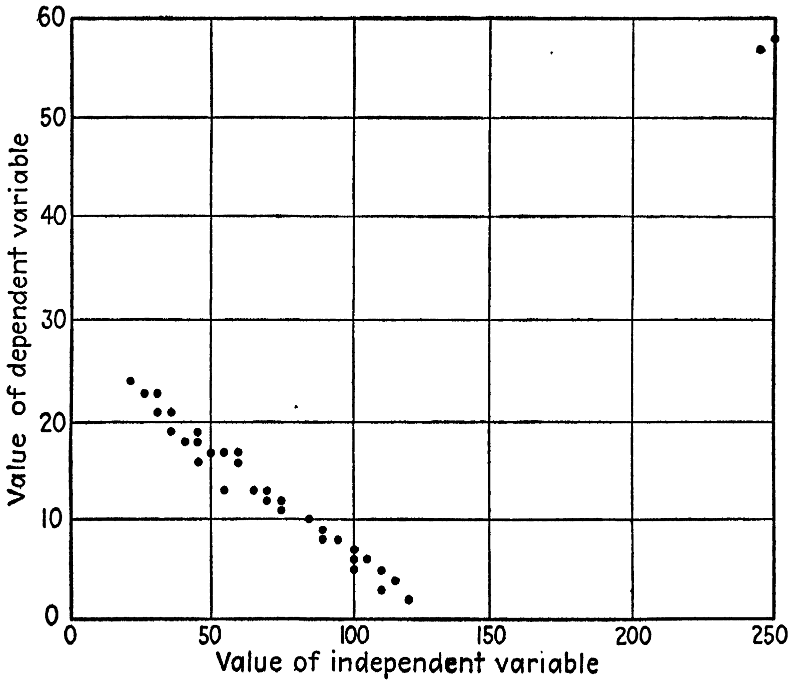
g. Compute r. Find the geometric mean of the two regression coefficients. Compare the value of r with this geometric mean. Explain.

L28. There is perfect positive linear correlation between the readings on an accurate Fahrenheit thermometer and those on an accurate centigrade thermometer. Suppose we wish to find a formula (the regression equation) for estimating the Fahrenheit temperature from known centigrade temperatures. The formula will have the general form:

$$F = a + bC$$

We know that the freezing point of water is represented by the temperatures 32°F. and 0°C. The boiling point is represented by 212°F. and 100°C. Using these two points as a basis, compute the equation by the method of selected points. Look up the equation for converting from one temperature scale to another in your physics text or in a book of tables. Compare your result with the methods there given.

L29. Study the scatter diagram on the adjacent chart and comment on the nature of the scatter. How would you handle this problem if you were investigating the relationship between the two variables pictured? Why?



L30. The following table gives the length, and average number of nails per pound, of nails of various sizes. Sizes are given in pennies.

Size (in pennies)	Length (in inches)	Average Number (per pound)
2	1.	876
3	1.25	568
4	1.5	316
5	1.75	271
6	2.	174
7	2.25	161
8	2.5	106
9	2.75	96
10	3.	69
12	3.25	63

Make up a formula for finding the length of a nail from its size in pennies. The formula should be of the general form:

$$L = a + bS$$

where L is length in inches and S is size in pennies.

L31. The article on Waves of the Sea in the fourteenth edition of the *Encyclopaedia Britannica* states, "It is clear that the height of the waves finally produced in the open ocean is in direct, simple proportion to the velocity of the wind. The larger waves occurring at short intervals which chiefly attract attention have about eight feet of height, reckoning from trough to crest, for each 10 miles-per-hour velocity of wind, but their apparent height is less when the ship is borne upon two waves."

Write a formula for determining the height of waves, in feet, for various wind speeds in miles per hour.

L32. High-speed photography has made it possible to determine the wing speed of various insects. In the case of drosophila, the fruit fly, "increasing the temperature skyrockets the rate from 6000 beats per minute at 50°F. to double that at 84°F."[1] Assuming that the relationship is linear, compute an equation for estimating wing speeds (in beats per minute) from temperature (in degrees Fahrenheit).

L33. For 17,127 college women at the ages of 16 to 21 years the correlation between height and weight was $r = +0.4215$. For 23,122 college men of the same ages the correlation[2] was $r = +0.5056$.

a. Interpret each of the coefficients.

b. Using the z-transformation, determine whether or not the correlation is significantly greater for men.

[1] Data from studies of D. L. E. Chadwick reported in *Science News Letter*, Aug. 5, 1939, p. 83.

[2] Data from H. S. Diehl, The Heights and Weights of American College Men and Women, *Human Biology*, Vol. 5, pp. 455 and 609.

L34. Under some circumstances sleeping bats breath irregularly, taking a number of quick breaths and then passing a longer or shorter breathless interval. Observation of a hoary bat for a period of a little over an hour[1] recorded the number of breaths in each series and the length of the ensuing breathless interval in quarter minutes. The observations follow:

Forty-two breaths, 22 quarter minutes breathless; 39 breaths, 25 quarter minutes breathless; 36 breaths, 21 quarter minutes breathless; 37 breaths, 24 quarter minutes breathless; 32 breaths, 20 quarter minutes breathless; 12 breaths, 11 quarter minutes breathless; 27 breaths, 23 quarter minutes breathless; 40 breaths, 42 quarter minutes breathless; 33 breaths, 19 quarter minutes breathless; 27 breaths, 19 quarter minutes breathless; 28 breaths, 22 quarter minutes breathless; 29 breaths, 18 quarter minutes breathless; 33 breaths.

One might suppose that a large number of breaths would make it possible for the bat to hold its breath unusually long, and hence a positive correlation might be expected between the number of breaths in any series and the length of the ensuing breathless period. Or one might imagine that a long breathless period would lead the bat to take an unusually large number of breaths at the close of the period. In this case he would expect a positive correlation between the length of a breathless period and the number of ensuing breaths. These two correlations would reverse the dependence of the variables. They would also put different items in the correlated pairs, in the one case pairing a number of breaths with the length of an ensuing breathless period and in the other case paiiing the length of a breathless period with the number of ensuing breaths. Work out the correlation both ways and determine in which case the relationship is the greater.

L35. It has long been known that katydids and crickets alter their rate of chirping with changes in the temperature. If we let N represent the number of calls per minute, the entomology textbooks give the following formulas for ascertaining the temperature:

$$\text{For katydids: } T = 60 + (N - 19)/3$$
$$\text{For crickets: } T = 40 + N/4$$

In both formulas, T is the temperature in degrees Fahrenheit.

An actual study of katydid calls with the accompanying temperatures yielded the following pairs of observations:

T	N	T	N	T	N
67	35	60	21	58	17
74	62	65	30	62	24
66	36	58	33	59	22
66	34	60	25	68	39
76	49	69	39	62	25
76	63	62	30	67	32
62	28	61	38	55	15
62	31			54	12

[1] Data from H. M. H. Kimball and J. T. Nichols, A Long Island Hoary Bat, *Journal of Mammology*, Vol. 21, May, 1940, p. 215.

a. Try computing the temperature in each case from the number of chirps, using the formula above. Find the errors and the standard error of estimate.

b. Compute a regression formula by the method of least squares. Find the standard error of estimate. How much more accurate is the least squares formula than the one given in the textbooks?

c. Plot the data to see whether they are linear.

SECTION M

MULTIPLE AND CURVILINEAR CORRELATION

M1. The following table shows the annual average wholesale price per pound of raw cane sugar (96 degree centrifugal) in New York City, and the annual average retail price per pound of granulated sugar in the United States for the years 1925 to 1933 inclusive.[1] Plot the price data on a scatter diagram. Fit a second-degree parabola by methods of least squares. Compute the index of correlation. Add the curve to the scatter diagram.

Year	Cane Sugar Price (cents)	Granulated Sugar Price (cents)
1925	4.3	7.2
1926	4.3	6.9
1927	4.7	7.3
1928	4.2	7.1
1929	3.8	6.6
1930	3.4	6.2
1931	3.3	5.7
1932	2.9	5.1
1933	3.2	5.4

M2. Compute the standard error of estimate for the regression equation found in Prob. M1.

M3. Correct the index of correlation found in Prob. M1 and the standard error of estimate found in Prob. M2 for the number of cases and the number of parameters. Explain.

M4. Try fitting to the scatter of Prob. M1 any other types of curves which seem to have the proper characteristics.

M5. How does the index of correlation differ from the coefficient of correlation?

M6. Suppose that we compute the index of correlation and the coefficient of correlation from the same data. What will be the relative size of the two coefficients (if we do not correct for the number of parameters in the regression equations)?

[1] Data from the *U. S. Department of Agriculture Yearbook*, 1934, p. 479, Tables 145, 146.

M7. The following table[1] shows the season's average farm price per pound for each of two grades of tobacco grown in Connecticut. The prices are in cents per pound. The grades of tobacco described are Connecticut Valley Broad-leaf Tobacco, Type 51; and Connecticut Valley Havana Seed Tobacco, Type 52. Data are for the years 1919–1932.

Year	Average Farm Price (cents)	
	Broad-leaf	Havana Seed
1919	44.8	31.8
1920	39.2	36.4
1921	19.9	23.0
1922	30.0	29.3
1923	35.0	35.4
1924	20.0	19.2
1925	18.9	16.2
1926	26.0	27.2
1927	21.0	23.8
1928	21.0	24.2
1929	27.4	31.1
1930	25.1	21.9
1931	14.1	13.0
1932	11.5	8.8

a. Plot the data on a scatter diagram. Determine whether the relationship is linear or curvilinear. If curvilinear, determine what type of curve will best fit the data.

b. Draw a freehand second-degree curve and compute its formula by the method of selected points.

c. Fit a second-degree parabola by the method of least squares. Add this parabola to the scatter diagram.

d. Compute the index of correlation and the coefficient of correlation. Compare them. Comment.

e. Correct the results of Prob. M7*d* for number of cases and number of parameters.

M8. Bradford B. Smith investigated factors affecting the price of cotton[2] and at one stage in his analysis he made use of the regression equation:

$$\log P = 1.548(\log I) - 1.705(\log S) - 0.051$$

[1] From *U. S. Department of Agriculture Yearbook,* 1934, p. 490, Table 157.

[2] *U. S. Department of Agriculture Technical Bulletin* 50. The equation referred to appears on p. 25.

In this equation P represents the price of cotton, S represents the supply[1] of cotton, and I represents the price index. Study the regression equation, and describe the nature of the regression surface. What is the nature of the relationship between "supply" and price when the price index remains constant? If the price index is 140 and the "supply" is 15 million bales what price would you estimate from this equation?

M9. In the study just quoted Smith found that the relationship between the three factors mentioned was such that the coefficient of multiple correlation was 0.955. Interpret this answer. Is the coefficient positive or negative?
Explain.

M10. Draw diagrams showing the regression surface for two independent variables with each of the following types of relationship:

a. Multiple linear.
b. Multiple curvilinear.
c. Joint.

Describe the nature of the difference in these relationships.

M11. In a study of labor efficiency in planting and harvesting operations on Connecticut dairy farms,[2] Hammerberg found that the relationship between the number of man-hours taken to mow a ton of hay (X), the size of the field on which the hay was grown (A), and the yield of hay per acre (B) could be described by the equation:

$$X = 3.98388 - .770900 \, (\log 10A) - 1.104315B$$

In this same problem he found:

$$R_{x.ab} = 0.610$$
$$S_{x.ab} = 1.0428$$

a. Interpret each of these results.
b. Interpret each figure in the regression equation.
c. Which of the three types of correlation mentioned in Prob. M10 is represented here?

M12. Assuming that the values of Prob. M11 have not been corrected for the number of cases and the number of parameters, and that the number of hayfields studied was 1200 (this figure seems to be approximately correct for the study cited according to a statement on page 48), make the corrections and compare the corrected figures with those given before.

M13. Suppose that we have a dependent variable X_1 and two independent variables X_2 and X_3. Their values in 10 cases are as follows:

[1] In technical economic theory one would prefer to use the term "quantity available" rather than "supply" at this point. I follow Smith's terminology here, however.

[2] *Storrs Agricultural Experiment Station Bulletin* 172, p. 47.

X_1	X_2	X_3
32	3	2
18	2	4
52	5	2
6	1	5
42	4	3
48	6	9
80	9	8
32	3	7
52	5	2
68	7	4

Find the multiple linear regression equation which describes this relationship. Test your result by estimating values of X_1 for the given combinations of x_2 and X_3, comparing your estimates with the actual figures.

M14. On account of the nature of the hypothetical case given in Prob. M13 it should be easy to find the regression equation by the method of selected points. Try it, and compare your answer with that obtained by the method of least squares.

M15. Charlier made a study[1] in which he tried to explain the variations in July temperatures by variations in the temperatures of the preceding 4 months. If we let X_1 be the deviation of a given July temperature from the average of past July temperatures, and if X_2, X_3, X_4, and X_5 represent, respectively, deviations of June, May, April, and March temperatures from the average temperatures for past Junes, Mays, Aprils, and Marches, we get the regression equation:

$$X_1 = 0.617X_2 + 0.125X_3 + 0.070X_4 - 0.041X_5$$

a. If March is 5° below normal, April is normal, May is 2° below normal, and June is 10° above normal, what temperature (in deviation from normal) do you expect for July?

b. Comment on the significance of the fact that one of the signs in the regression equation is negative. What does it signify?

M16. Suppose we are trying to estimate the price of 92-score butter at New York City. We are using as independent variables from which to make our estimates two other variables; namely, the net receipts of creamery butter at New York and the *U. S. Bureau of Labor Statistics Index Number* of wholesale prices. We symbolize the variables as follows:

X_1 = butter price
X_2 = butter receipts
X_3 = price index

[1] Data here given are quoted from Forsyth, "Mathematical Analysis of Statistics," pp. 227–228, John Wiley & Sons, Inc., New York.

When the monthly figures for these three variables were actually correlated[1] for the two periods February, 1918–May, 1921, and February, 1923–May, 1926, the following results were found:

	1918–1921	1923–1926
$R_{1.23}$ =	.8019	.5965
$b_{12.3}$ =	−.1083	−.0650
$b_{13.2}$ =	+.2574	+.1562

Interpret each of the figures given. Comment on the signs of the regression coefficients.

M17. The following data are supposed to represent concomitant values of three variables. X_1 is dependent, and X_2 and X_3 are independents. The data are hypothetical.

X_1	X_2	X_3	X_1	X_2	X_3
64.3	5	9	32.2	2	9
77.8	6	3	101.7	8	4
53.7	4	2	89.1	7	3
34.2	2	7	112.3	9	7
98.2	8	1	56.4	4	6
89.7	7	4	78.6	6	5
65.7	5	8	33.6	2	2
23.3	1	2	86.7	7	1
55.8	4	3	78.4	6	4
78.4	6	6	55.7	4	8

a. Handle this as a problem of multiple linear correlation, working out the usual correlation results.

b. Handle the data as a problem of multiple curvilinear correlation, discovering first the type of curves called for.

c. Describe the shape of the correlation surface which is represented by the regression equation found in part *b* above.

M18. The data in the table on page 213 represent 25 observations of three variables.

a. Using X_1 as the dependent variable, compute the multiple linear regression equation, the standard error of estimate, the betas, and the coefficient of multiple linear correlation. Interpret the results.

[1] The results of this analysis are given in the mimeographed bulletin entitled "Research Method and Procedure in Agricultural Economics," which was published by the Social Science Research Council in August, 1928. The data are from pp. 282*ff*.

b. Compute the multiple curvilinear regression equation, using second-degree parabolas. Compute the standard error of estimate and the index of multiple curvilinear correlation. Interpret the results.

c. Correct both the linear and the curvilinear results computed above for number of cases and number of parameters. Interpret the results.

Observation Number	X_1	X_2	X_3	Observation Number	X_1	X_2	X_3
1	62	10	10	14	83	80	30
2	20	20	50	15	93	90	50
3	79	50	10	16	4	10	90
4	55	70	50	17	29	30	40
5	57	80	80	18	21	50	90
6	24	10	40	19	76	80	40
7	11	20	70	20	99	90	40
8	48	50	30	21	45	20	20
9	99	70	10	22	10	30	80
10	84	90	70	23	46	70	70
11	5	10	80	24	61	80	70
12	48	30	20	25	118	90	20
13	35	50	50				

M19. Draw a picture showing in perspective how the regression surface would look which was described by the regression equation of Prob. M18*b*.

M20. In Prob. M18*b* we computed a multiple curvilinear regression equation. This equation described the relationship between three variables, and from it we could compute values of X_1 for various values of X_2 and X_3. Suppose that we wish to find an equation from which we can compute the values of X_1 for various values of X_2 if X_3 always has a value of 50; that is, X_3 is no longer to be thought of as a variable, but as a constant, and we wish to get an equation describing the relationship between X_1 and X_2 when X_3 is held constant at a value of 50. This new equation can be derived directly from the more complicated equation computed in Prob. M18*b*. Derive it.

M21. Some stars have the peculiarity of varying regularly in brightness, passing through a regular cycle in their apparent changes in brilliance. One type of such variable stars is called "Cepheid variables." For such stars it has been discovered that there is a definite relationship between the period of pulsation and the actual luminosity of the star. (A star may be very luminous intrinsically yet appear dull because of its great distance. The relationship here is between the period of pulsation and the actual

intrinsic luminosity.) The table below shows A, the period of pulsation in days, and B, the luminosity with the sun = 1, of 9 Cepheid variables.[1]

Star	A	B
RR Lyrae	0.6	125
SU Cassiopeiae	2.0	260
Polaris	4.0	460
δ Cephei	5.4	700
η Aquilae	7.2	1,000
ζ Geminorum	10	1,700
X Cygni	16	3,200
Y Ophiuchi	17	3,500
I Carinae	36	9,600

Find an equation for estimating the luminosity of a star from its period of pulsation. What luminosity would you expect for a star whose period of pulsation was 12.5 days?

M22. There seems to be evidence that all spiral nebulae are receding from the earth and that those farthest from the earth are receding fastest. The following table gives the distances in light-years of spiral nebulae in seven constellations, and likewise gives the speed of their recession from the earth in miles per second.[2]

Constellation	Distance (light-years)	Recession (miles per second)
Virgo	6	560
Pegasus	24	2,400
Cancer	29	3,000
Perseus	36	3,300
Coma	45	4,700
Ursa Major	72	7,400
Leo	104	12,200

a. What is the relationship between distance and velocity? Find an equation from which you can estimate a nebula's velocity of recession from its distance in light-years.

b. The greatest velocity yet measured for a spiral nebula is 24,300 miles per second for a nebula in the constellation of Boötes.[3] What would you estimate to be the distance of this nebula? Comment on the problem of extrapolation involved.

[1] Data from "Worlds without End," p. 173, by H. Spencer Jones, The English Universities Press, London, 1935.

[2] *Ibid.*, p. 209.

[3] *Ibid.*, p. 210.

M23. In 758 autopsies the skull length and the brain weight were measured.[1] The following table divides the skulls into groups according to their length and gives the average brain weight for each class of skull lengths.

Skull Length (millimeters)	Mean Brain Weight (grams)	Number of Cases	Standard Deviation of Brain Weights (grams)
146–169	1220.00	10	96.26
170–179	1236.45	155	108.13
180–189	1284.17	360	125.50
190–199	1334.54	207	120.75
200–216	1438.46	26	162.30

We note that, as skull length increases, the brain weight also increases. If we plot the data, making allowance for the inequality of class intervals, we note that the relationship is definitely curvilinear.

a. Draw a freehand curve from which you could estimate brain weight from skull length.
b. Compute a formula for estimating brain weight from skull length.

M24. The atmospheric pressure P in inches of mercury at various heights h above sea level in miles is given in the following table:

h	0	1	2	3	4	5	6	7
P	29.9	24.5	20.1	16.4	13.4	11.0	9.0	7.4

Find a formula for expressing this relationship.

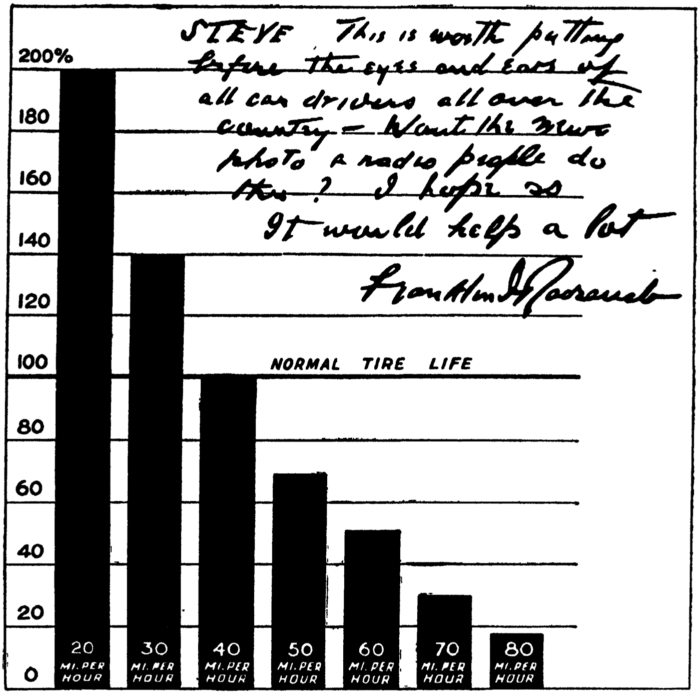
M25. The weight of water vapor W in a cubic foot of saturated air varies with the temperature T. Weights (in grains per cubic foot) and temperatures (in degrees Fahrenheit) are given in the following table:

T	−20	−10	0	10	20	30	40	50	60	70	80	90	100
W	0.17	0.28	0.48	0.78	1.24	1.94	2.85	4.08	5.74	7.98	10.93	14.79	19.77

Find a curvilinear regression equation by means of which one may estimate the weight of water vapor from the temperature.

M26. The newspapers of Sept. 13, 1942, carried a chart released by President Roosevelt in an effort to encourage wartime economy of automobile tires. This chart, based on data of the Baruch Rubber Committee, appears herewith. The release that accompanied the chart indicates that, if we

[1] F. W. and E. M. Appel, Intracranial Variation in the Weight of the Human Brain, *Human Biology*, Vol. 14, February, 1942, p. 53.



Tire wear vs. car speed. (*From the Automobile and Rubber Industries Tire Committee of the S.A.E. War Engineering Board, Report on Interim Tires and Treads, Vol.* 1, 1942.

take the tire life of a tire driven at 40 miles per hour as 100, the tire life of tires driven at other speeds will be

Speed	Life
80	18
70	30
60	50
50	70
40	100
30	140
20	200

Work out an equation for estimating the life of a tire driven at various speeds.